On Mathematics, Music and Film

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PUBLISHER'S FOREWORD

We are pleased to present, "On Mathematics, Music and Film", the second monograph by Evan Cameron to be published in the CINEMA STUDIES series. The present work was composed by Mr. Cameron as a thesis in partial fulfillment of the requirements for the degree of M. S. in Film, in the Film Department of the School of Public Communication, Boston University.

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Gerald Noxon

Bridgewater,
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Introduction

To me, film is a means of making and recording music. Not 'music' in the sense of 'aural stimulation' alone, but rather in the sense in which the word was used by Pythagoras and Plato in ancient Greece: music as the rhythmic ordering of all things, the balance and proportion of anything.

This paper, then, is both an introduction to the film and a first step toward the structuring of a vision. For I have lived with images in my mind, fragments of moving colors and sounds, with which I could not work for want of a notation by which to describe and control them. It is in this sense that the task of this thesis was not chosen by me, but rather forced upon me by the presence of images which I could neither forget nor understand.

The goal of this endeavor, thus, is simply put: to have made clear to myself the mathematical (i.e.,musical) structure of the images I have known, and to have derived therefrom a set of procedures useful in the process of composing such images into a film. I have not sought for a 'notation' for film, in the sense of providing a system in which to formulate *production* scripts. Rather, I have sought for a set of procedures by which to describe and control images in the process of *composing* a film. By the end of Section III below, the extent to which I have been successful will be apparent.

If matters are pursued at some length in the forthcoming pages which strike you, the reader, as being only faintly relevant to film, perhaps you will understand that my conception of the task of the filmmaker is somewhat more stringent than most, and that the matters with which I have concerned myself are essential to doing that which I wish to do with film. It is too much to expect that the issues raised below will prove of interest to the majority of filmmakers, for the majority of films by filmmakers has proven of only passing interest to me. Rather, it is a very few films by a handful of filmmakers by which I have been enchanted and toward which this thesis is directed. And if, by the end of it, I shall have shown you how I have come nearer to the goal of being able to compose like them, I shall be satisfied.

Section I: On the functions of mathematics and art

1. Before one can compose anything, one must first know something of the *sort* of thing one is going to compose. But the task of coming to know what sort of thing anything is,involves either, or both, of two chores: (a) giving the thing a *name* (which chore is not always trivial); and (b) coming to recognize the *function* of the thing (i.e.,coming to understand the relationships in which the thing exists with other things of its sort).

The task itself is commonly designated 'research', and 'research' has become equated in our pragmatic society with 'scientific research' (i.e.,research whose results can be confirmed or disconfirmed by the scientific method of experiment and observation). The technique of scientific research has often been restricted to manipulating known entities into new and untested relationships, which were then subjected to testing. If the tests were satisfactory, further tests were devised; if unsatisfactory, the new relationships were either modified for retesting or rejected.

If it were not apparent prior to our century, however, the advent of relativity and quantum mechanics ought to have demonstrated that the manipulation of known entities (i.e.,variables) into new and untested patterns is neither the principal nor the essential task of research. The theory of relativity did not give new names to new combinations of several older variables, as one might coin the word 'event' to mean 'any point in space and time'. Nor had the older variables of electromagnetics and gravitational theory been recombined in fascinating and productive new ways. 'Space-time' simply, was *not* a new name for 'space and time'. And Einstein's theory of gravitation was no more a recombining of Newton's theory of gravitation with Maxwell's electromagnetic equations than had Newton's or Maxwell's discoveries been a recombining of the known variables of mechanics, and electricity and magnetism, in their time. Rather, the theory of relativity postulated a new sort of entity (space-time) whose essential characteristics could not be defined by recombining the variables of classical pre-relativity mechanics. That certain features of the new conception could be pointed-to by using the names of older variables was essential so that men might come to *understand* the terms of the new theory by getting some idea of how the older structures would have to be reinterpreted under the new conception. But notice: although older terms could, in some instances, be redefined in terms of the new conception, the new conception could not be interpreted in the older terms.

Similarly, when situations occurred in the examination of nuclear structures which caused men to ask 'Is the electron a wave or a particle?' (etc.), a new conception was required (which has yet to be fully articulated). Quantum physicists found that they could not define their new conceptions in the older terms of 'mass', 'momentum', 'wave', 'particle', etc. Some of the older terms could be redefined and used properly in certain circumstances; but it was the new terms which provided the logical foundation for the analysis and specification of the old, not the reverse.

The history of research has been largely determined by events of the above kind. Yet one area of research, the mathematical, has played a curious game with the trend. As new conceptions arose (eg., irrational numbers, complex numbers, analytic geometry, the calculus), the development of the field followed much the same pattern as other fields with a singular twist: each new development was seen as an *addition to,* not a *redefinition of,* older conceptions (and truths). Thus, complex numbers were conceived as an addition to the real numbers, as had been the irrationals (when finally accepted) to the integers, etc. When it was discovered that the length of the diagonal of a square could not be expressed as an integral ratio of the lengths of the sides, a new kind of number was eventually admitted to the haven formerly occupied by the integers alone: irrational numbers. But the discovery was not considered as a means of *redefining* integral numbers. Rather, '1', '2', '3', etc., and the fractions remained as before, with room being made in the continuum for numbers such as $\sqrt{2}$ (= 1.414...). Even with the discovery of analytic geometry, it was decided that geometry and algebra had thereby been united, *not* that geometry and algebra had thereby been redefined.

It is not altogether surprising, therefore, that the *logical* advances of the late 19th and 20th centuries found their purported application in the foundations of mathematics. As philosophy had worked with propositions for centuries, considering them to be true or false as they did or did not reflect (in some sense) the objects of the universe, but never considering the objects of the universe to have been *redefined* by the discovery of a new facet of language, logicians saw the constancy of the concept of 'number' in mathematics as amenable to their methods. If the axiomatic method of logic could produce a timeless definition of number, then the timeless truths of mathematics (as expressing propositions true of numbers) would have been reduced to truths of logic.

Godel's celebrated discovery of the necessary incompleteness of any axiomatization of arithmetic shot this hope. But the challenge remained: could all truths of mathematics be expressed using logical notations alone? The work of Russell, Whitehead, Church, Quine, et. al., has clearly demonstrated that such is the case, *given that mathematical statements are expressions of propositions about numbers, and that new mathematical discoveries are discoveries of new propositions timelessly true of numbers,* not discoveries requiring that the notion of 'number' be thereby redefined.

When mathematicians balked at the new mathematical logic, sensing that it had little to do with creativity in mathematics in any sense, they had only themselves to blame. Had they queried more closely their foundations, and, once a new discovery had occurred, not assumed that '2 + 2 + 3 = 7' must *mean* the same as previously (being an eternal a priori truth), they might have made it a precedent of tradition that mathematics was itself constantly redefining its older conceptions in light of newer ones. Instead, they made dubious assumptions at each step about the nature of the continuum, and when a new number was discovered (or function, or approximation technique) simply made room for it among all the others.

It is one of the assumptions of this paper that mathematical objects (numbers) have changed their meaning as many times in the history of mathematics as has the concept of 'particle' in physics, and that since the advent of the calculus the notion of the differentiation-integration function has remade the sense of mathematics (and its objects) in a profound fashion. This, of course, is not a paper in mathematics, but in film, though it is one of the purposes of this paper to show how the latter is best conceived as a special case of the former. I shall only briefly describe the fundamental mathematical notions with which I am concerned so that the tenor of the above assumption is more-or-less familiar, and then move as directly as possible into the direct applications to the structuring of a film. (To do this and yet remain intelligible to filmmakers is an unenviable task: if one should remain fluent enough to be read, one would necessarily be inexact in what one says. I have tried to strike a balance, but there is only so much that can be said in English about relations without ambiguity and vagueness, and the decision not to use symbolic notation may prove to be of more trouble than it was worth. But the choice has been made after consideration, and I shall not vary from it.).

2. Mathematics has been shown to be reducible to symbolic logic in the following sense. Any mathematical statement expressing a proposition which is true or false can be restated without loss of rigor as an expression consisting of only three notational devices: the ' $\downarrow$ ' sign of truth-functional logic; the notations of quantification theory; and the sign '$\in$' of

class membership. As the theory of relations has been shown to be reducible to the notation of classes, and the converse, the above is equivalent to saying that the notations of truth-functional logic, quantification theory, and relation theory (class theory) are sufficient to transcribe any mathematical proposition.

But more can be said. Truth-functional logic is itself a system of *relations* in the sense that it is reducible to expressions of relations between propositions which, in turn, are related to either, but not both, of the truth-values 'true' and 'not-true' (i.e.,'false'). Quantification theory also is a system of relations, in the sense that it is reducible to expressions of relations between objects of two sorts (variables and predicates).

Mathematics, thus, ought to be considered as the generalized theory of relations, in the sense that a theory of relations general enough to encompass the logic of propositions, quantification theory, and the theory of classes, would be adequate to express any mathematical proposition. The question, of course, is whether or not the notion of a generalized theory of relations is a confusion. Certainly, those who have labored to clarify the above reduction of mathematics to the logic of propositions, quantification theory, and the theory of classes have done so because they believed that the three primitive notions, necessary and sufficient under the final reduction to express mathematical propositions, were as basic as possible (in some sense which I don't think could be further specified). The fact that the three basic notations are themselves *relational* notations, however, leads me to expect that a new approach might lead to a single primitive of which the others could be construed as special cases. In a sense this would require a radical re-orientation of thought away from propositional logic as the standard of rational clarity. (How many would care for such a re-orientation is of no concern at present.)

3. Mathematics, as presently conceived, can be reduced to logic and the primitive '$\in$' of class membership. Mathematical equations, expressing relations of a particular kind between objects of a particular sort, are thus conceived to be expressing truths about classes of objects. What are the objects? Numbers. And, indeed, the entire program of contemporary mathematical logic began, and has been sustained, largely as the attempt to clarify the notion of 'number' – the objects with which mathematical expressions are taken to be concerned.

But a curious fact emerges when one examines the definitions of number so conceived. "Any objects will serve as numbers as long as the arithmetical operations are defined for them and the laws of arithmetic are preserved." (Quine)[1] Thus, certain variations occur between systems of set theory as to what is to count as the number 0, 1, 2, etc. That unlimitedly many alternative versions of set theory are possible which are consistent and adequate is admitted. But all involve intimately the notion of classes of classes. Thus, (e.g.,) Zermelo took 0 arbitrarily as Λ, the null class (the class having no members), and defined the successor of a number x as $\{x\}$, the unit-class of x (i.e., the class having x as sole member). For Zermelo, therefore, the natural numbers become:

$$\begin{aligned} \Lambda &= 0 \\ \{\Lambda\} &= 1 \\ \{\{\Lambda\}\} &= 2 \\ \vdots \quad & \ \ \vdots \quad \vdots \end{aligned}$$

etc.

All other systems, though varying in detail, involve equally at some point the notion of Λ (the null class) as basic. *But the important point is not that the basic class have no members, but rather that the fact that it has members, if any, be considered irrelevant to the purpose at hand.* That is, any class having members, no matter how many, could be taken as basic for the purpose of defining the real numbers if the fact that it *has* members were considered irrelevant. For example, if a, b, c, . . . r be considered irrelevant,

$$\begin{aligned} \{a, b, c, \ldots r\} &= 0 \\ \{\{a, b, c, \ldots r\}\} &= 1 \\ \{\{\{a, b, c, \ldots r\}\}\} &= 2 \\ \vdots \qquad\qquad & \ \ \vdots \quad \vdots \end{aligned}$$

etc.

becomes equivalent to the system sketched above.

The point is that the natural numbers, even on contemporary accounts, do not require *objects* of a particular sort in their definitions. Any objects will do, as long as they are considered irrelevant to the *function* being performed on them. To be a number, then, is to be a function (i.e., a relation) of a certain kind, and any objects will serve to elucidate the definition (even numbers themselves) so long as their presence is considered irrelevant.

Indeed, it is precisely because objects of any particular kind are irrelevant to the definition of functions (i.e., numbers) that functions, in hand, can be used to define *objects* (as will be seen in the following section).

4. I shall consider the basic function (relation) to be that of differentiation-integration. As the operations of differentiation and integration are defined to be operations performable only on functions, I can construe them as functions of functions (of functions, etc.).

Given the notion of differentiating a function (and the function which we know as 'multiplication-division' taken for granted), how could the natural numbers 1, 2, 3, ... be defined? Consider any object specified by 'x'. The natural numbers can be defined as follows:

where

$$y = x \quad , \quad 1 \overset{\text{def}}{=} \left(\frac{dy}{dx}\right)$$

$$= xx \text{ (i.e. } x^2\text{)}, \quad 2 \overset{\text{def}}{=} \frac{1}{x}\left(\frac{dy}{dx}\right)$$

$$= xxx \quad , \quad 3 \overset{\text{def}}{=} \frac{1}{2x}\left(\frac{d^2y}{dx^2}\right)$$

$$= xxxx \quad , \quad 4 \overset{\text{def}}{=} \frac{1}{6x}\left(\frac{d^3y}{dx^3}\right)$$

$$= xxxxx \quad , \quad 5 \overset{\text{def}}{=} \frac{1}{24x}\left(\frac{d^4y}{dx^4}\right)$$

$$= xxxxxx \quad , \quad 6 \overset{\text{def}}{=} \frac{1}{120x}\left(\frac{d^5y}{dx^5}\right)$$

etc.

If the natural numbers be thought of as a series 1, 2, 3, . . . , the general function defining each member n + 1 of the series in terms of the preceding member n would thus become:

$$\text{where } y = x^{n} \cdot x, \quad n+1 \stackrel{\text{def}}{=} \frac{1}{n!\,x}\left(\frac{d^n y}{d x^n}\right)$$

(where it must be remembered that 'n' stands for the last number *previously defined* in the series. That is, I have not used the notion of any number (undefined) to define another, and there is consequently no breach of rigor.)

The point being stressed is that, on this account, the number 2 *is* the function expressed by the symbol '$\frac{1}{x}\left(\frac{dy}{dx}\right)$', where y = x.x (i.e., x^2); similarly with the numbers 1, 3, 4, 5, . . . The object x can be anything; indeed, it could even stand for the function being defined (e.g., the number 2 itself), for whatever x is, is irrelevant to the function being performed on it. That is, x *is* what the function being performed on it says it is. (An essential difference should be apparent, therefore, between what a function (an x) is, on this account, and what the subject of an English sentence or a variable of quantificational logic is, on other accounts. Although it is impossible to express the notion in a subject-predicate language like English without pseudo-nonsense resulting, the sense of the notion is something like this: there is no ambiguity or breach of rigor, on my account, in taking a function (e.g., the number 2) as the object x in its *own* definition!)

Since numbers have been defined to be functions on this account, mathematics (instead of being, as on contemporary accounts, the collected expressions of the propositions true of numbers) becomes the expression of functions of functions (of functions, etc.) . . . i.e., the general theory of relations.

Each of the natural numbers can be defined in the above fashion as a *function* of the differentiation-integration function. As functions, so defined, they can become the object of other functions. Indeed, by being so defined they are already the function of a more general function than the one which defined them uniquely, this general function being

$$\text{(I)} \quad \text{where } y = x^{n} \cdot x, \quad n+1 \stackrel{\text{def}}{=} \frac{1}{n!\,x}\left(\frac{d^n y}{d x^n}\right)$$

This function is such that, given any natural number n as defined, the succeeding natural number n + 1 can be determined. In fairly-common terminology, the natural numbers are *ordered* by this general function.

The notion of an ordered function will be of some importance in this paper, and I wish to be certain that the idea is intuitively clear to the reader. To be *ordered,* a function

(a) must be determined by a function α from a function β previously determined by the function α ;

(b) must determine another function ϕ by the funtion α.

For example, the number three (3) is a function determined by function I from the number two (2), a function previously determined by function I, and the number three determines the function which is the number (4) by function I (which determined it). Thus, the number three is ordered by function I, and exists in order between the numbers two and four *as ordered by function I.* In this sense, the natural numbers cannot exist apart from the determination of function I.

