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SPEAKERS

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Welcome. In this lecture, we're going to go through two examples of using L'Hopital's Rule to compare the growth rates of two functions. So the first one we'll do will be the natural log of X compared with E minus πX , which note is actually a polynomial. Sometimes you might look at it and not be sure, but that's part of why I wrote it was to give you a little practice with realizing that that kind of thing is a polynomial, π and E are still constants. And then in the other example, we're going to compare E to the X with X squared. So let's just get started. And the first step right of using L'Hopital's Rule is to check that you have an indeterminate form, so you can. And we're going to be computing, right, so in order to compare the growth of these two functions, we want to compute the limit as X goes to infinity of, and then we just compare the ratio of them. So we have the natural log of X , and we're going to divide that by, and we have E minus πX . Right? And then if you take the limit of the top one, right, so if I was to take the limit of the top, I would get infinity. And if I was to take the limit at the bottom, I would get minus infinity. And so what that indicates to me is I do have an indeterminate form. So this is an indeterminate form. I can apply L'Hopital's Rule. So can apply, and then remember, we abbreviate L'Hopital's Rule with LH.

Okay, perfect. So onward, this is going to equal. So using L'Hopital's Rule, it's nice to indicate with LH that you use L'Hopital's Rule, and then we have to remember, we have to keep the limit every time. You should write the limit until you've actually taken the limit, just so that you don't make a mistake. And then, right, we're still taking it as X goes to infinity. And now the thing that we have to remember is to very carefully, remember that we're going to take the derivatives of the top and the bottom separately. So no quotient rule, I'm just going to take the derivative of the top and then I'm going to take the derivative of the bottom. So the derivative is at the top, so I'm taking the derivative of the natural log of X , if I'm taking the derivative of the bottom, I'm taking the derivative of E minus πX . Okay? And then we can go, we can just keep on going with this. So this is equal to, and then we have the limit as X goes to infinity, and we can just take those derivatives. So the derivative on the top, that's going to be 1 over X . And then on the bottom, if I take the derivative of the bottom, well, I just pick out the coefficient of X . Because this is a constant, so this goes to zero, and then I just pick out the coefficient of X . So this should just be minus π . Okay, and so this is going to equal.

Well, what does this equal? The X kind of falls down to the bottom right, so let's again, not forget to

write our limit here. So we have this is equal to the limit as X goes to infinity. Right? And because we have 1 over X on top, that X ends up in the bottom. And so this entire thing becomes, well actually maybe that will be to think, so that it's clear where that came from, I can actually just keep that color scheme there. So you can kind of see that that X came from the 1 over X on top, it dropped down to the bottom. And this limit, we know is actually equal to zero. Right? So now we're good, we're done. We don't have any indeterminate forms. We have zero. We're done. We know that. And then we look back up, right? I have to look all the way back up and I have to see what I was taking the limit off. So we have that the limit as X goes to infinity of $\frac{1}{X}$ is zero. I was a little worried that might be a little tricky to see. So we have the limit as X goes to infinity. Trying to put in a little bit of carefulness. So the limit as X goes to infinity of, and then I have the natural log of X divided by E minus πX . Right? Because I looked back and that's what I was initially computing. But with all of these equalities that actually ended up giving me that this was zero.

Right? And then so this is kind of, but what does that tell us? Because when we're really interested in is the comparing the growth. Okay, so that's just that. But then we know that that tells us that so this is true and, right, if it's zero that tells me that the bottom grows faster than the top. So I get that G of X equals E minus π X grows faster than, so this grows faster, right, then, this is still always good to check that everything's on the board. I have to do all these things, I want for you to be able to see everything I'm writing. So I always try to check a lot. So I have F of X equals the natural log of X . Okay, so that's the, the, actually, in reality, we were oops that was supposed to be green, I don't know how that ended up orange. Got carried away, where I was focusing on explaining to you that I'm always checking what can and can't be seen, to try to make sure that you can see everything you need to see. And then I lost track of the color. So this is the natural log of X . Right? So we know that, right, because this one is on top, this one's on bottom, and the the limit is zero, which tells me that the bottom dominates. Okay, so that's good. Now let's try another one. This one's a little bit different. You're going to see why. So let's go ahead and get started with it. So I'm going to take the limit. Again, I want to compare the growth rate of these two. So I want to take the limit and of the quotient. So I want to take the limit as X goes to infinity of, so I want E to the over X squared.

