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SPEAKERS

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Welcome. In this lecture, we're going to talk about how you can compare the growth rate of different functions. So let's just get started. So to compare how functions, to compare how functions, so if I had the functions, F of X , F of X and G of X , how they grow. Then what I can do is I can find, this is what we'll do. We're going to find the limit as X goes to infinity of F of X divided by G of X . Okay? So here's some that we've already seen. So we have already seen.

Okay, so we've already seen that, so here's the first one. That if I take the limit as, I'm feeling unhappy with all the blue markers. So as I take the limit, as we, of, so we have, this one's going to work, okay. So we have the limit, and on the top, we're going to have $2X$ cubed minus 1. And we can divide this by, maybe I have 1000, and then I have X squared plus πX . Right? This is an example of a rational function, we have a polynomial on the top and a polynomial on the bottom. So we know that if we take the limit as X goes to infinity. The other thing we know is that the degrees, oops on the, yeah the degree on the top is bigger than the degree on the bottom, and the sign is positive. And so this is going to be positive infinity, okay? So this is just going to equal infinity, which is going to tell me that, so we have that F of X equals $2X$ cubed minus 1 grows faster. Okay, because right the, as you go towards infinity, it must be getting bigger faster than the bottom where we couldn't actually have gotten to infinity like that. Okay, so it dominates. It's the top one, and the limit's infinity. So it dominates, so it grows faster. Great. Here's a, the next kind of limit you've seen before, and we can interpret it here. So we're going to have the limit as, and then we have X approaches infinity of, and then on top, we're going to have $2X$ cubed minus 1.

Again, but on the bottom we're going to have, so now I'm going to put $14E$ squared minus πX cubed. Again, this is a rational function. I know I made it look really funny. This is a constant. This is a constant. So this is a rational function of degree three, because it's the biggest degree there. So this is going to give me, okay, so then because this, the top and the bottom have the same degree, I'm going to take the ratio leading coefficients, which is 2 over minus π , right? So this is equal to minus 2 over π . Right? And because we've got some kind of finite number there is going to tell us these have the same growth. Okay? So finite, so similar growth. In terms of categories, it's the same growth category system.

Okay, great. And now we're going to look at one more here, which is, I'll do that over there, actually. So we have 3, I have more of these blues. None of my blues for some reason are working. This one's going to work. Okay. So now we're going to have the limit as X goes to infinity. And then we have E to the X minus E squared divided by, and then we have X squared minus π . Okay? Again, this is a rational function. But remember, this is a constant. That squared has nothing to do with a degree, that's a constant. The X has degree one, this has degree two, so the bottom has bigger degree, which means that my limit is actually zero. Okay? Which is actually going to tell me that, that the bottom one must grow faster. Okay? So zero, so, right, that limit was zero. So, G of X equals X squared minus π , or what was on the bottom grows faster.

Okay? So here it becomes a question which is that what do we do? So question, what if when taking the limit we end up with, so what we call in indefinite form? Okay? So i.e., so what are some indefinite forms? So, here are some indefinitely forms. So we could have zero over zero, we can have plus or minus infinity over minus plus infinity. Or we could have the signs of the infinities be the same. We'll go over that more in the next lecture. Okay? And the answer today is going to be L'Hopital's Rule. So the answer is L'Hopital's Rule. Right, and we'll cover that next time.

Okay, so this is just you can compare function growth rates by taking the ratio and then taking the limit as you go to infinity. We've already seen this for a rational function. So polynomial over a polynomial, we know how to take those limits. So we know when we're comparing polynomial growth, actually, it's just determined by the degree. This is where you could really notice from here, is that if one of the degrees is larger, that one has larger growth. And if the degrees are the same, then you actually have some kind of finite number, okay? And the other thing was what to do if you end up with zero over zero or infinity over infinity or something like that, and we're going to handle that in the next lecture with L'Hopital's Rule. Okay, so I hope that was helpful, and I will see you in the next lecture.