A *random* series of natural numbers, thus, is not a series of unordered numbers, but an unordered series of (ordered) numbers. That is, it is an ordering of the natural numbers according to some function β which is irrelevant to the purposes at hand. For any finite series of numbers, it can be shown algebraically that there exists a function which determines it, in the sense that given any number in the series the next can be determined by the function. Thus, the notion of *randomness* does not entail lack of *order,* but rather lack of *relevancy* of the order. (But I shall have more to say on this later.)

5. In most mathematical textbooks, discounting the non-essential idiosyncracies of individual authors, a *function* is defined equivalently to the following:

> A function is a correspondence that associates with each number x of some given set of numbers one and only one number y. [2]

The notion is then expanded to include multi-valued functions as the occasion demands.

This definition, of course, is not available to me as I have defined 'number' in terms of 'function' above. Indeed, for me numbers *are* functions, and *sets* of numbers (eg. the natural numbers) are for me functions of a function; hence, on my account, the notion of 'function' is logically prior to either the notion of 'number' or of 'set', in virtue of which the customary definitions of 'function' are couched.

Implicit in the above discussion of defining the natural numbers was the notion that the differentiation-integration relationship (function) was to be taken as basic. That is, all functions, on this account, are to be viewed as determined by it, *including itself,* in the sense that a *function* is anything which can be determined by the differentiation-integration function.

If this seems slightly illegitimate, let me remind you of my earlier assumption concerning advances in mathematical knowledge. Once a new concept has been discovered, it is futile to attempt to define it in older terms (if indeed it is a new concept, and not simply a new name for a combination of older concepts). The task must rather be to redefine as many of the older terms in terms of the new concept as possible. Of course, the only terms available to me are older terms, particularly when writing in English, in which the new notion must be advanced. In English, the closest I can come to describing the fundamental notion of mathematics (as I see it) is to call it 'the differentiation-integration function'. But it must be remembered that by 'function' I do *not* mean what has customarily been meant by the term (i.e. a notion determined by the notions 'number' and 'set'), but rather a new and more general notion *in virtue of which* 'number' and 'set' (and therefore 'ordered number') can be determined.

(The attempts of 19th-century mathematicians to determine the differentiation-integration function in terms of older notions, as developed in the theory of the *limit* of a function, seems to me singularly unenlightening. Not that the function cannot be determined in terms of the notion 'limit', but that the notion 'limit' is as foreign to the classical notions of 'addition' and 'multiplication' as is the function to be determined. The use of three dots '. . . ' to indicate infinite series, as in

$$1 + \tfrac{1}{2} + \tfrac{1}{4} + {}^{1}/_{8} + \ldots \overset{\lim}{=} 2,$$

or the use of phrases such as 'for every number $\epsilon > 0$' in the definitions of 'limit of a function', seems to me to cover a multitude of sins (in the sense that new notions are being advanced under the guise of being older ones). In my judgment, the only satisfactory way of approaching the notion of 'limit' is *thru* the notion of the differentiation-integration funtion (e.g., as used to determine 'extrema' of functions, etc.), and not the reverse.)

In this paper, I shall use the terms 'define' and 'determine' synonymously. Thus, to speak of a number being 'defined by a function' is to say that a number is being 'determined by a function', and likewise for all things not readily understood as numbers. In general, I think, I shall prefer to use the word 'determine', for it is etymologically related to the noun 'term' (e.g.,to determine 'to specify a term for'), and hence connotes the process of giving a new name to something already understood. It is in this sense that the numerals '1', '2', '3', etc., are construed to be new names (terms) for the functions by which the numbers are determined. To define a number, then, is to determine it by giving the function whose name it will be, and *by whom* it is said to be determined.

6. It was mentioned earlier that a random series of numbers is not a series without specified *order,* but rather without specified *function.* For example, the number series

(I) 1, 1, 3, 12, 2, 7, 9, . . .

possesses a definite order, in that it begins with a one, followed by a one, followed by a three, followed by a twelve, etc. What the series does not *prima facie* possess is a general function in virtue of which each member of the series is determined by any other number in the series (in the sense that, given any number of the series, there does not *prima facie* exist a function in virtue of which the following, or preceding, number (s) can be determined). In contrast, the series

(II) 1, 3, 5, 7, 9, 11, . . .

not only is ordered but is also determined *prima facie* by the function notatible as

$$n + 2$$

in the sense that, given any number of the series as n, the following number can be specified by adding two to it (and the preceding number specified by subtracting two from it).

It can be proven algebraically, however, that any finite integral series of numbers possesses an unlimited number of functions in virtue of which the members of the series are determined. Thus, although the function or functions which determines the members of the first series (I) above remains unknown to us, and therefore the series *prima facie* is without a determining function, such a function does exist. Randomness, therefore, does not mean without function, but without *specified* (or evident) function.

Perhaps the matter can be shown more clearly by noticing that a series of non-random numbers is non-random because each member of the series in turn can be determined from the series of natural numbers by a function. For example, to say that series II above is determined by the function n + 2 is to assume that it makes sense to speak of adding two to each member of the series in turn, where 'adding two to n' is defined in terms of the series of natural numbers whose order is taken as given. If we did not first know that 5 + 2 = 7, it would make no sense to speak of the number 7 in series II as having been determined by the function n + 2 from the number 5. But we can only know that 5 + 2 = 7 by having firstly taken the ordered series of natural numbers

$$\text{(III)} \qquad 1, 2, 3, 4, 5, \ldots$$

as given (and hence non-random), and having secondly defined the function (of functions) 'addition' in terms of it. Given this, series II can be defined as that series whose members are determined in turn by the members of the series of natural numbers III according to the function:

$$n + (n-1)$$

That is, applying the above function to the members of the series of natural numbers III in turn yields the members of series II in turn, as a quick check by substitution will indicate.

To have said, therefore, that series II was determined by the function n + 2 was simply shorthand for saying that series II was determined from series III by the function n + (n–1). And to have said that series I was *prima facie* undetermined was shorthand for saying that there is no specified function analagous to n + (n – 1) by which series I can be determined from the members in turn of the series of natural numbers III.

(There is no mathematical reason, of course, why series I could not have been taken as given (hence as non-random), operations akin to 'addition' defined in terms of it, and series III declared to be *prima facie* random relative to series I. The important thing is the definition of the operation 'addition', of course, for any two series for which an equivalent operation could be defined would be identical (i.e., the differences would be purely notational); and thus, if the addition operation were defined for series I, the series would *be* identical with the series we now *express* by the numerals '1', '2', '3', '4', etc. (i.e.,the series of natural numbers III).

What must be noticed from the above is that the *order* of anything depends upon the existence of something by which it is *functionally* determined. To say that something is ordered is to say that it possesses a *metric,* a yard-stick, by which the relative position of any part of the thing to any other part can be known, for having a *metric* is the result of being functionally determined from something taken as *ordered* (i.e. as functionally determined).

It was argued earlier that the differentiation-integration function determines itself – a notion quite beyond the capacity of the dyadic (i.e. two-place, subject-predicate) meaning structure of English to express. It is the contention of this paper that works of art, being (and hence being determined by) such functions, must possess an *intrinsic* metric, in the sense that they are their own functional determinants, and, hence, carry with them the yardstick consisting of their elements-in-order in virtue of which the function of each element in that order can be measured.

7. In determining the natural numbers above, it was *assumed* that the multiplication-division function was already in hand. That is, the natural numbers were defined in terms of the notion of multiplication-division , not the reverse (as is customarily the case in mathematics texts). This procedure can be justified, on my account, if and only if the multiplication-division function can itself be determined by the differentiation-integration function which I consider as basic, for, if this can be done, the above use of the multiplication-division function was, as intended, a notational shortcut for a much longer (and less familiar) expression using only the notation of the differentiation-integration function.

Although I shall not attempt to show in detail how the multiplication-division function can be shown to be determined by the differentiation-integration function, I shall sketch briefly how the determination could be carried out.

That multiplication and division are extensions of (i.e.,determinable by) addition and substraction, in the ordinary sense of the words, is familiar to all. Assuming the same, if it can be shown that the addition-subtraction function can be determined by the differentiation-integration function, the fact that the multiplication-division function can likewise be determined will be considered proven. The task, then, is to show how the addition-subtraction function is determined by the differentiation-integration function.

The fundamental theorem of the calculus, discovered independently by Newton and Leibnitz, can be stated (in customary terms) as follows: If a function f is continuous (the meaning of 'continuous' I shall not discuss here) in a closed interval, then:[3]

(1) the function f has anti-derivatives in this interval; and
(2) if F is any anti-derivative of f, and a and b are numbers within the interval, then

$$\int_a^b f = F(b) - F(a).$$

'a' and 'b' in the expression above stand for numbers. On my account, however, numbers are a particular kind of unction. Thus, I shall evaluate the symbol

$$\int_a^b f$$

as the definite integral of f evaluated from function a to function b, leaving it unspecified as to whether or not the functions are natural numbers. (That is, a and b are to be understood as functions determined by the differentiation-integration function). With this proviso, I can determine the difference between two functions, α and β, as follows:

$\alpha - \beta \overset{\text{def}}{=}$ the definite integral of the *derivative* of F evaluated from a to b, where F is any function which, when applied to functions a and b, yields $F(a) = \beta$ and $F(b) = \alpha$.

In symbols roughly compatible with ordinary usage:

$$\alpha - \beta \overset{\text{def}}{=} \int_a^b y, \quad \left(\begin{array}{c} y = \frac{dF}{dx} \\ F(a) = \beta \\ F(b) = \alpha \end{array} \right)$$

As mentioned earlier, the above discussion is not intended to do more than sketch a proposed determination of the addition-subtraction function. That it would require much more thought and care to be precise and non-circular is apparent (if for no other reason than that, if a and b *are* natural numbers, then apparently the determination of the numbers a and b *is* circular in a sense I should wish to avoid). But I think the above is clear enough to show how the problem could be attacked, if this were a paper in the fundamentals of mathematics, which it isn't, and how the multiplication-division function can be expected to be determined by the differentiation-integration function. (Given that $\alpha - \beta$ can be determined as above for any functions α and β, then

$$\alpha =_{df} \beta + \int_a^b y,$$

and the multiplication-division function, of course, can be determined from the above as a special case of the addition-subtraction function. For example, if in customary terms β should equal $\int_a^b y$, then $\alpha = \beta + \beta =$ the *double of* β, etc.)

8. I shall say no more about the particular mathematical notions involved in this paper, much less take the time to clarify the functional relationships which exist between the notions already mentioned. I do this for two reasons: (a) such an examination would lead me too far from the main course of this thesis; and (b), I am as yet unable to clarify for myself the precise relationships involved (and expect that a detailed notation would be necessary to accomplish a further understanding).

From the discussion so far, however, I think it should be apparent that I maintain:

(a) To be is to be a function (i.e.,to determine other functions and be determined by a function);

(b) To be a function is to be a number.

In English, (a) could be paraphased with fair accuracy as follows: To be is to *have* a function. Ordinary linguistic usage shows a curious reflection of the above thesis in our tendency to ask about an object 'What is its function?', meaning 'What's its purpose?' or 'What does it *do*?' To ask for the function of an object in ordinary life is to ask how it is determined within a particular context of use, and we have all received the answer at one time of another, 'It doesn't do anything', meaning *within the context of the purposes under discussion,* that the object doesn't exist (i.e. that it is irrelevant that it be there or not). Indeed, strictly speaking, if we use the context of purposes under discussion as the only possible focal point of the adverb 'where', the object isn't any*where* at all (i.e. it serves no function), and, as Gorgias pointed out circa 400 B. C., to be no*where* is *not to be.*

If, then, I equate being (a function) with being a number, by the expedient of defining all numbers as those things determined by any function, I have taken a long road back to a position advanced by another Greek circa 400 B. C., Pythagoras, when he maintained that everything was a number.

And that it not be misunderstood, let me put it explicitly: I am not playing word-games here. Anything which is can be defined, in the sense of being determined by a function. *What* it is depends upon the function which determines it. And functions are defined to be numbers (though, of course, not all functions are *natural* numbers).

As I shall discuss later, a work of art is a function which determines its own elements. To be an element of a work of art is to have a function (i.e. to be determined by the function which is the work). Not to be a function in a work of art is to be artistically irrelevant (i.e. to be artistically nonexistent), and a work is artistic to the extent it contains no irrelevant elements.

9. The English language (indeed, any natural language) is subject-predicate in form, with variations on the theme. That it is adequate for expressing the functions of ordinary life in most instances is not to be denied, but that it ought to be taken as the standard of clarity for deciding whether any notion is clear seems at best suspect, particularly when its inability to express complex relations is considered.

I have spoken above, in English, of the differentiation-integration function, meaning that notion which can only be expressed in English as the conjunction of two notions:

$$(1) \quad \int f = F + c \ .\equiv. \ \frac{dF}{dx} = f$$

$$(2) \quad \int_a^b f = F(b) - F(a)$$

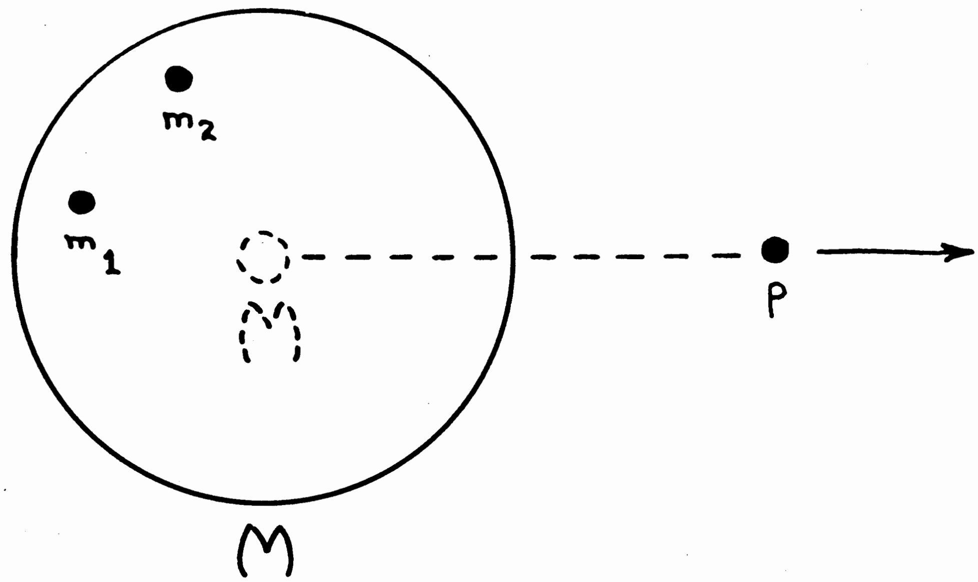
That this notion cannot be adequately expressed in English becomes apparent when one attempts to speak of what a 'function' is, for to say that a function is anything which is determinable by the differentiation-integration function is to be *circular* in one's argument. Yet to speak of functions in general without regarding the differentiation-integration relationship as a function, indeed as the determining function of all functions *including itself,* is to miss the point. (It should be apparent from the last sentence that the logician's dodge of a metalanguage-object language distinction, in which the former is used to speak of the latter, will be of no avail here. The basic function, as basic function, *does* determine itself.)

In short, the English language cannot express a *triadic* relationship except as a succession of dyadic relationships (eg.,'x is left of y', 'y is left of z', and 'z is left of x'). Nor can a mathematics which is based upon a notion of triadic relationships as conjunctions of dyadic ones suffice to determine triadic functions (witness the inability of physicists to solve the Three-Body problem, as discussed below).

Yet the notion of the differentiation-integration function, as I conceive it, is *essentially* polyadic, in the sense that functions are determined by at least two other functions: the function to which they are related by the covering function, and the covering function itself. Thus, the integral of a function F is not determined solely by the nature of the function F, but also by the nature of the differentiation-integration function which orders them both.

The English language, in virtue of its subject-predicate meaning structure, is equally restricted in its ability to describe polyadic functions constituting works of art. Any element of a work of art is determined by every other element of the work, not only in the sense that it exists in specifiable relationships to every other element, and they to each other and it (a set of dyadic relationships), but in the sense that, if it were other than it is, the relationship between any *other* two elements would be different from what it is. That is, it is precisely correct to say of any element in a work of art that it is functionally determined by all the other elements in the work, but also that *it* functionally determines the function of all the other elements in the work.

