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equal, exponential function, rational, multiplied, base, divided, input, write, output, circumstance, integers, accumulate, play, happen, 5th, warm, equalities, grow, simplify, little bit

SPEAKERS

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Welcome. Exponential functions are one of the most important concepts in mathematics for almost anything that you care about in life, whether it's how you accumulate money or accumulate debt. Whether you care about how a population grows, or how a disease spreads, or pretty much most things that grow, you need to understand whether they're growing exponentially and what that means. So I wanted to kind of play a little bit with an exponential function. There's the exponential function, which is E to the X , but we're going to talk about an exponential function as one where you have A to the X , just to kind of build a little bit of what's going on with it. Okay. So an exponential function is going to look like F of X equals A to the X , we have F of X equals so we F of X . So the thing I'm inputting is my X , and then what I'm outputting is some kind of A to the X . And this A that you're going to see down here is called the base. So this is the base, that's just the name for it. The most common base is the number E . Okay? And so then we just kind of look at that. And let's just kind of explore and play as to what's going on with this function. Okay, so what's going on with this function?

And there are many different ways to play with this. We're not going to play with its growth yet. That's a very important thing, what's to come later in the course, but we are going to play with understanding a little bit about how it operates. Okay, so let's look at the first circumstance. So let's let's first look at what happens if let's let's look at what happens if this exponent is a positive integer. So if X equals N is a positive integer, what's happening with this function? Okay? So if it's a positive integer, then what are we looking at? Well, when we are looking at when I take A to the N , this is going to equal right, I'm going to be multiplying A by itself N times. So A gets multiplied by itself N times. Okay? So for example, if I had 2 cubed I'm looking at 2 times 2 times 2 three times right?

Okay, so let's think just at this stage already about something that's going on, which is that if I increase my input by one, that means that I've multiplied the output by a single copy of the base. Maybe I'll even point that out. So this says that, it says many things but one of them it says is if I increase my input by one, I'm writing this in orange on purpose because I'm talking about the input. So if I increase oops, my input X by one right, then right, my output is multiplied by, right, my output actually ends up getting multiplied, and what it gets multiplied by is my base A . Okay, so that's one thing that's, you could pick up on from what happens with integers. Okay? Now what are some like, other kind of like working from here, right, I know what happens with integers, and I can move a little

bit further. Okay? So that's kind of like one thing this is happening. So the next thing to kind of realize is happening is that if I take, what happens if I take my base, right, and I take it to the zeroth power, right? Then I get out one, that always works. Okay? So what does this tell me, this tells me that always for an exponential function, I have that F of zero is going to equal one. That's always always going to happen for an exponential function. So for all exponential functions. But it has to be an exponential function. This is obvious, or not obviously, this is not necessarily true for other functions.

Okay, and then the next thing that we can kind of build off of this for, right, so we know that if I have, so if I take my base, and I take it to a minus N power, right? Well, we actually kind of know what happens with this. This is the same thing as taking, right 1 over A to the N . Right? So what does that give me? That gives me that F of, so if I take a minus N in there, so like a negative integer like this, and this is going to give me out, 1 divided by F of N , right. And this again, works for all exponential functions. So both of these things hold for all exponential functions, and maybe I should have. So both of these holds for all exponential functions F . Okay, both of those two things, just working off of these properties, okay. Now, so what's going to happen, so this is kind of like a warm up.

Okay, what's going to, I'm going to have you simplify. This is going to, you're going to be like, how in the world is this related? But maybe you'll see the connection and if not, then you'll see in a moment hopefully. Okay, so we're going to simplify where we have. So simplify 8 to the minus 1 divided by 3 to the minus 5 .

Okay, so let's kind of work through this a little bit. So what's going to happen? I'm going to, this is going to equal well A to the, so maybe we'll write this in one other way, which is that this is A to the minus 1 , times 1 divided by 3 to the minus 5 . Okay, and then let's keep going. So this is going to equal 1 over 8 , times 1 divided by, I know this is like, a little bit funny, 1 over 3 to the 5 th. Okay, I can write this is on the bottom, I can flip it over to get to the top. So this is going to equal one eighth times 3 to the 5 th, which is equal to 3 to the 5 th divided by 8 . Okay? So what really happened was that we just kind of flopped, we switched those. Okay, so that's kind of our warm up. And now, why did I do that warm up? Which is another circumstance. So this is kind of a, let's keep our warm up in a box so that we don't confuse it with the other stuff we're doing. Which is that if X is rational, so if, and what in the world is rational mean? Okay, so X is a rational number. Right? So what do we mean by that? So, a rational number. What can we mean by that is i.e., X is going to equal some kind of P over Q , trying to minimize my overlapping colors. Okay, so it's going to equal P over Q for some integers. So for some, or there exists some integers so that this is true. So for some integers P and Q . So if we're in the circumstance of a rational number, then let's look at what happens. Then we have, so let's take our A to the X , so we have our A to the X , to the X , and what do we get? We get, so this is equal to, so we get A to the, and then let's do our P over Q , the P divided by Q , right, which is equal to A to the P times 1 over Q , right? And then this is equal to, well, there's two different ways to write this. Okay? So it's going to equal so we can do parentheses. I'm kind of out of colors, aren't I? Okay. We're first going to do the A to the P inside. So this is equal to A to the P to the 1 over Q . Right? And, right, so this is equal to this. And right, this is also equal to, you can also swap the order of those, so you can go A to the 1 over Q , and then go to the P . Okay? So each of these has another way to write it. So this one equals the square root, not square root, sorry, the Q th root of A to the P . And this one here is equal to the P th power of, and then on the inside, we have the Q th root, Q th root of A . But we needed to do something here, I'm saying that these are equal. We did need like for these equalities, right, we need that Q is, is greater than zero.

Okay. Now, what if I'm not rational like this? So if X is, if X is what's called irrational, right, not rational, then what I'm going to do is fill in the holes. So my outputs going to fill in the holes. Okay? So you kind of get a picture where it's like X , and we have Y . Right? And I'm going to look at Y equals 2 to the X . My base of 2 and then I'm always looking at my input becomes the power X . Okay, the first thing we happen to know on here is that this is, right, we know that this is 1, right? And so what actually ends up happening here, okay, is that before it's, you know, if I'm only looking at the rational values, it's almost like a dotted line. It's like missing a whole bunch of points. And the irrationals are very close to the rationals. Okay, so you basically, you could take limits or something like this. But the real point is that, well, if I put all the rationals, you could kind of see where it goes. Okay? So that's just giving you a little bit of kind of hands on, getting a feel for what's going on with exponential functions. So, I hope that made some sense and I will see you in the next lecture.