Okay? And if I took those limits of the top and the bottom, I would get, so the limit as X goes to infinity of E to the X is actually equal to infinity. And then the limit on the bottom if I took the limit as X goes to infinity of X squared, I again get infinity. So I'm a very happy camper because I have an indeterminate form, and can use L'Hopital's Rule. So indeterminate form, right, which tells me that I can use L'Hopital's Rule. So can use L'Hopital's Rule. Right? I checked that, always need to check that. So let's keep going. So this is equal to the limit as X goes to infinity of, and then I, you know, I'm very careful about this to always remember, hey, write it out. Careful not to skip any steps so that I don't do something silly. So I actually write out each time I'm taking the derivative of the top separately from the derivative of the bottom. No silly mistakes will take over my work. Okay? And then I can just compute these derivatives. So I'm taking the limit as X goes to infinity, and the derivative of the top is just E to the X , the derivative of the bottom is $2X$. Great. And then looking at this, if I took the limit of the top, I would get infinity. If I took the limit of the bottom I would get infinity, right, because I don't know what to do with this limit. This isn't obvious because the top and the bottom are both infinity. So I do indeed get infinity over infinity again, just taking the limit of the top and then taking the limit of the bottom. Okay, so again, I got the indeterminate form and can use L'Hopital's Rule. So indeterminate form again, which tells me that I can use L'Hopital's Rule again. That's good news. And I will go ahead and do that. So then this is going to equal, so I guess this was a L'Hopital's Rule and

then we have another L'Hopital's Rule here. So I have to again take the derivative of the top and bottom separately, but this time of this. So I have the limit as X goes to infinity of the derivative of the top separate from the derivative of the bottom.

Derivative of the top separate from the derivative of the bottom, right? And so this is going to equal the limit as X goes to infinity of, and so the top. So this is E to the X . On the bottom, this is 2. Okay, so what happens? So now if I take the derivative, sorry, if I take the limit at the top I get infinity. If I take the limit of the bottom I get 2, so infinity definitely dominates over 2. Okay, so this is equal to infinity. Right? So this tells me that it took, it took doing L'Hopital's Rule twice, but this told me that so now I go all the way back to the very start that this is the limit I was taking. That I was taking the limit as X goes to infinity of, and then I was taking it of E to the X divided by X squared, right? So I was taking that limit, and in the very end I got infinity. Right? Which is going to tell me that, right, what does that tell me? That tells me that, and thus, we have, what do we have? We have that F of X equals, too many pens in my hand. So I have F of X equals E to the X grows faster, right, than what was on the bottom. So then G of X equals X squared, okay? My limit on the ratio is infinite and so top grows faster than the bottom. And the takeaway of this particular example is that, so takeaway from this particular example, the example two, okay? So what is my takeaway? Is that as long as we keep an indeterminate form, we keep getting an indeterminate form. So as long as we keep getting an indeterminate form, getting an indeterminate form.

Oh it does fit, check that out. Okay, so as long as we keep getting an indeterminate form, we can keep applying L'Hopital's Rule. Until like, we actually know what the limit is, so you can keep applying L'Hopital's Rule. And then you could think about that, and you could probably guess what would happen if I had a degree five polynomial? Right? Because looking at this, right, looking at what we did here, you know, every time we did L'Hopital's Rule, so every time we took the derivative, right, the degree went down. The degree went down by one, every time we did L'Hopital's Rule and took that derivative. And so what that would actually give me is that, I'm trying to make this a little bit darker. So that would actually give me that, and E to the X just stayed the same. So I could have done that with actually any polynomial, I just would have had to apply L'Hopital's Rule a whole bunch of times. And so here's kind of like the key really cool important remark. So remark is that these examples aren't coincidences. I chose them carefully. So these examples aren't coincidences.

They actually tell me something, and what they're going to tell me is that logarithmic growth, growth is less than polynomial growth which then in turn, right, and actually let's look at this for a second. Because like, why was this example general? Because if I had 1 over X , and I had any polynomial here, the X , you know, now all of a sudden I have like in this step, I would have had 1 on the top, and I still would have had a polynomial on the bottom, actually one of even bigger degree. Okay? So this still would have ended up zero no matter what the polynomial was on the bottom, right? And we could have had a similar thing here. Like I was saying, that E to the X , no matter how many times you take the derivative, it doesn't change. So you keep on getting infinity on top. And with a polynomial, every time you take the derivative, your degree goes down by one. Eventually, you end up with it going down to a constant, which then means that you're infinite. Okay? So this one also works and we always have that polynomial growth is less than exponential growth. And actually, this is very significant, the extent to which they grow differently is very significant. And this is something that we'll kind of look at examples of, okay? So, I hope this made some sense, and I will see you in the next lecture.