Perhaps a physical analogue will make the point clearer. Newton proved that the mass of the earth (M) could be considered as concentrated in a point at its center for purposes of describing the gravitational effect of the mass of the earth upon objects (masses) at varying distances from its surface (discounting the effects of the sun, etc.). He did this by using the new techniques of the calculus (integration). Thus, he was able to describe how the gravitational effect on a particle of mass p would lessen as it moved further from earth.



Newton was able to do this because the relationship between the particles of mass making up the earth's mass remain invariant regardless of the position of mass p relative to them. That is, the relationship of particles m_1 to m_2 of the earth's mass determines the variable gravitational effect on particle p regardless of its distance from them, but particle p, regardless of its distance, could effect no change on the relationship between particles m_1 and m_2. Since all particles of the earth's mass are in the same situation, Newton was able to integrate the result and treat the problem of gravitation as essentially a two-body, rather than a many-body, problem. Had the motion of particle p been significant in altering the relationship between particles m_1 and m_2, however, the integration could not have been effected, for there was (and is) no known way of determining polyadic functions which cannot be described as conjunctions of dyadic functions.

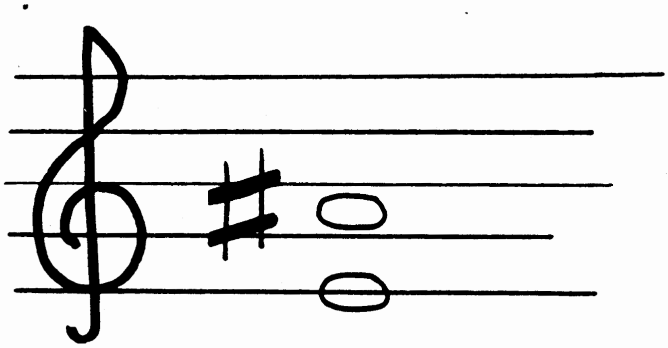
That is, conceive of three bodies related in such a fashion that the action of any one of them affects the motion of both of the other two non-identically. (The situation is the same as if, in the diagram above, a change of position in particle p were to effect a change in the positions of m_1 and m_2 relative to each other.) It is impossible, under these circumstances in general, to specify a function which correctly describes the overall activity which would result if one or all of the members of the system were displaced in some manner. Various approximation techniques have been devised to enable physicists to work with such systems, and solutions to severely restricted versions of the three-body (and $n > 3$ - body) problem have been given by Lagrange, Poincare, etc. But the fact remains that no general solution to the problem of three or more bodies exists; and the work of Poincare has shown that a general solution, if found, would have to involve functions which are neither algebriac nor uniform – and no one knows, given the classical definition of 'function', what that could even *mean*.

The problem of describing polyadic functions, therefore, goes right to the root of contemporary mathematical thought, and, given the essentially *dyadic* determination of functions now current, is apparently unsolvable. (The procedures used to deal with such systems in the physical world (for they exist all about us) are approximation techniques determined by extending the notion of classes of dyadic functions to their limits (e.g.,limits of series, etc.).)

It is no wonder, therefore, that the English language, with its subject-predicate meaning structure (essentially a dyadic function) has been found incapable of expressing precisely the patterns of functions constituting works of art, no matter how fluent the author or how careful the exposition. The fault does not lie in the inability of *men* to use their languages precisely enough, but rather in the structural inability of the *languages* themselves to be capable of such expression. An essentially dyadic function *cannot* describe an essentially polyadic (more-than-two) function, for it lacks the structural means of conveying that which is essential in the latter.

Each element of a work of art does not exist in a context determined by the other elements alone, therefore, but in a context partially determined by its own presence. That is, if an element x should exist in the context of the work of art in a particular functional relationship to elements y and z, and they in turn to each other, substitution of element w for x would not only change the relationship between that element and elements y and z, but would *also* change the functional relationship determining y and z themselves.

Particular examples from the arts could be multiplied indefinitely to illustrate the above, though I think two will suffice. The function determining the two pitches constituting an interval of the augmented fourth changes depending on the musical context in which they are heard (as any modern textbook discussion of the 'tritone' will emphasize).



(a 'tritone')

The musical context, of course, consists of other pitches existing in various functional roles relative to the pitches of the tritone. Thus, a change in any one (or, more blatantly, any combination) of the contextual pitches would not only change the context in which the tritone is heard, but would also change the functional relationship existing between the pitches *constituting* the tritone. (The very habit of some theorists, bespeaking a particular harmonic tradition, of referring to the various pitches in this frequency relationship by one term, 'augmented fourth', or 'tritone', regardless of the *functional* relationship involved within the context of the work, and the resulting lack of compositional pertinance of such analysis, has regrettably lessened the interest of active composers in the results of the search for whatever genuine physiological determinants of artistic measure there may be. E.g.,the works of Helmholtz in particular.)

Similarly, the placing of a particular shot rather than another after two given shots in a film not only would effect a different functional relationship between the first two shots and the third, but also would effect a change in the functional relationship determining the first two shots themselves. That is, how one would take the first two shots themselves would depend upon the nature of the third, while how one would take the third shot would depend upon the nature of the first two. (Again I point out: although my account is intentionally circular, the circularity is the result of attempting to describe an essentially polyadic function by a combination of dyadic functions (sentences). To those who would say 'So much the worse for art', I reply 'So much the worse for sentences', and go on about the business at hand.)

10. I have argued that the structural nature of English (or any other natural language) is such that no general analysis of the patterns of art is possible within the language, for the latter involve polyadic functions essentially while the former is limited in virtue of its dyadic meaning structure to the description of complexes of dyadic functions.

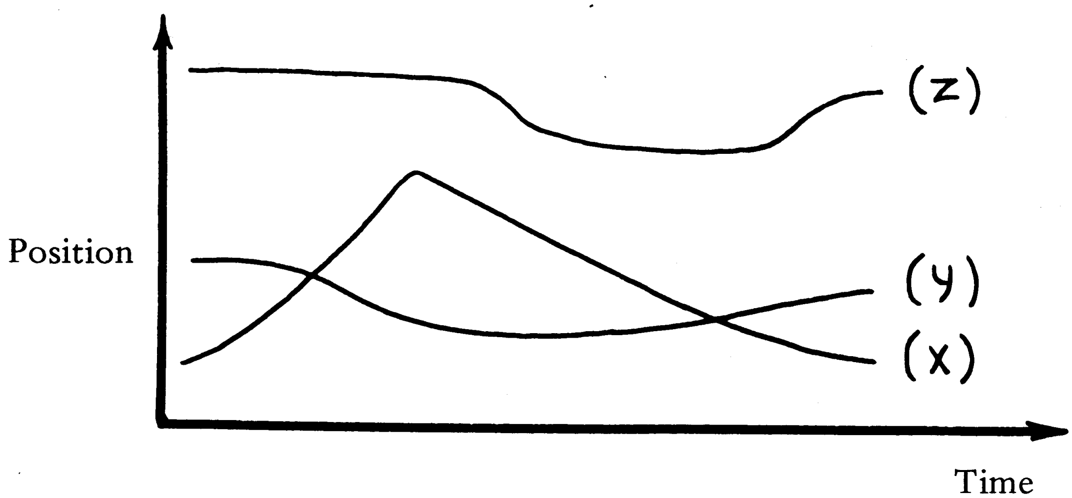
Logic, of course, began as the study of the formal characteristics of natural languages; and the symbolic logics of today, developments of the former, have increased the scope of complexity over which the logical analyses of dyadic functions hold. But, as I have futher indicated above, it is hardly accidental that the mathematical functions upon which contemporary physics are constructed, i.e.,those functions which have been reduced to logical notation by Russell, Whitehead, Quine, et. al., are inadequate to determine any situation which involves more than a complex of dyadic functions (e.g.,the Three-Body problem in particular, and polyadic functions in general).

The upshot of the preceding discussions, consequently, is that neither English (nor any other natural language) nor mathematics, as currently understood, are adequate to the description of those polyadic functions which are the essential determinants of a work of art. That is, although the pragmatic success of approximation techniques (and I include probability 'functions' under this heading) gives good reason for maintaining that such functions are certainly in effect, they cannot now be expressed in mathematical notation for they remain undiscovered (i.e. unknown). Simply put, the mathematical foundations of artistic functions are, at present, unthinkable (i.e.,conceptually unconstructed), and will remain thus until a major mathematical advance is achieved: the solution in general to the problem of polyadic functions.

A corollary of the above, of course, is that no notational system derived from a mathematical notational system can be adequate to the expression of even the simplest functions of a work of art, for even the simplest functions are essentially triadic and hence beyond the scope of mathematics as it is presently conceived (i.e.,as an extension of the complex dyadic functions of logic, which are in turn derived from the subject-predicate meaning structure of the natural languages). As approximation techniques are technologically effective in describing the physical world, however, and as music has had a fairly workable notation based on such a technique for several centuries, one might hope that such a technique might be developed for the cinema despite the lack of a mathematical foundation for art in general. (And, as we shall see, such is the case.)

The fact that approximation techniques have been developed in mathematical physics to deal with the problem of 3-or-more bodies, and that such techniques are effective, gives credence to the assumption that the activities of 3-or-more bodies *are* functionally determined though the function is not now known and hence cannot be expressed (it is apparent that something beyond the classical meaning of the word 'function', though including it as a special case, is intended here, as previously explained).

Notice, to say that the *function determining the activities* of 3-or-more bodies cannot be expresses is not to say that the *activities* cannot be expressed. If, for example, three bodies, x, y, and z should be moving in a one dimensional space such that, for any time t, the position of any one of the three bodies relative to a zero position can be specified, a graph of the relative positions of the three bodies against time might appear as follows:



To be able to produce such a graph (or its equivalent), however, is *not* to be able, in general, to produce the *function* which determines why the positions of the 3 bodies graphed against time are as they are. In particular, even though it is possible to graph the positions of 3-or-more mutually interacting bodies against time, it is in general impossible (given the present state of mathematical knowledge) to specify a function by which the relative positions of the bodies are determined. For many utilitarian purposes, however, lack of knowledge of such a function is relatively unimportant, for the presence of a graph of the positions provides sufficient information despite lack of functionality to accomplish the purpose at hand.

The nature, purpose, and effectiveness of musical scores provides a striking example within the world of art of the above. What is required of a musical score is that it provide the *performer* with information sufficient to enable him to produce at the appropriate times the appropriate sounds. The fact that he cannot express the *function* determining the sounds in that order, and that the score does not express it (in precisely the same sense in which the above graph of the positions of the 3 bodies does not express the function determining them), is not a devasting handicap to musical *production* anymore than the lack of knowledge of a general solution to the problem of n-bodies handicaps astrophysicists in their launching of satellites into prescribed orbits, and for the same reason: it is the *composer* of the musical work and the *creator* of the universe who must have been essentially concerned with the *function* determining the positions of the sounds and satellites respectively, not the performers of (or within) the work. (I am, of course, using the phrase 'creator of the universe' as a metaphoric analogue to the subject 'composer' – and in no other sense.)

The assumption that the universe is functionally determined (i.e. has 'unity', in Sciama's reparsing of Mach's principle)[4], is an empirical hypothesis awaiting confirmation like any other. The existence of *effective* approximation techniques provides evidence confirming the assumption. In the same sense, the existence of *effective* musical scores provides evidence confirming the assumption that the elements of a musical work are functionally determined, even though the function cannot be expressed (for exactly the same reasons, I have argued, that the functions determining n-bodies cannot presently be expressed).

But a musical work differs in one important sense from the universe vis a vis our perspective as human beings, for it is possible to say in the former case precisely where the focal point of responsibility for being aware that a function *does* determine a musical work lies, even though that function cannot be expressed by him or anyone else. The responsibility rests on the *composer* of the work. And that it is a *responsibility,* in the sense that the function determining their works exists as an objective against which their works are to be measured even though it cannot be named (i.e. expressed as the subject of sentences), has been consistently maintained by the greatest of composers, Bach thru Beethoven to Stravinsky, in the face of consistently naive, careless, and pretentious arguments against the *reasonableness* of this attitude. I have given my reasons above for holding that the lack of means of expression is mathematical in nature, and hence probably temporary. To those who still wish to maintain that whatever cannot be the subject of sentences cannot be taken seriously (for it really *isn't),* I say: Bach took it seriously *in* the Goldberg Variations, as did Beethoven, et. al., *in* their works, and their testimony, *given the evidence of their works,* strikes me as somewhat more relevant to understanding their works than protests without payoff. To invert a phrase of St. Paul: "Faith without evidence is misunderstanding."

11. I wish to stress the difference between the notion of art and mathematics being presented in this paper and a contrary notion sometimes advanced in similar terminology.

It is sometimes a matter of dispute between rival groups of artists or critics whether or not art *embodies* mathematics. Although one can only guess at the meaning of the word 'embody' when used in such contexts, the polemics in general seem either to be directed at, or to issue from, those individuals who wish to maintain that there is a world of mathematical relationships which exists in some sense apart from the phenomenal world of things, colors, sounds, feelings, etc., and that art-objects differ from things of the phenomenal world in that they are structured so that their patterns of colors, shapes, sounds, etc., become instances of these mathematical relationships.

Perhaps the crucial difference between the above view and that advanced in this paper can be put in the turn of a phrase: on their view, works of art *embody* mathematical relationships; on my view, works of art (like everything else) *are* mathematical relationships.

It will be recalled that 'numbers' (the *subjects* of mathematical propositions in the classical view of mathematics) were defined above as functions of the differentiation-integration function, where the letter 'x' stood for anything (be it colors, sounds, feelings, *or even the function itself)* determined by the function so defined. (The circularity of the definition was noted, and applauded, in passing.) To the extent that colors and sounds can be functioned (in a sense, given in patterns), on this account, they *are* numbers; and hence, in the classical sense, the work of art *is* a mathematical proposition (function).

The difference between the world about us and a work of art is not that the latter embodies mathematical relationships while the former does not, but rather that a work of art exists as functionally determined (though we cannot express the function), while, no matter how broad our sense of the functional determination of the facets of the world about us, there always exists facets which do not seem to be determined by any function. To the extent that the world about us is seen as functionally determined (e.g.,as by Jesus, Aquinas, Newton, Einstein, Weyl, Schrodinger, etc.), it *is* seen as a work of art, and it *is* a mathematical function.

(The curious distinction found in English grammar which permits a work of art, unlike a sentence, to be 'true *to* life' but not 'true *of* life'[5] supports the position taken above, in the sense that: the function determining a work of art is less complex than that of the world (if such there be), although there are *aspects* of the world which can be seen as determined by the same function as the work of art if we choose to neglect the aspects not so determined; the work of art may then be said to be 'true to life'. But this is so *only* because we have chosen to restrict the aspects of the world under consideration. There is no similar restriction on the aspects of the world in saying 'This pencil is black', for the procedure of *designating* and *classifying* does not entail that I consider the world as bounded by the measuring device of the proposition I assert. That is, I do not thereby disregard other aspects of the world by asserting a true (of life) proposition; I simply don't assert them. A work of art, in contrast, is true (to life) only to the extent that the world is seen (or felt) as being exhaustively determined (i.e.,bounded) by the same function which determines the work of art.)

12. The composer of music, or any work of art, cannot be said to *know* the function which determines his work of art, for the function cannot be expressed, only *sensed.* It is for this reason that the training of a composer consists essentially in two activities, neither one of which is equivalent to acquiring knowledge of the sort detailed in textbooks on harmony, counterpoint, etc.

(a) Examining compositions of other composers; and

(b) Composing works of his own.

Theories of harmony and counterpoint maybe useful to the composer, not because they can provide him with a knowledge of the functions of his art (for no such knowledge is mathematically possible at present), but rather because they may provide him (1) with a rich and familiar vocabulary in which to couch the insights into function (i.e.,awareness of function) which he derives from an examination of the compositions of others and himself, and (2) because they often provide him with sources of examples and exercises thru which he can develop his functional sense.

