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SPEAKERS

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Welcome. In this lecture, I want to talk about the function inverse of A to the X . So I don't mean 1 over A to the X , I mean the function inverse. So we'll talk about this and you'll see what I mean, hopefully. So we have X , and we have Y . Right? And then we have this graph of this function, right, so this is Y equals, and then I think my color scheme has been A to the X . And if it hasn't been, this is what it's going to be for now. And we happen to always know this is, you know, we might as well put this on here for helpfulness, this that this crosses here, right, at 1 , perfect. Now, something that you might notice about this, looking at this, is that it passes the horizontal line test, right? This passes the horizontal line test.

So we have inverse, like an inverse function, so have a function inverse. Right? And what I mean by that is we have this function, so we have $\log$, and then we have A , and then if you input into here Y , you get out X where we know that, so you get out X , right, precisely when, so this corresponds to, this symbol means that these two things correspond. I'm actually going to use it. It's not a symbol to get nervous about. It just means that these two things happen in precisely the same circumstance. So then we have A to the X equals Y . Okay? So this is how this function, you can think of this as being how this function is defined. This is how I think about it, right? That like, yes, there's rules for logarithms, and you should know them and so on. But as I'm going to say a million times, or have said a million times, which is that how do I think of the logarithm? I think about this relationship, because I understand exponential functions much better than the logarithm. And something I want to point out here is this is still called the base. So this is still called the base.

Okay, so that's good to know. Now, in being an inverse, it still satisfies all of our inverse properties. Okay? So satisfies, and this satisfies all of our function inverse properties. So it satisfies all our function inverse properties. So let's kind of go through some of them. And then for example.

Okay? So the first one will be, let's just kind of go through them. That if I want to get between the graphs, like how do I get between the graphs? I'm going to redraw that over here, so I can do a little bit more with it. So here's my X , here's my Y . And then that was my Y equals A of X . Right? So that

was Y equals A to the X . Right? Well, how do I actually get through it? Like, how do I get over to the graph over here? So I want to look at, I want to reflect across there's this line. Right? This is the X equals Y line. So this is X equals Y . And I reflect across there in order to get the graph, okay? So how am I going to do it? I'm going to reflect across the X equals Y line. So the X equals Y line. And then I get the graph of the other function, and look at this, with the way I'm writing it. And then if you want your variables to be correct, you know, to kind of switch them everywhere. So this is Y and this is X . Sometimes it helps to actually think of variables as colors, so that then you can just call them whatever you want, it doesn't matter. So then what is my graph going to look like? So now I want my graph to actually kind of be orange here. Okay? So this is the graph of X equals. And then as I have over there, so this is $\log A$ of Y where I input that. Okay? So I have, right, so these are kind of how these correspond. And the fact that this intersects this axis at 1 doesn't change, right? If this one, then has to intersect this axis at 1, right? Okay. So that's the first thing and then we also have, the next property is that when I take $\log$ of A of, right, all I'm doing is I'm composing the two functions, A to the X .

Let me keep my colors consistent. Sometimes it can be a little bit of work, then what I'm going to get out is that this is equal to X . Right for all X in the real numbers. So for all X in the real numbers. Remember that when I write this, I mean every real number X is what I'm saying. So I'm requiring that X is a real number. So that always works. And then I also have that if I take A and I go to say, of $\log$, and then base A of X , that this is going to equal X for every X . So it can't be every real number, it has to be greater than zero. For every, now I'm requiring that X is greater than zero. Okay, why? Because my outputs, if you look at this, my outputs are always greater than zero. So this function is never, like that graph is never going to cross over here. So I don't know how to input anything less than zero. Okay? So what's going on here is that we have, right, we started with this function, Y equals A to the X , we realize this passes a horizontal line test, this has a function inverse. The function inverse is $\log$, like $\log A$. And then don't think of these, like this is your input variable, this is your output variable. If it confuses you, a lot of times you can put like, sometimes I'll put like hearts and stars or something, so that you don't think that just because this is an X this is your input, and this is, just because this is a Y this is your output. It's not like that, but I want to know the relationship between these, so I wrote it that way. This is your input of the function, this is your output of the function, this is still called the base. And because this is a function inverse, it does satisfy the function inverse properties. One of them is that the graphs are you reflect across the line X equals Y . So the graph will look like that.

And then you also have that if I first apply A to the X , and then I apply a $\log$ of A , I get out X . That what it means to be a function inverse, and you can go the other way around. But then it has to actually, your X has to be defined putting it in there, which is that first I apply $\log A$ to X , now I apply A to that that, and I get out X , right? And this is something that kind of look at that this is going to need to happen because this graph doesn't ever go over here. So we don't actually know how to input anything over there. Okay? And once again, don't get confused by the X and Y thing. Think of them as inputs and outputs. My horizontal is my input, my vertical is my output. This is my input, that's my output. But by kind of looking at these correspondences, you can start with using the same variables so that you can see the relationship. But then in the end, you know, switch it back so that you know what is your input in your output. So usually, you would write $\log A$ of X . Okay, so it's a very important function. So it's good to learn them as well as you can. And I hope this made some sense, and I will see you in the next lecture.