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derivative, natural log, chain rule, integrating, equal, substitution, dx, du, function, differentiate, respect, students, integral, minus x, write, mistake, match, x dx, absolute value, carefully

SPEAKERS

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Welcome. In this lecture because the natural log and also the function like $1/u$ derivative are such, you know, like common functions and very important functions, it's worthwhile to go through a little bit carefully some of the stuff that students will very often make mistakes on. Particularly, they forget that they have to use the chain rule to differentiate, and then substitution to integrate. I jokingly call it lawnifying. I've must have heard a student say this at some point, where whenever when they're integrating whatever is on the bottom, they just stick inside of the natural log. I've seen, probably more students do that than not do that. So this is, if you're doing that you're in good company. But it means that slow down, carefully, look at what you're doing, and make sure to use substitution. Okay? So the things to first remind you of that we're going to kind of be needing are that if I take the derivative with respect to X of, and I'm going to take the derivative with respect to X of the natural log of X . So I have the natural log of X . Then this is actually going to equal, so I'm going to just write this as $1/u$, and then I'm going to do this with parentheses to kind of emphasize that this is 1 over something, and these have to match up. Okay? And, right, going the other way, I have the integral of, and then I have $1/u$, and then I have here X . And then I'm integrating with respect to X , that same X , nothing, there's nothing else in here, except that exact X that I'm integrating with respect to, okay? Then this is going to equal, and I have the natural log of the absolute value of this X . And then I try really hard to remember not to leave out the C . Okay, so let's look at this derivative situation when it gets a little bit more complicated. So if there's anything more complicated than that same X , as you're differentiating with respect to inside, you need to use the chain rule. Okay? So you must, this isn't like an option, use the chain rule to differentiate, to differentiate.

Okay, so let's do, so let's look at an example. So let's say we have the derivative with respect to X . And then I have the natural log of, and on the inside, I have $1 - X$. And I look at this and I say, okay, this X does not match up with the entire inside, okay? Which means I have an inside function, right? This is my inside. And I have to use a chain rule. So I have, right, I take the derivative of the outside first, right? So I can go $1/u$, and then now I have to put in that $1 - X$ that was on the inside. And then I can take, now I have to go and take the derivative. Now I can take the derivative with respect to X of, and then I have $1 - X$. Okay? And then, so now we can just write this. So the derivative of $1 - X$ is -1 . I do this one in particular, because this is one of those things where the impact of the chain rule is so small, that it's just like a sign that students think that the sign is a mistake, and that they didn't actually need to use the chain rule there. But you did, because that

minus 1 does belong there, i.e., so this is equal to minus 1 over 1 minus X, or maybe 1 over X minus 1. It's another way to write it if you want. So my, so in summary, the derivative with respect to X of the natural log of 1 minus X is going to equal, and then my answer is going to be minus 1 over 1 minus X. That minus was not a mistake, the minus came from the fact that I had to use the chain rule. Okay? Now let's look at going in the other direction. So now we go, so we need to use substitution when integrating, so need to use substitution when integrating.

Let's see if I can succeed in getting this on here. There we go. Look at that I did, okay. So let's look at an example. So we're going to take actually the same thing, almost. I'm going to leave out the sine, but otherwise, I'm going to take exactly the same thing, so you can kind of see what happens. So I'm going to have 1 over 1 minus X, DX. And here, I'm going to do a U substitution, with this being my U, right? So this part here is my U. So I have U equals 1 minus X, when I take the differential, right, so this is my DU. Well, I have to take the derivative of that, and the derivative of that is minus 1, right? So I get minus 1 from taking the derivative of that times DX. Okay? And then I can multiply both sides by minus 1 and I ended up getting minus DU equals DX. Okay? So now I can continue. So this is going to equal the integral of 1 over and then U, DU with that minus sign there. So times minus DU, so minus DU, okay? Because I could replace DX with minus DU, I replaced 1 minus X with U. This is what this told me I can do over here, so I did that. I can write this a little bit more simply, this is just equal to, right, minus the integral of 1 over U DU. But now, because these match up, right, these U and these DU match up, now I can just go ahead and this is going to equal the natural log of the absolute value of U plus C. Okay? Or at this stage, so maybe I'm going to kind of want to, so this is, right, so that it kind of matches up with what I have up there. And this is that U, okay? And then, well, I'm almost done, except that there was no U here. So I need to put that back in, right, when we're doing the substitution. So I'm going to get minus, and then you get the natural log of, and then I put in for U my 1 minus X, because that's what U actually is, and then plus C. Never forget the plus C. Okay? So on this side gets kind of the opposite of that. I get the integral of, right, so for both of these I end up with a minus that's coming from that derivative of this thing that's on the inside. They both have the same thing on the inside, right? DX, is going to equal, I'm going to box this.

Okay, and then this is going to equal, so I have minus and then the natural log of the absolute value of 1 minus X DX. Sorry, 1 minus X plus C. Okay, so let's box that one too. So the moral of today is that a lot of times students, it gets really tempting to just stick whatever you have, you know, to take these formulas and whatever's inside of the natural log whatever's underneath 1, just stick it in the other side, and that won't be a problem. But this is precisely the circumstances where we need to use the chain rule to differentiate, and the substitution rule when integrating. Because the derivative of 1 minus X is not 1, it's minus 1. Okay? So, and that ends up showing up in here, and it ends up showing up here. Okay? And so that is where you end up with a mistake if you didn't use the proper methods. Okay? There's an inside function, right? Here's your inside function. Here's your inside function. This is 1 over U is your outside, here's your inside. It's LN of U is the outside, and 1 minus X is the inside. Okay, so I hope that made some sense, and be careful when you're differentiating or integrating things with the natural log or 1 over X, and I will see you in the next lecture.