There are two principal dangers in speaking as I have of 'awareness', 'insight', 'functional sense', etc. Firstly, there is a tendency among some to equate acting upon awareness with acting *irrationally,* in the sense of acting emotionally (i.e. by force of habit) or non-cognitively (i.e.,without regard to evidence). To act upon awareness, however, is not to act irrationally but to act *non-rationally* (i.e.,to act 'unratioed', in the classical Greek and mathematical sense – to act without knowing of any general premise from which the particular act can be deduced). Neither Bach nor Einstein acted irrationally in composing their thoughts and works, though both surely acted non-rationally.

Secondly, and closely tied to the first, there is a tendency among some to equate acting upon awareness with acting without preparation (i.e. without time-consuming methods of evaluation and measurement, or without critical judgment). The notebooks of the greatest artists refute the contention that this is, in fact, how composers (not now referring only to composers of music) have worked. And the fact that the greatest composers have been *students* of composers, and hence that the craft not only can but must be *learned,* refutes the contention that functional insight is not the product of extreme effort, perseverance, and work. Newton did not discover the laws of gravitation by being hit in the head by an apple, though he may have been hit in the head by an apple *while* discovering the laws of gravitation. At that moment, any number of events would have triggered the crucial insight; without the preparation, none would have.

Although a composer cannot hope for a notational system which will express the *function* determining his work of art (for such a function can only be sensed at present, not expressed), is it the case that some methods of creation are more likely than others to make evident to the composer whether or not the work in progress is being functionally determined, and to what degree? If the evidence of teaching methods of the great artists is an acceptable clue, the answer seems to be 'yes'. Painters often teach their students to block-out sections of the canvas, or turn the canvas upside-down, to destroy natural prejudices of vision which may be hindering the functional sense. Musical composition majors, exercising in counterpoint or harmony, are often told to check each voice against the others individually so that any discrepancy from the functional norm (here specified by rules) can be noted and corrected immediately. The point of these exercises is not to teach the student rules, but to give him practice in *sensing* functionally which will serve him well in later years when no rules can explicitly be formulated.

Diverse as the teaching methods of the arts may seem to be at first glance, all are engaged in articulating a simple admonition to their students:

(a) Learn to sense which of the elements of your ideas are more functional within particular contexts than others; and

(b) Learn to sense when a weaker element can be replaced by a functionally stronger element in context.

By constantly being forced to compare and contrast elements in context with genuine alternatives, and to judge their functional value, the student is being given practice in sensing functionally.

From this clue it can surely be seen that the most advantageous notational system for a working composer would be one which combines a means of expressing the elements (and hence contexts) of his ideas with maximum clarity with a means of comparing and contrasting elements in context with maximum flexibility. Only within such a system could the ideas of a composer be developed and expanded to their full functional value.

Every major composer has developed his own method of composition, some using notebooks, others scraps of paper, sketch pads, etc. The method I shall outline in Section III of this paper, though peculiar in its details to the problems of film composition, could be extended in its essentials to any other art, and seems to me to possess advantages of clarity and flexibility to the composer far beyond the methods indicated above.

But first a necessary digression awaits us, for as yet the elemental functions of the cinema have not been discussed.

Section II: On the limits of Cinematic Space

1. As indicated above, to be in order (i.e.,to be ordered) is to be determined by a function from a given function taken as defining what 'order' is to mean. To be random, in contrast, is to be unordered by any specified function relative to a given function taken as the standard of order.

For ease of speaking informally, let me use a noun-word to designate any function taken as *given* (i.e.,as the standard of order) in virtue of which the ordering of other series may be functionally determined. Such a function, which I shall call a *space*, is taken as given in the sense that it is considered as undetermined by any specified function, and hence is the standard of order for all other functions. To paraphrase T. S. Eliot, a *space* is where one starts from.

This section is concerned with examining a particular space and the functions determinable with respect to it: *the cinematic space.* Just as numbers were determined above as functions of the differentiation-integration space, so the concepts which usually are discussed in film theory (color, shape, motion, dramaturgy, rhythm, etc.) must be functions determinable with respect to the basic cinematic function. (Just as numbers were determined to be those functions which were determined by the addition and multiplication functions, which in turn were determined by the space of differentiation-integration, so the aspects of contemporary theory of the cinema are functions of functions . . . . of the cinematic space.)

The notion of the cinematic space cannot be expressed non-circularly anymore than the notion of the differentiation-integration space. As I have argued, the tendency to attempt to *define* the latter in terms of numbers was misconstrued, for it is the latter in virtue of which numbers ought to be defined. However, this is not to say that one *learns* of the differentiation-integration space prior to learning of numbers; one learns of one in learning of the other. (Although to say that 'learning elementary arithmetic is learning the calculus' may seem strange, the strangeness seems to me to arise from a misconception of what numbers and the calculus *are.*) Similarly, one learns of the cinematic space while learning about colors and sounds and movements, etc., for it is by exercising with these elements that one develops one's *functional sense* relative to the cinematic space.

Thus, just as the elements of arithmetic are numbers defined as functions of the differentiation-integration space, and just as the budding arithmetician develops his *functional sense* by playing with numbers, so the elements of a film are color-relations and sound-relations defined as functions of functions of . . . the cinematic space, and the budding cineaste develops his functional sense by playing with color-relations and sound-relations.

(I shall assume without discussion that color-relations and sound-relations are distinct in the sense that, although a functionally determined color-relation would be modified by the substitution of one color for another within any context, it makes no *sense* to speak of a functionally determined color-relation being modified by the substitution of one *sound* for another within a context functionally determining both colors and sounds.)

Colors and sounds (the terms will often henceforth be used as abbreviations for polyadic color-relations (color-functions) and polyadic sound-relations (sound-functions)) can be determined by an unlimited number of functions of functions of . . . the cinematic space, just as numbers themselves can be determined by innumerable functions. In the history of the art, certain of these functions (of functions, etc.) have tended to predominate for one reason or another, to the extent that fairly significant *names* have been given to these families of functions of colors and sounds: 'dramatic', 'documentary', 'comic', 'poetic', etc. A dramatic, or documentary, etc., film is a film having a certain kind of form; that is, it is a combination of colors and sounds determined by a certain familiar function (of functions of . . .) of the cinematic space rather than some others. By playing with colors and sounds, and arranging them so that they are determined by one such family of functions rather than others, the student develops his *functional sense* relative to this family of functions of the cinematic space. And, in a society where it is profitable to make films *only* in accordance with one or another of these families, there is much to be said for developing predominantly this sort of sense in the student.

But the fact that these families of functions constitute just one part of the functional spectrum of the cinematic space leads me to ask: what are the limits on colors and sounds in terms of the cinematic space in general? That is, what are the *limits* on the functional possibilities of the cinematic space?

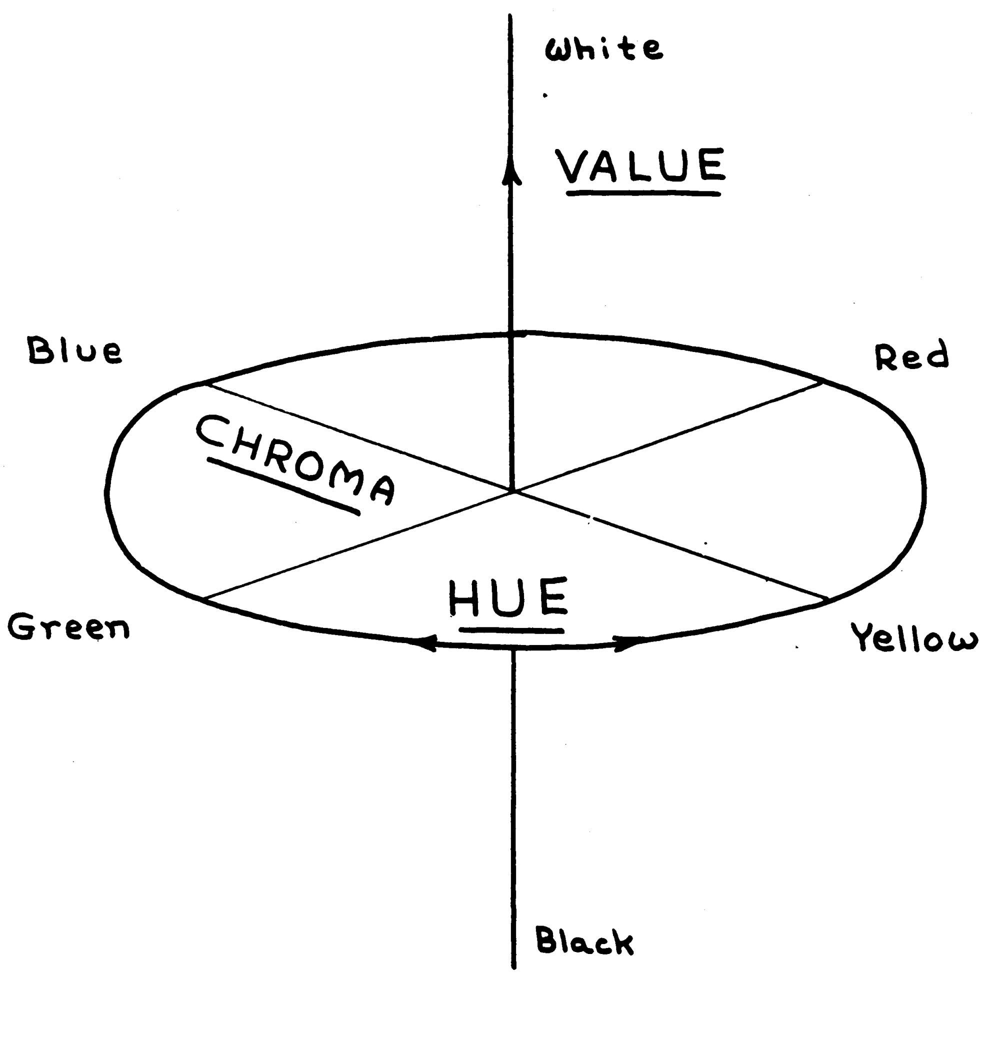
It is at this point that the analogy between the cinematic space and the differentiation-integration space becomes useless. For if everything is a number, as argued above, than the cinematic space itself is but a function of functions of . . . the differentiation-integration space. That no limits can be set upon the differentiation-integration space is apparent, for there is nothing apart from itself in virtue of which it cannot be measured: it is the fundamental standard of order in virtue of which all functions are to be judged, including itself. But, although the cinematic space exists only as it determines and is determined by *its* functions (like the differentiation-integration space), if it could none-the-less be taken, not as a space, but as the function of *another* space whose limits we either know or take as given, its own limits could then be measured (i.e., shown) functionally.

And there seems adequate evidence that the cinematic space need not be taken as a space, but can be evaluated (i.e., taken as a function determined by) the space of *experimental psychology*. Thus, it is likely that certain limitations of the cinematic function can be recognized by considering the known limits of the space of experimental psychology. I shall consider the limitations on the cinematic functioning of colors and sounds in the following discussion in this sense.

The contemporary viewpoint of experimental psychology on perceptual colors and sounds may be summarized as follows:

A. Colors.

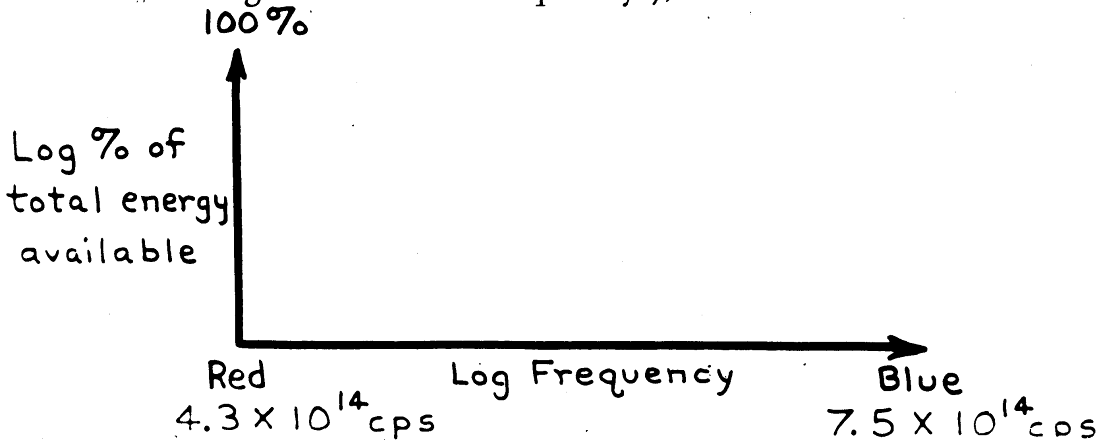
As the work of Munsell, Ostwald, and others have shown, all perceptual color-functions can be expressed as functions of points in a 3-dimensional color solid having *value* as its vertical dimension, *chroma* as its radial dimension, and *hue* as its circumferential dimension.



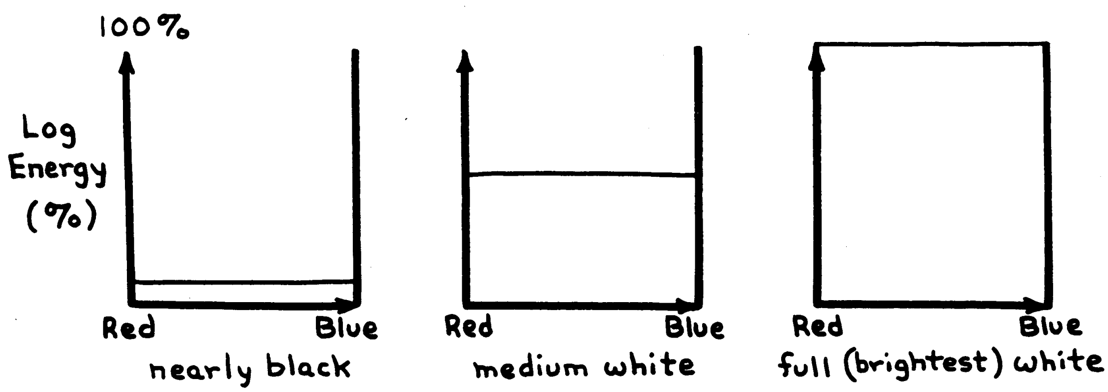
Hue is that quality of difference in color-points which we usually designate by labeling one 'red' rather than 'yellow' or 'green', etc. *Chroma* is that quality of difference in color-points which we usually designate by calling one 'purer' or 'more vivid' or 'more saturated' than the other. (As is evident from the diagram, chroma is a measure of a color-point's distance from the achromatic axis of the greys, a measure of its chromaticity or relative chromatic-ness.) *Value* is that quality of difference in color-points which we usually designate by calling one 'brighter' or 'less dark' than another (in Ostwald's phrase, a measure of how much 'black' or 'white' a color-point contains).

There is an intrinsic metric in the perceptual color solid in this sense: The hues located diametric to each other in the solid are *complementary* in that, when mixed in proper proportions on a Maxwell color-wheel rotating rapidly, they blend to achromatic grey. Given any complementary pair of hues as measuring points, it is always possible to determine perceptually whether or not two color-points chosen at random are on the same or opposite sides of the hue circle as divided by the complementary pair, to determine to which member of the complementary pair each color-point is closest, and also, if both are on the same side, to determine which is closer to either member of the complementary pair. Similarly, given any plane perpendicular to the *value* line as a reference, it is always possible to determine perceptually whether the value-planes of two color-points chosen at random are on the same or opposite sides of the value space as divided by the reference plane. Similarly, given any cylinder of points having equal *chroma* as reference, it is always possible to determine perceptually whether the chroma-cylinders of two color-points chosen at random are on the same or opposite sides (inside or outside) of the chroma space as divided by the reference cylinder. (I am, of course, assuming that all differences given above do not fall below the threshold levels of perceptual discrimination.)

If, as was indicated earlier, to be ordered (i.e.,to have a metric) is to be functionally determined, the perceptual color solid must be functionally determined. But by what function? Although traditional mind-matter distinctions would lead us to consider both the human perceiver *and* the physical world as joint determining functions of the color solid, and although the present state of bio-physical research into the nature of perception is inconclusive, the evidence seems to suggest that the preponderance of the function must be determined by the nature of the physical stimulus. At the very least, the human perceiver cannot perceive more metrical *variety* in the color-solid than is made available in the stimulus. (i. e.,the mind cannot create a greater variety in the colors it perceives than there exists in the stimulus.) Indeed, in the case of color (and sound) the evidence suggests a very close isomorphism between the metrical variety of the physical stimulus and that of the color solid. For most practical purposes, therefore, the space of experimental psychology vis a vis color-perception can be taken, not as a space, but as a (logarithmic) function of the *physical space.* In the case of color, if one considers the usual diagram of the electro-magnetic spectrum for visible light (with the horizontal axis converted from the metric of 'wavelength' to that of 'frequency'),

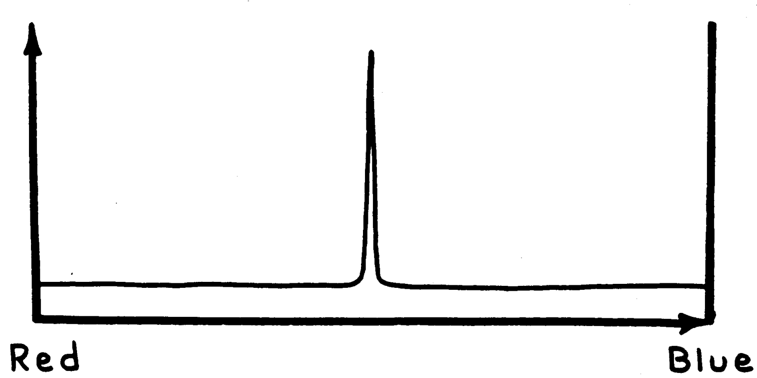


whenever equal amounts of energy are present at all frequencies, the resulting light is seen as achromatic. Thus, diagrammatically, an energy line parallel to the frequency axis indicates an achromatic perceptual stimulus.

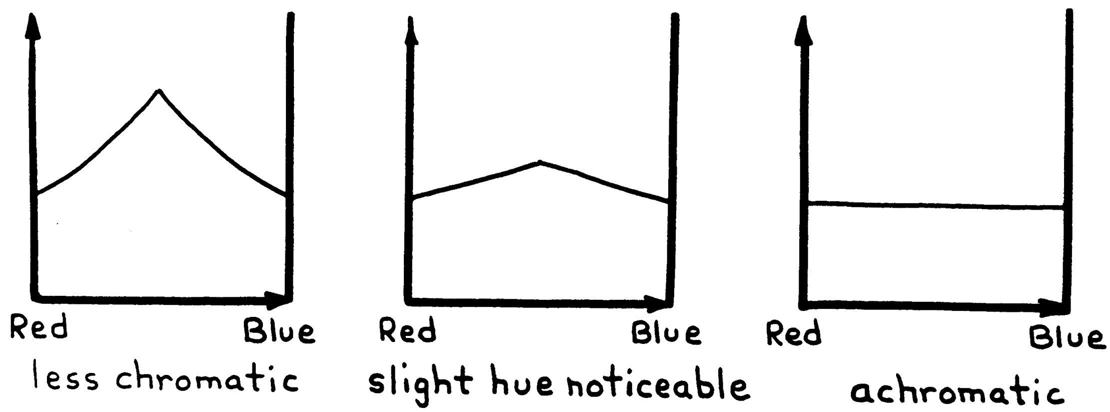


The relative height of an energy line parallel to the frequency axis, therefore, indicates roughly the same perceptual situation as is measured on the *value* scale of the perceptual color-solid.

As the spectrum color at each frequency is, by definition, the most saturated color possible at that frequency, a single vertical energy line at any given frequency would represent a source of the most saturated color possible at that frequency.

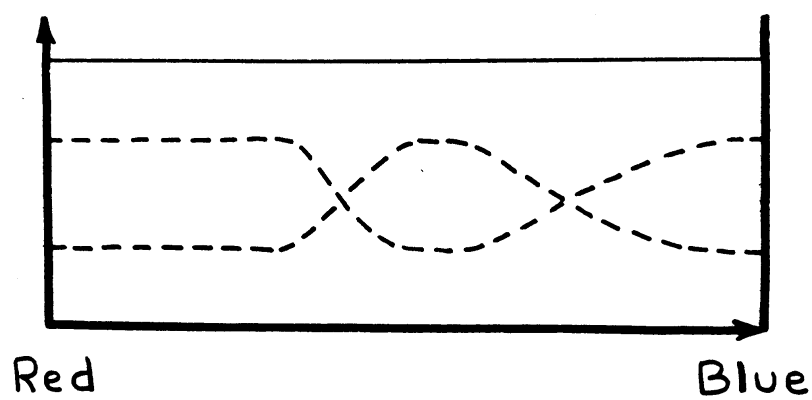


As the energy line which peaks in the vicinity of any given frequency becomes less vertical and more horizontal, the color at that frequency becomes correspondingly less chromatic (i.e.,more achromatic).



Thus, the relative *restrictedness* of the range of *peak* energy indicates roughly the same perceptual situation as is measured on the *chroma* scale of the perceptual color solid.

The relative *position* of the energy peak (or combination of peaks), of course, indicates roughly the same perceptual situation as is measured on the *hue* scale of the perceptual color solid, complementary hues being those whose energy lines, when added together, form a composite energy line parallel to the frequency axis—hence achromatic.

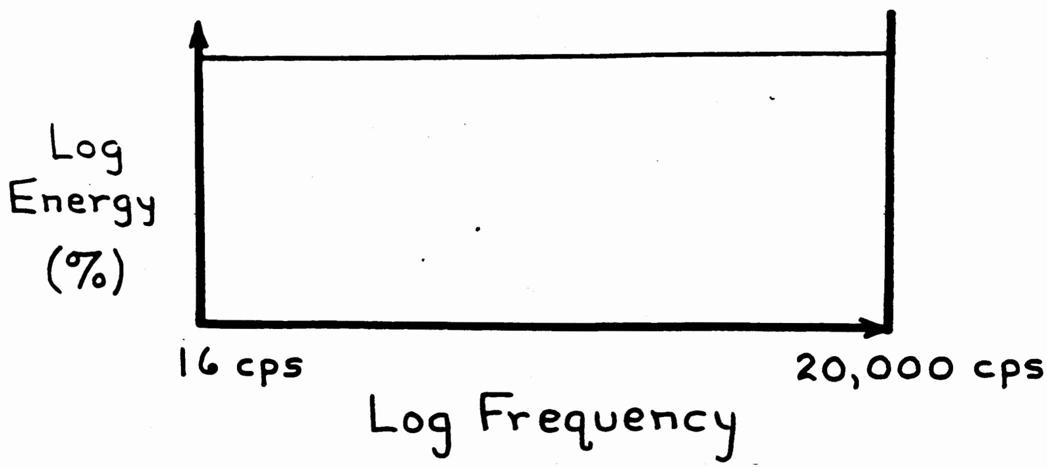


B. Sounds

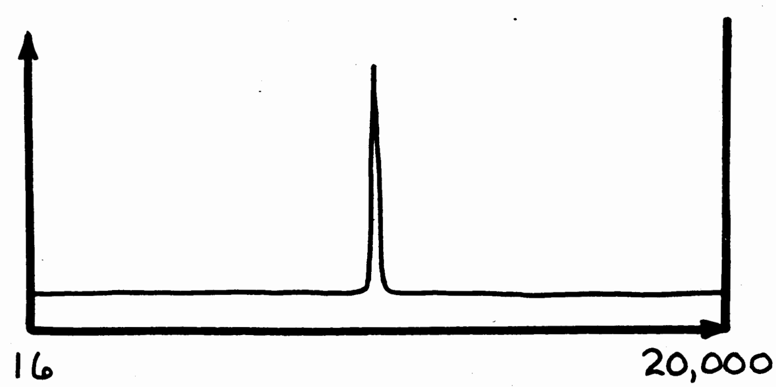
The evidence of a very close isomorphism between the metrical variety of the physical stimulus and that of the color-solid is repeated in the case of sound. That is, for all practical purposes, the space of experimental psychology vis a vis sound perception can be taken, not as a space, but as a (logarithmic) function of *physical space*. Since the notion of a sound-solid has never gained much headway (for reasons which will become apparent below), I shall firstly describe the physical stimulus, and then secondly describe the perceptual correlates.

When the sound spectrum is contrasted with the spectrum of light, one obvious difference is apparent. The *range* of possible frequencies in the spectrum of light is a factor of *less than 2* times the lowest perceivable frequency ($7.5 \times 10^{14} = 4.3 \times 10^{14}$ times < 2 (i.e.,times less than 2)), while that of sound is *more than* 2^{10} times the lowest perceivable frequency ($20{,}000 = 16$ times $> 2^{10}$ (i.e. times more than 2^{10})). This difference is of crucial perceptual importance, and I shall speak more of it below. In other respects, however, the mathematical analyses of the physical stimuli are identical in either case.

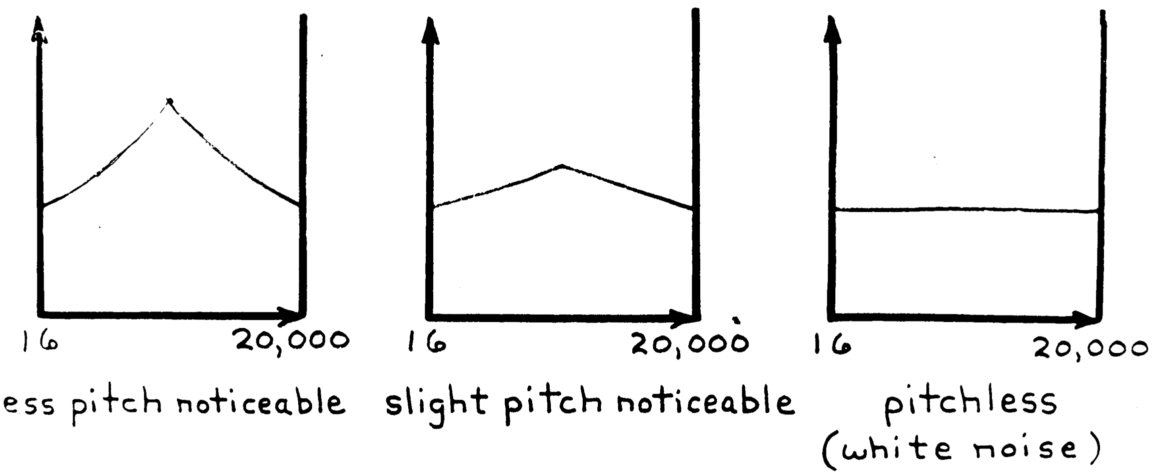
Whenever nearly equal amounts of energy are present at all frequencies, the resulting sound is called 'white noise', and is mathematically equivalent to the achromatic (i.e. white) light of the visible spectrum.



As the spectrum sound at each frequency is, by definition, the purest possible sound at that frequency, a single vertical energy line at any given frequency would represent a source of the purest sound possible at that frequency.



As the energy *peak* in the vicinity of any given frequency becomes less vertical and more horizontal, the sound at that frequency becomes correspondingly less pure (i.e.,more like 'white noise').



The relative position of the energy peak (or combination of peaks) to the frequency axis is also determinable.

Thus, the physical stimulus of perceptual sound provides the basis for *at least* a three-fold metrical variety of perception strictly analogous to that which was specified for the color-solid. And, indeed, corresponding to the scales of *hue, value,* and *chroma* in the color-solid, there would exist the scales of *pitch, loudness,* and *purity* (or clarity of pitch) in the mythical sound-solid.

Sound, however, possesses another scale in virtue of the *range* of its frequency spectrum which light cannot possess. It was mentioned earlier that the range between the lowest and highest frequencies of sound varies by a factor of more than 2^{10}. That is, as I proceed from the lowest to the highest frequency along the spectrum, I encounter at least ten frequencies which are integral multiples of the lowest frequency (i.e.,16 cps times 2, 16 times 3, 16 times 4, etc.), many which are integral multiples of the second lowest frequency (i.e. 17 times 2, 17 times 3, etc.), and so on. As the greater of any one of two of these frequencies contains the lesser as an integral factor, I ought not to be surprised that there exists in the physical stimulus of sound the functional basis for an additional perceptual metrical scale corresponding to nothing in the physical stimulus of light or the perceptual color-solid. That is, since no frequency in the spectrum of light can be an integral factor of any other, but many of the frequencies in the spectrum of sound are integral factors of each other, it is not surprising that the latter are *perceived* to be functionally related in a manner in which the former could not be. Such sets of integrally-multiple frequencies constitute the *tonal* families of pitch, and are the basis, as Helmholtz has shown, for the sensation of *tonality* (very roughly, the 'C'-ness of a pitch, or its 'A # '-ness).

The hearing of sounds, consequently, unlike the seeing of colors, involves perceiving in a dimension of the sound-space which has no correspon-ding scale in the color-solid. The following correspondences hold for both perceptual colors and sounds:

Color-solid	*Sound-solid*
Hue scale	Pitch scale
Value scale	Loudness scale
Chroma scale	Purity scale (i.e. clarity of pitch; lack of noise)

But for the *tonality* scale in the sound-solid, there is no corresponding scale in the color-solid.

. . . .	Tonality scale

Perceptual sound-space, therefore, is four-dimensional, unlike the three-dimensional color-solid, and does not thus lend itself easily to conceptualization as a visual model. For this reason, and because musical composers until recently have been relatively uninterested in the 'purity' scale of perceptual sound (except as conceived pragmatically under the blurry notion of the respective 'timbres' of the various musical instruments), the notion of specifying a precise perceptual sound-solid has proved of little interest. Until this is accomplished, however, the hope of ever orchestrating precisely the sounds of locomotives and the shuffle of feet in films will remain exactly that: a hope.

3. I have presented a somewhat truncated version of the contemporary notions of perceptual color and sound as they are conceived by experimental psychologists, and given a rough indication of their contemporary status as functions of physical space. Now I wish to indicate wherein the above account of perceptual color and sound is, if not wrong, at least sufficiently confusing to warrant clarification.

That the color-solid is *not a* 3-dimensional space in the ordinary geometrical sense of the word becomes readily apparent when it is noticed that the dimension of *hue* has no zero (or center) point. That is, the 3-dimensions of the color-solid are not independent of each other, for if the fact that a perceived color has hue is indicated by 'H*' (and likewise 'C*' and 'V*' for 'has chroma' and 'has value' respectively), and if the fact that a perceived color lacks hue is indicated by 'H-' (and likewise 'C-' and 'V-' for 'has no chroma' and 'has no value'), of the following eight possible combinations of hue, chroma, and value,

1. H*, C*, V*
2. H*, C*, V-
3. H*, C-, V*
4. H*, C-, V-
5. H-, C*, V*
6. H-, C*, V-
7. H-, C-, V*
8. H-, C-, V-

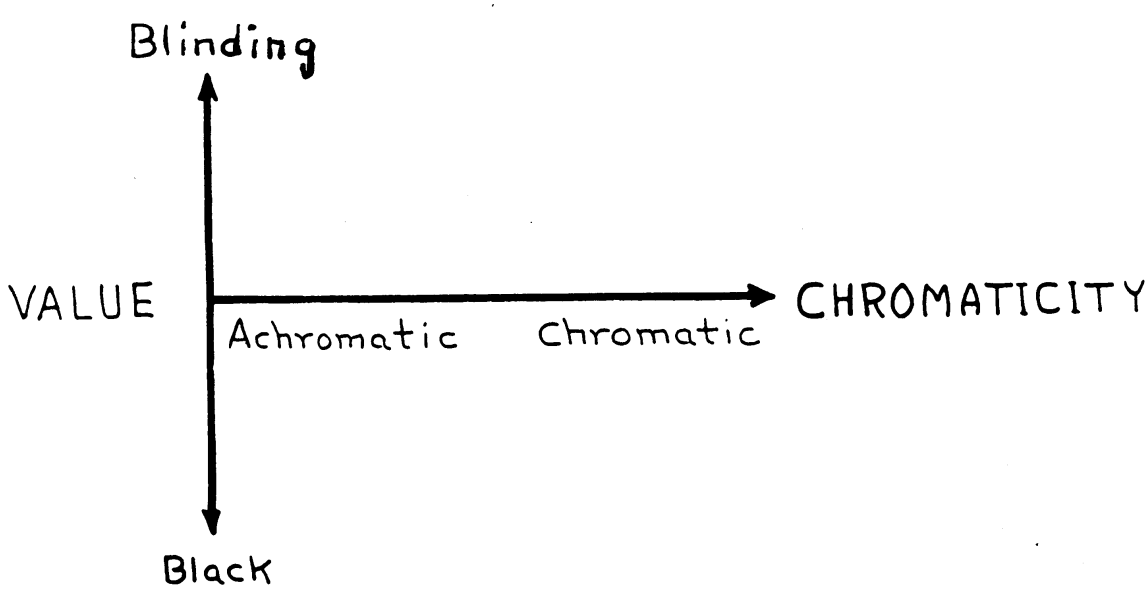
only (1) and (8) indicate possible functions of physical space! To be perceived as a color (i.e. to be visually perceived at all), a stimulus must, therefore,

(a) have Value *; and

(b) have either both Hue * and Chroma *, or both Hue - and Chroma - .

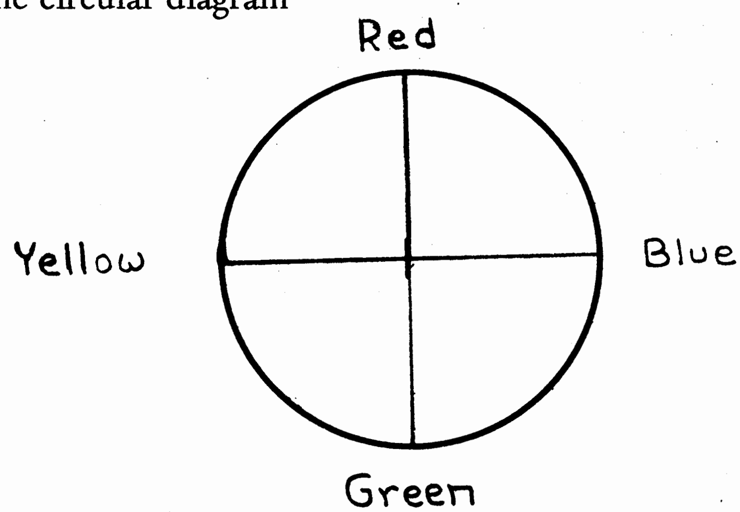
Although any color may be indicated as a point in the color-solid, therefore, the tendency to *construe* the color-solid as consisting of 3 *metrically* independent functions is incorrect. Hue and chroma are metrically **interdependent,** but (jointly) and metrically independent of value, in a sense which is totally obscured by referring to perceptual color as a function of *hue, chroma,* and *value.*

Metrically, perceptual color would more accurately be determined as a function of *value* and *chromaticity* (the latter being understood as a measure of the increasing possibility of *complementarity).*

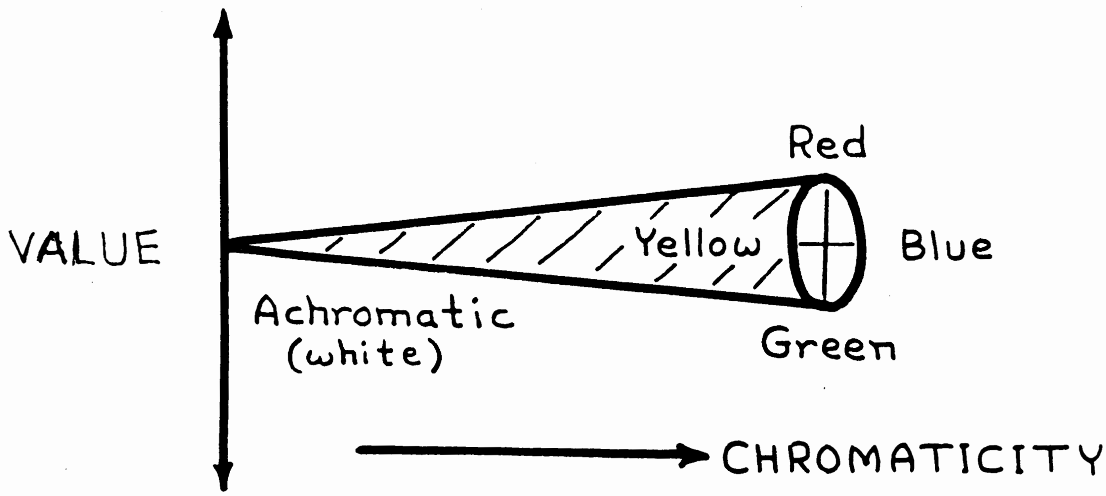


Failure to understand perceptual color in this sense may lead to serious inaccuracies in making choices as an artist. For example, all too many artists have *written* as if they considered whiteness and blackness to be on a metrical par with redness and greeness (regardless of whether or not they allowed this misconception in practice to affect their work). But to say that one color is 'whiter' than another is not to say that it is less black, but that it is less chromatic (i.e.,more achromatic), where 'black' is understood to be the relative absence of energy (relative darkness). Whiteness, thus, is not the absence of blackness, but of chromaticity. To be white is to be achromatic; to be *anywhere* on the scale of chromaticity (from most chromatic to achromatic) is to be necessarily not-black; and to be black is to be necessarily neither chromatic nor achromatic. As a practical example, to contrast the effect of a red-and-black to a green-and-black as color-functions is to judge them metrically as differing in relative *value,* not as differing in relative *chromaticity.* Chromatically speaking, black is as neutral to red as to green.

If the circular diagram



is helpful in remembering what a complementary color function is (i.e. remembering that each complementary function automatically determines the metric of the elements of any other complementary function, in the sense that, given a complementary function, it is always possible perceptually to determine (a) whether or not two arbitrarily chosen elements of other complementary functions (i.e.,two hues) are on the same or opposite sides of the circle as divided by the poles of the given complementary function, (b) to which of the two poles either is closest, and (c), if both are on the same side of the circle, which is closer to either of the two poles), and if it is desirable to incorporate a graphic reminder that *decreasing chromaticity* means less and less relevance for the complementary function in the metrical structure of the perceptual color function, the above *value-chromaticity* diagram might be modified as follows:



Here I have simply shown graphically what the chromaticity axis metrically *means* in terms of perceptual color: to be more chromatic is to be metrically more determined by the complementary function. And the degree of chromaticity of a color *is* metrically independent of its value.

The compositional upshot of the above is that the aesthetic relevance of *hue,* by and large, has been greatly overemphasized by most persons concerned with the subject. It is only when two perceptual colors are of nearly the same chromaticity (i.e. are at approximately the same metrical distance from 'white' as judged relative to a third color) that their metrical relationship vis a vis the complementary function (i.e.,their hue difference) becomes relatively important. Or, put another way, given a perceptual color of a certain chromaticity, the fact that a 3rd color exists at a distance several times farther away on the chromaticity axis than a 2nd color is of much greater importance in sensing the functional color relations holding between the three colors than the fact that the 3rd is red and the first green. (By the same argument, the fact that the 2nd is red relative to the first would be of more importance than the fact that the 3rd is red relative to the first.) Differences in perceptual colors, therefore, are first and foremost determined by the metrical functions of *value* and *chromaticity* (i.e.,distance from 'white'), becoming functions of their hue-relations only as they metrically approach each other along the chromaticity axis.

(It is no accident that most filmmakers, faced with emulsions of limited chromatic sensitivity and the necessities of projection, have resigned themselves to being able to operate only within a narrowly confined segment of the chromaticity axis, and thus to being concerned principally with *hue*-relations. But, note, it is also no accident that the most color-sensitive filmmaker of all, Antonioni, painted the grass and trees green in *Blow-up,* not to change their *hue,* but to increase their *chromaticity* function vis a vis the remaining colors in the shot(s).)

Thruout the above discussion, of course, (in line with the notion of 'function' as presented in Section I) it has been assumed that *hues* are those elements determined by a complementary function, *chromaticities* those elements determined by a chromaticity function, and *values* those elements determined by a value function, all in turn being determined by the (a) color-function. That is, it is *nonsense* to call a color 'red' except it be functionally determined (i.e. related to a second color) by a complementary function which determines the metrical position of the second color relative to red and *its complementary.* To be 'red' is to be *given* as complementary to 'green', and to have established a metrical field in virtue of which the hue of any other color may be measured. It is, thus, *nonsense* to talk of a red-patch apart from the metrical field (i.e. the complementary function) which determines it as 'red' – i.e.,as complementary to 'green', and as existing, thus, in a determined metrical position relative to any other hue. (It is some such sense as this, I think, that the results of Land's experiments will be ultimately explained.)[6]

4. An analysis similar in most respects to that given above for color-functions can also be made for sound-functions, as one might expect.

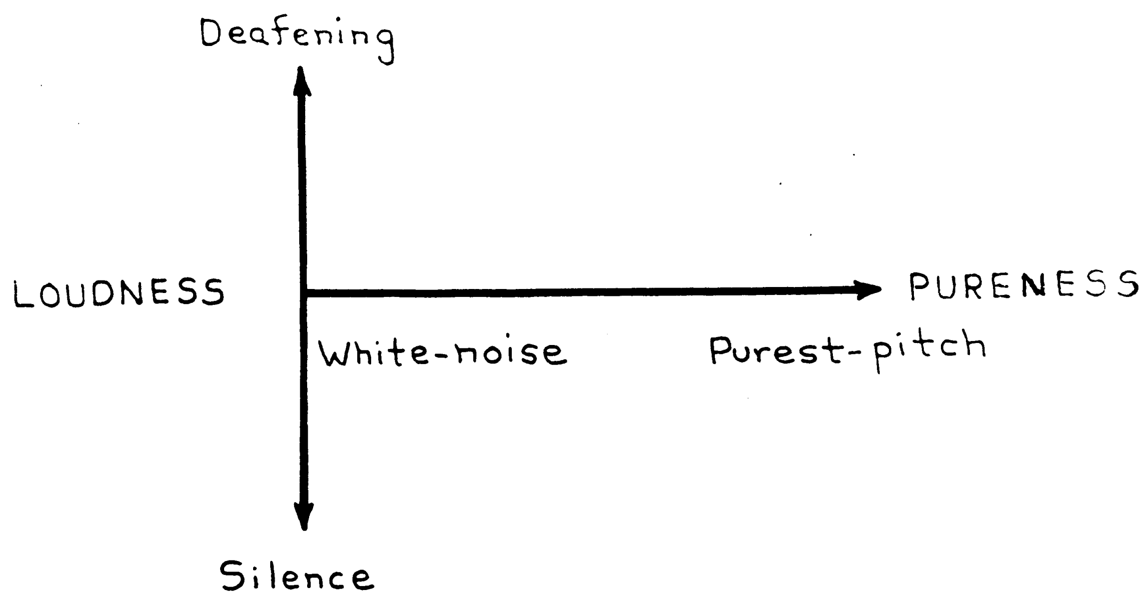
Pitch and *purity* are metrically interdependent, and metrically independent of *loudness* in the same sense as *hue* and *chroma* were interdependent, and independent of *value.* For a sound to be, the stimulus must

(a) have Loudness *; and

(b) have either both Pitch * and Purity *, or both Pitch - and Purity -.

Thus, although any sound could be indicated as a point in a sound-solid previously described, the tendency to construe the sound-solid as consisting of 4 *metrically* independent functions is incorrect. That *tonality* is dependent upon *pitch* I shall assume without further argument. But that *pitch* and *purity* are metrically interdependent, and metrically independent of *loudness,* is obscured by referring to perceptual sound as a function of *pitch, purity,* and *loudness.*

Metrically, perceptual sound would be more accurately determined as a function of *loudness* and *noise* (the latter being understoond as a measure of the increasing possibility of *dissonance;* i.e.,the emergence of pitch from noise).

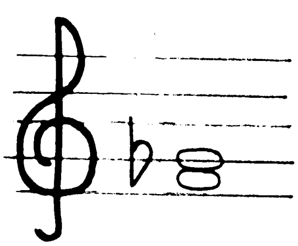


Due to the historical accident that white-noise, until recently, has been a relatively uncommon perceptual occurrence, workers in the field of sound have *not* tended to confuse 'white-noise' with the opposite of 'silence' (as their visual counterparts, 'white light'and the opposite of 'blackness', have been confused), but rather have understood that 'sound', whether noisy or pitch-like, is the opposite of 'silence' and is a precondition for anyone perceiving *anything* on the pureness axis, whether noisy or pitch-like.

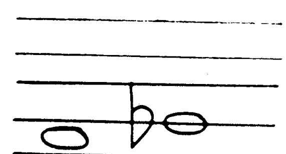
By the same historical accident, men discovered how to build instruments capable of producing relatively pure pitches long before they were able to build instruments capable of producing ranges of relatively unachromatic hues. Indeed, so easy has it been to construct instruments which can produce a variety of controlled and progressive noises (while in the playground of light exactly the reverse has been the case), that the notion of 'complementary noise' has played almost no part in the development of musical traditions, although a mathematically identical physical situation exists for both colors and sounds. That is, two noises whose energy levels would add to white-noise would be *complementary* in exactly the same sense as two colors whose energy levels add to white-light. Due to the historical paucity of controlled-noise instruments in contrast to the abundance of controlled-pitch instruments, complementarity in sound has been considered only relative to its function over that part of the *pureness* axis furthest removed from white-noise; and, indeed, a special term has been traditionally employed to describe the subtle complementary effects which occur in the range of greatest pureness of sound: *dissonance.*

Dissonance, as a physical factor of the sound stimulus (not to be confused with the varieties of *pleasure* which various cultures or individuals may find in stimuli of widely-varying dissonance), has been understood since the time of Helmholtz as being a factor determined by the relation existing between the *overtone* series of two notes. As was mentioned earlier, the spectrum of stimulus sound frequencies varies from lowest to highest by a factor of about 2^{10}, such that there exist many frequencies which are integral multiples of others. When a note is sounded on a musical instrument like the piano, the strings not only vibrate at the fundamental frequency but simultaneously at many of the frequencies which are integral multiples of the fundamental (though not with equal amplitudes at each). These higher frequencies which are integral multiples of the fundamental are called 'overtones', and the family of overtones a 'harmonic series'. (In some instruments, like drums and rods, which vibrate when struck in a variety of frequencies which are *not* integral multiples of the fundamental, the family of frequencies generated constitute a 'non-harmonic' series.)

When two notes of varied fundamental frequencies are struck simultaneously, the resulting two families of overtones may have many frequencies in common (as in the case of A and E), or few (as in the case of A and A#). The more frequencies the two families have in common, the *less dissonant* the resulting sound (and the converse). Thus, A and E (in traditional terms, a 'perfect fifth') are much less dissonant than A and A# (a 'minor second'). It is also apparent, from this anlysis, why the dissonance of the notes of a minor second is greater when the notes are sounded simultaneously than in succession,



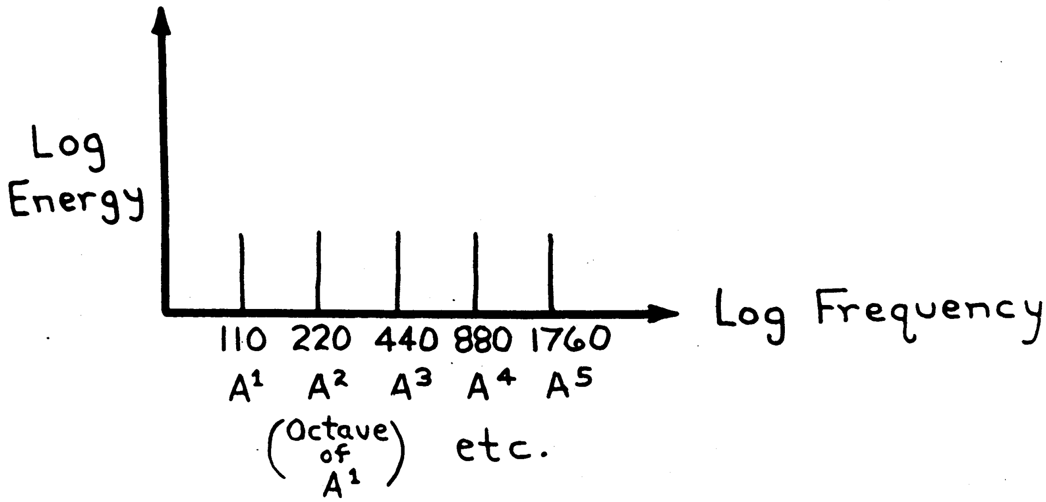
than when sounded



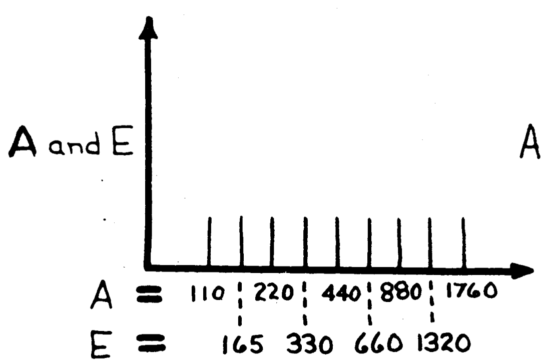
for in the latter instance the family of overtones of the G^b does not begin to sound until the overtone family of the F has begun to diminish in amplitude, and hence the condition necessary for the most direct sound confrontation between the two overtone families has been greatly curtailed.

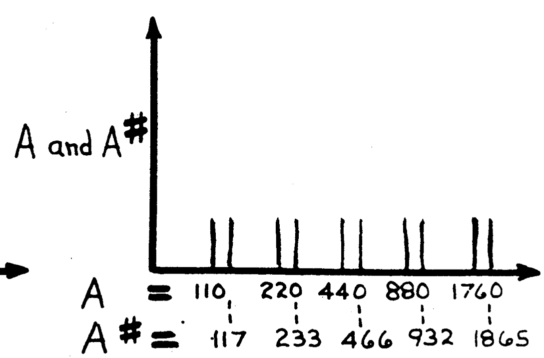
Helmholtz argued, and most have agreed, that the effect of dissonance arises when two overtone families have few frequencies in common because the possibility of two or more of their overtones producing *beats* is then greatest. ('Beats' are the effect which one notices when two strings of a single note on a piano are slightly out-of-tune: a constant fluctuation of amplitude caused by the fact that the energy waves of two *nearly-identical* frequencies sum to produce an energy wave with a frequency too low to be heard as a pitch, but having a noticeable even though very slow fluctuation in amplitude, i.e. a 'wow-wow-wow...etc.'. Piano tuners, and muscians with trained ears in general, are thus able to *tune* their instruments by listening for, and eliminating the conditions giving rise to, beats which occur when playing combinations of notes which ought not to beat audibly.) That this explanation has much to offer when speaking of sounds occuring at that part of the *pureness* axis furthest removed from white-noise, I shall not deny. But the fact that the production of beats is restricted to frequencies which differ from each other by not more than (approximately) 130 cycles per second (the human ear being unable to discriminate beats produced by frequencies further apart than this) seems to limit the effectiveness of the explanation as an account of dissonance in general, though it is surely adequate to describe the *metric* of such acute dissonances as those due to instruments producing families of overtones.

The above conclusion is further strengthened when it is observed that the metric of acute dissonance can be explained, and with it the metric of dissonance in general, as the result of a single determination of the *complementary* function. It was noted above that the overtone families of A and E have more frequencies in common than the families of A and A#, and hence the latter sound more dissonant when sounded together. If the overtone family of A (fundamental frequency: 110 cps) is plotted along the spectrum of log-frequencies of the sound stimulus (assuming equal energies for all indicated frequencies),

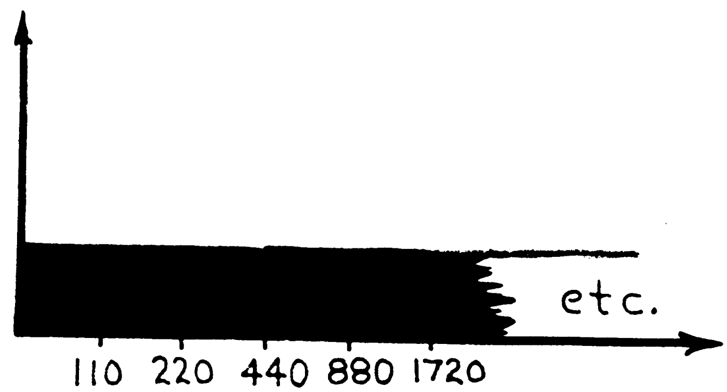


and the result of simultaneously sounding E (fundamental frequency: c. 165 cps), or, alternatively, A# (fundamental frequency: c.116.54 cps) is shown by superimposing their overtone families respectively on that of A,





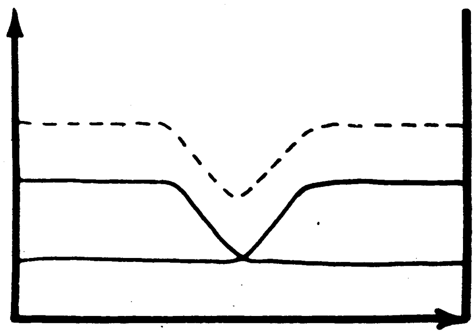
and if it is further recalled that *white-noise* is defined to be that stimulus which results when nearly equal energy is present at all frequencies of the spectrum,



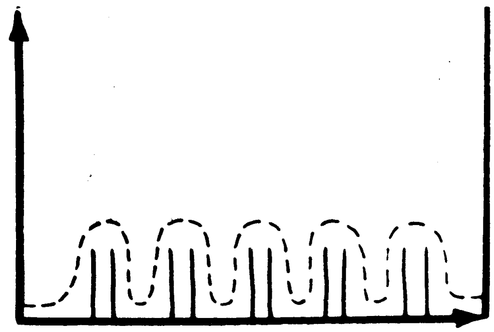
it becomes apparent that the situation determined by each A-A# is simply more like the situation occuring *about each A or A#* in the case of *white-noise* than is the situation determined *about either A or E* by each A-E. Indeed, given any two successive A's, it is impossible to find a pitch which, when sounded between them, produces a situation *less* like that of white-noise than the respective E, for it occurs, log-frequency wise, precisely half-way between the two A's.

A dissonant interval, thus, is simply a *noisier* interval (i.e. one whose overtone relationships are more cluttered about particular frequencies, and thus resemble more closely the situation about those frequencies which would be the result of white-noise). Remembering that the *complementary* function, as discussed above for colors, was a measure of the distance along the chromaticity axis away from the white-light (i.e. visual noise), it becomes apparent that *dissonance* can be explained simply as the effect of the complementary function, now applied to sounds, in those regions of the pureness axis furthest removed from white-noise: i.e. the effect of the complementary function at its subtlest.

Tonality, thus, becomes that factor of sound determined by the complementary function at its subtlest. The sharp boundary between *pitch* and *tone,* always a terminological obscurant, vanishes, and in its place we find that sounds are more or less 'tonal' as they are more or less subtly determined by the complementary function. That is, the difference between the sound of two pitches,



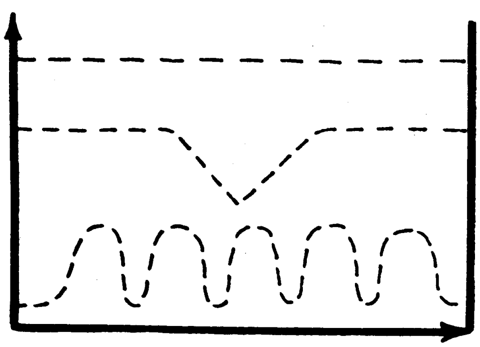
(a)



(b)

when both are noisy (a), or when both are tonal (b), is not a difference in kind, but only in degree: the degree by which they differ respectively from the horizontal energy line defining white-noise, (c),

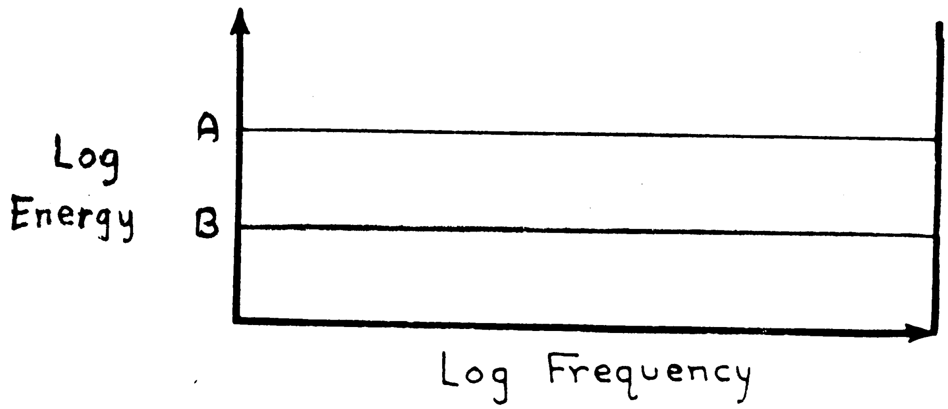
(c)



as determined by the complementary function. Dissonance, thus, is but extreme complementarity.

5. Although 'noise' has been discussed above as designating the white-end of the complementarity axis, in the sense that the less complementary of two colors or sounds is the noisier, in Section III it will be found useful to have extended the meaning of the term such that, of two white noises, the one having the highest energy level may be said to be 'noisier' than the other.

For example, the white-noise A in the following diagram would be said to be 'noisier' than the white-noise B.



(But I shall say more of this in Section III.)

6. Perceptual color and sound, therefore, can each be determined metrically by identical logarithmic functions:

(a) an *energy-level* function (called the 'value' scale for colors, and the 'loudness' scale for sounds); and

(b) an *energy-completion* function (called the 'chromaticity' scale for colors, and the 'pureness' scale for sounds).

By an *energy-level* function, I mean roughly that function which, given any perceptual color or sound as reference, allows us to determine, for any other two perceptual colors or sounds chosen at random, which of the two is more or less bright or loud than the other. Similarly, by an *energy-completion* function, I mean roughly that function which, given any perceptual color or sound as reference, allows us to determine, for any other two perceptual colors or sounds chosen at random, which of the two is more or less *complementary* to the given than the other (i.e.,which of the two when summed with the given, would come closer to *completing* a horizontal energy line, effecting either white-light or white-noise).

7. What is time, and what is space? The answers can be pointed-to by saying: 'That which is measured by a clock, and that which is measured by a rod of known length'. But what is a clock? The answer can be pointed-to by saying: 'Any periodic function'. *Any* periodic function? Yes; and, as Einstein re-

minded us, since to measure lengths we must *repeatedly* use a rod whose length is taken as given, rods are best considered periodic functions: i.e. spatial *clocks.* Thus, since 'space' and 'time' are nonsense apart from the notion of 'measurement', and spatiality, like temporality, requires clocks to be measured, the notion of 'clock' becomes fundamental. What is important, call it what you will, is that one thing which can be measured by clocks: let's say 'space-time'. But, then again, what is a clock? Apparently, as argued much earlier in this paper, the notion of *function* must be taken as given. A clock is a periodic (i.e. repeating) function.

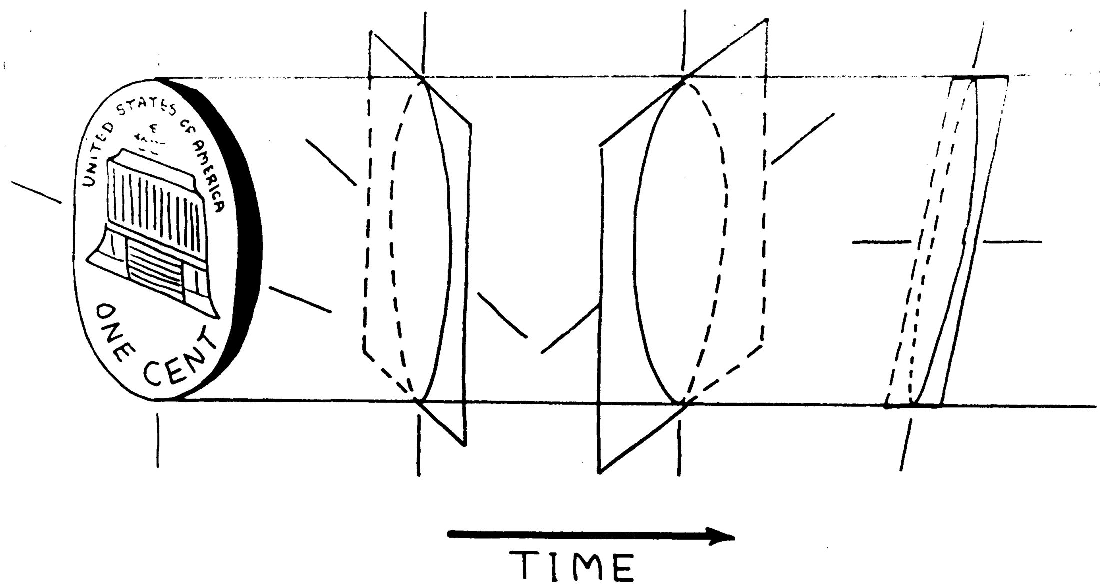
As was also argued earlier, to be ordered is to be determined by a function from a space taken as non-random. Hence, to be ordered in space-time is to be determined by a function from a periodic function (i.e.,to be correlated with a clock).

I am now going to assert something for whose truth I know not how to argue further: an object exists as a work of art if and only if it is determined by a function from a periodic function which it is itself. That is, a work of art must constitute a clock, a repeating function, in virtue of which every element in that work is ordered. Put another way, each element of a work of art must be determined by a function from that periodic function which all the elements are. Or, again, each element of a work of art must be correlated with that clock which they jointly constitute.

The upshot of the above is that every object, to be a work of art, must possess its own *metric,* i.e., its own measurement function in virtue of which every element, and combination of elements, including the work of art itself, must be measurable; and since to be a measurement function,(i.e.,a clock), one must be periodic, an object to be a work of art must be *intrinsically periodic.*

(It is in the above sense that works of art may be said to be 'timeless', for they can be measured, as works of art, only in terms of their own intrinsic metric which, being periodic, has no beginning or end. A Mozart sonata has neither a beginning nor an end any more than a sculpture by Rodin. We enter a gallery to see a work by Rodin, and having viewed it, depart along a corridor . To leave the work is not to have seen the end of the work, but to have isolated ourselves to a particular viewpoint along which the work eventually recedes into imperceptibility. The fact that we approach and leave a Mozart sonata by a different set of corridors, more abruptly, says much about us, but nothing about the sonata. The difference lies in *our* point-of-view and its abruptness, not in the metrics of the respective works themselves.)

That space-time is jointly a function of the *point-of-view* of a measuring device was Einstein's insight. To Newton, the existence of a penny lying on a desk could be represented as a 4-dimensional figure (3 in space and 1 in time),

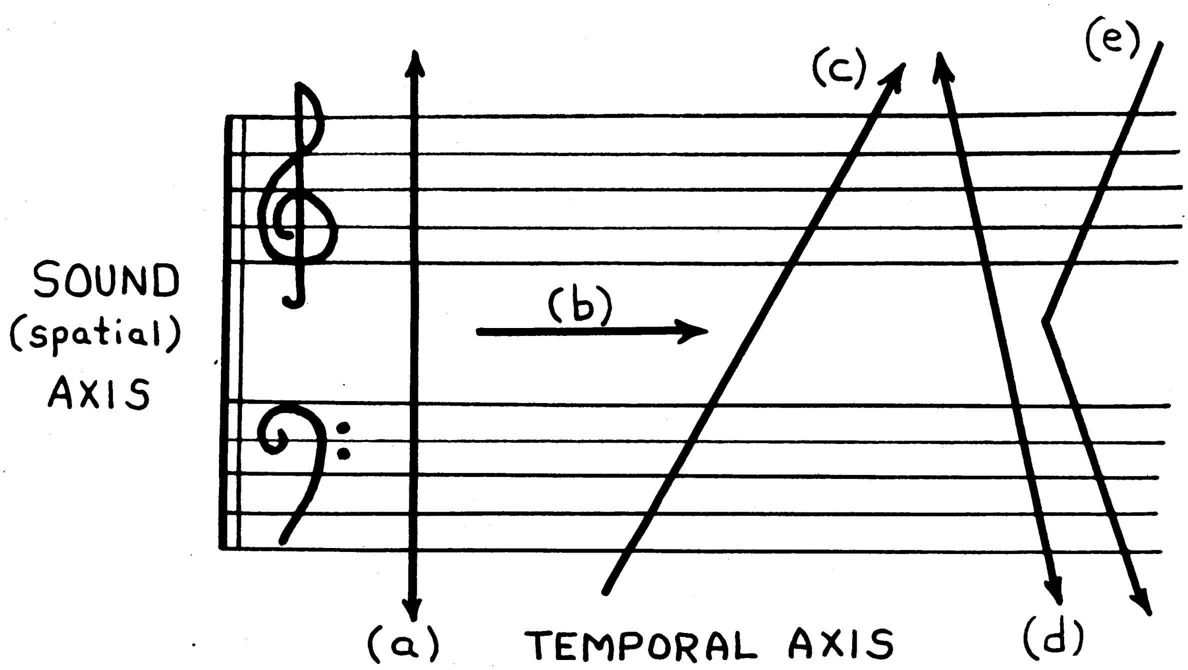


which could be *theoretically* measured from any number of viewpoints (eg. planes, above), all metrically correlatable. But Einstein saw that to change the viewpoint was, in many instances, to change the *metric* of that which was being measured. Indeed, some viewpoints possible in Newton's space and time were simply impossible, for to be in such a position would be to destroy *any* metrical characteristics of the object. Thus, space and time had to be replaced by a new notion, space-time, whose metric could be *determined* by the limits of measurement, i.e., by the clocks in one's possession.

(That I should have spoken above of the metrics of a sculpture and of a sonata as being essentially the same may have seemed peculiar, for, like an air conditioner whose omnipresent sound remains unnoticed, one tends to remain unaware of the acts of measurement upon which one's naive conceptions of time and space have been constructed. It is not until one looks to his acts of measuring themselves to discover what space and time are, that one finds them curiously unlike what one had assumed they must be. Indeed, one finds them not at all, but rather a peculiar amalgam determined by the limits of one's acts of measuring.)

8. What, then, is it like to encouter a work of art (in space-time)? That is, what sort of measuring act is it, what is the nature of its metric, what sort of clock is being used? For example, what does one measure when one hears

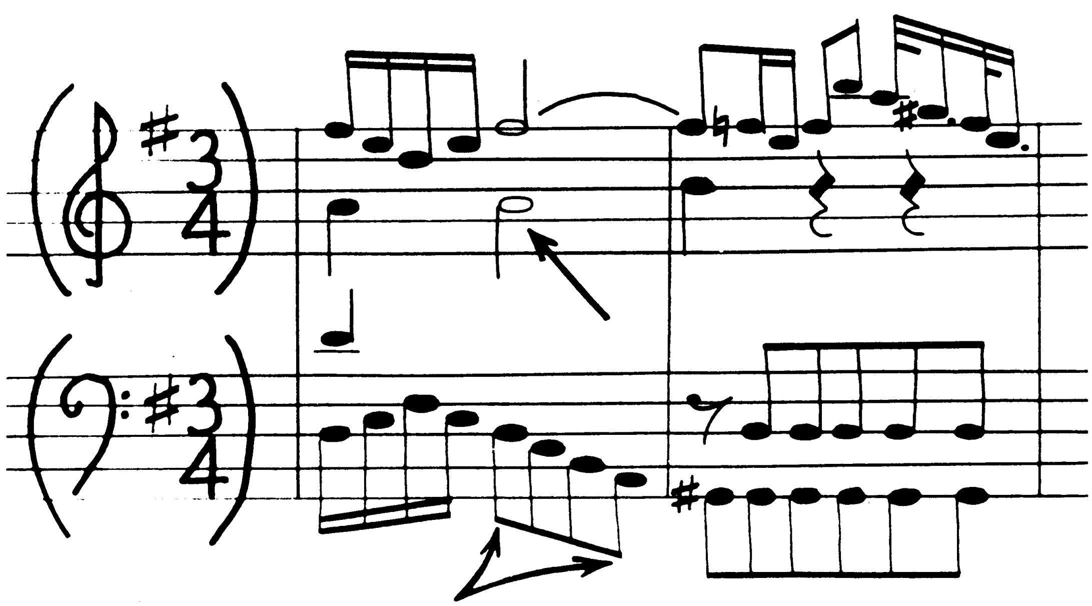
a work of music? By looking at a section of a *score,* one might imagine that one was to measure a 4-dimensional object in Newton's sense, measurable from countlessly different perceptual perspectives (e.g.,lines, below) vis à vis the sound and time axes.



But when one *hears* the piece of which the score is a notation, one can only hear (i.e.,measure) harmonic functions (a) and rhythmic functions (b), and hence the *countrapuntal function (a) and (b)* which determines them. The remaining perspectives are simply not indicative of any points-of-view which could be a *perceptual* possibility. (Notice, one could not sensibly assert, "Yes, in *practice* (a) and (b) are all we can hear, but the rest are perceptual possibilities", for what could be relevantly meant by a 'perceptual possibility' other than something which can *in practice* be a perspective of measuring? By denying that the rest could ever be perspectives of perceptual measurement, one would have denied that they could be *perceptual.*)

Thus, given that the *visual score* of a piece of music consists of marks on paper which can be perceived (i.e.,measured, functioned) from a variety of *visual* perspectives (e.g.,from back to front, bottom to top, sideways, by circles, by triangles, etc.), it is simply a *mistake* to assume that the piece of music can be *aurally* perceived from perspectives corresponding to any or all of the visual ones of its score. The relation of score to musical work is essentially symbolic, and only incidentally perceptual. (And this holds in general for all notational devices. A blue-print needn't *look-like* an engine to be a workable, and perhaps indispensable, aid in making one.) It is the temporal-and-spatial functions of the work of art which must determine whether or not a notated score is accurate, not the perceptual relations of the score which determine what the temporal-and-spatial functions of the work must be. Hence, it is to the perceptual measuring of temporal-and-spatial functions in the works of art themselves that one must turn to understand what sorts of clocks works of art are.

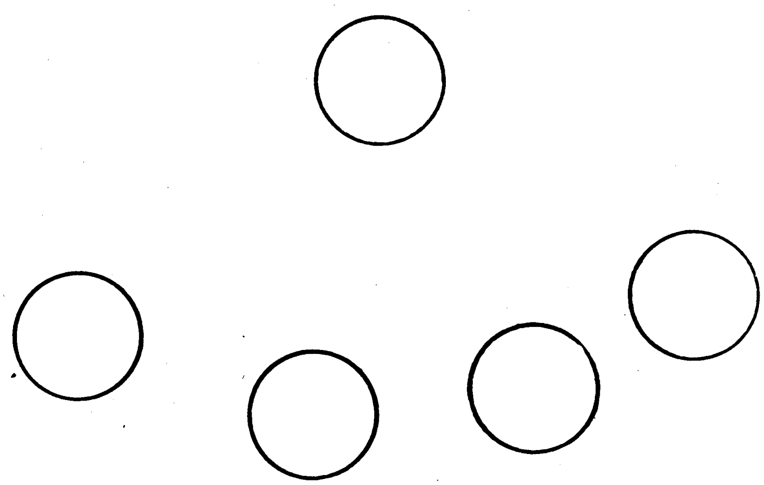
9. Consider, then, the following two bars from the Sarabande of J. S. Bach's Sixth English Suite:[7]



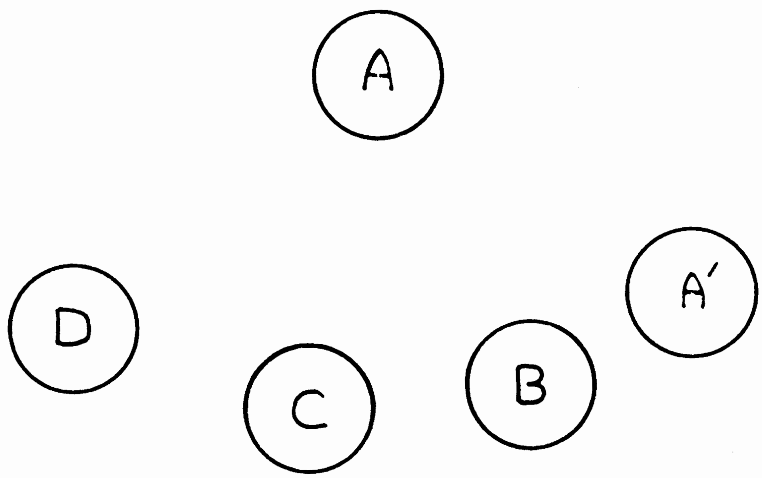
The half-note, A, pointed-to by the single arrow, is correctly said to be equal in *time* to the four eight-notes, low D thru A, pointed-to by the double arrows, for the sound which is the A is heard continuously *while* the sound of low D thru A below are heard in succession (alternating with very brief silences between notes). Thus, given that the four lower sounds are of equal temporal size (i.e. of equal duration), the half-note A can correctly be said to be four times the temporal size of any one of them. Notice what is happening from the perceptual perspective of a hearer: (1) two sounds (A and low D) are heard simultaneously; (2) the first continues to be heard, while the second ceases and a third begins (C) after a very short silence; (3) the first continues, while the third, ceasing when it has sounded as long as the second, gives way after a very short silence to a fourth (B); (4) the first continues, while the fourth, ceasing when it has sounded as long as the third (and second), gives way after a very short silence to a fifth (A′); (5) the first and fifth cease together, after the fifth has sounded as long as the fourth (and third, etc.).

Simply put, of course, something is changing while something else remains unchanged. And that, metrically, is what *time* is all about. But notice, to have thus measured time is to have assumed measurements in sound space (i.e. to have assumed that one knows what 'change from C to B' metrically means). To have measured a change of sounds, however, is to have assumed already measurements in *time* (i.e. to have assumed that one knows what 'change' means). It seems that to measure any object in space-time, i.e. to determine it metrically by a space-time function, is to presuppose a space-time function as given. And yet I have maintained that the metric of a work of art is *intrinsic* in the sense that it can be measured only in terms of itself. Are the two positions at loggerheads, or do they say the same thing?

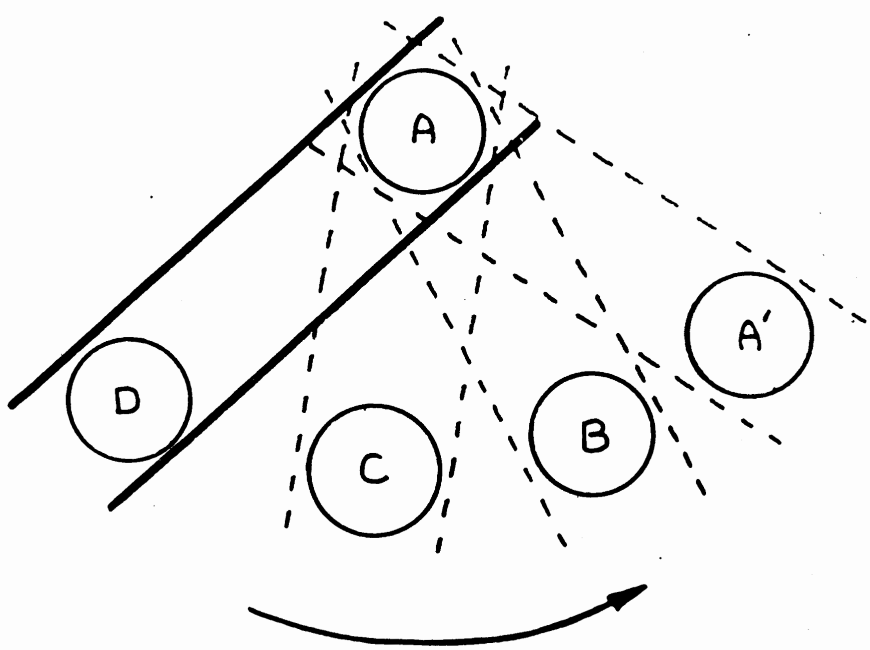
Perhaps an analogy will help to clarify the above. Take the following diagram,



and *construe* the circles to represent the sounds A, and low D, C, B, and A′ as follows:

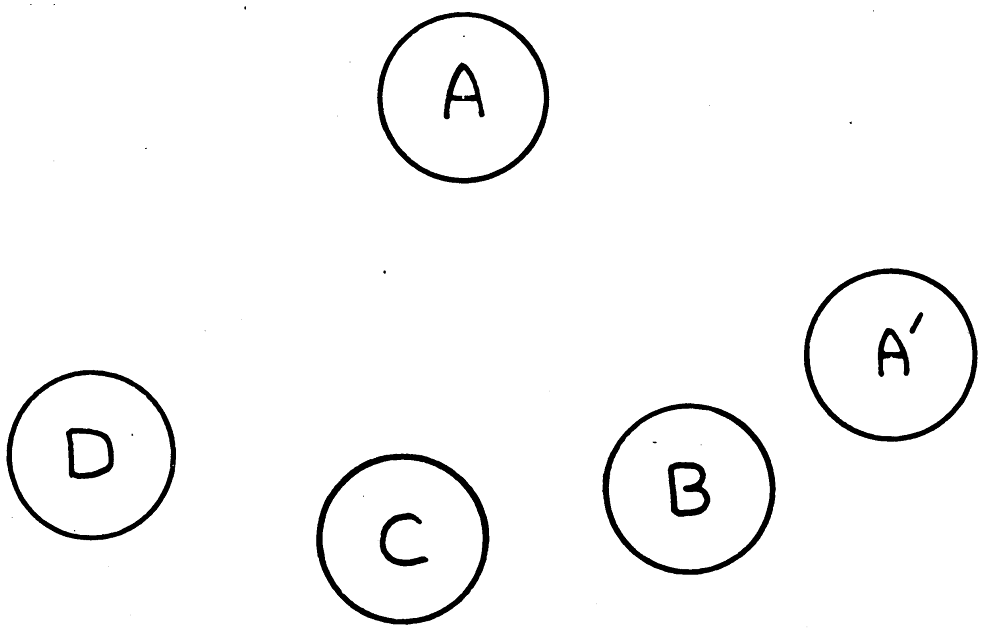


The sounds, although represented by circles and letters on paper which necessarily have an order, are not themselves to be considered as having any order. What is required to order them? Consider a *point-of-view,* which functions like the corridor of parallel lines below.



That is, the two sounds A and D are entertained; then, without A ceasing, D ceases, to be replaced *at the same place in the context of the point-of-view* after a brief silence with C ; then, without A ceasing, . . . etc. The situation described is that which occurs when one hears the passage scored above of Bach's Sixth English Suite.

But notice: it is *not* correct to assume that, because the drawn circles



were necessarily in *visual* order prior to constructing the *point-of-view,* and because the final diagram accurately represents a situation of perceiving sounds, that there exists a situation of non-perceived sounds corresponding to the drawn circles *without the point-of-view lines.* In fact, there does not. (To have assumed that there does would be either to have confused the necessary visual order of *part* of a diagram useful in describing a situation, or to have construed part of the diagram as representing a sound-space which *is* determined by some function, without specifying what the function is; i.e. to have surrepticiously assumed the presence of something akin to the point-of-view lines without drawing them into the diagram.) The drawn circles by themselves are meaningless (i.e. they represent no situation). In this sense, they are incomplete symbols awaiting further diagrammatic assistence before they can properly be taken to represent anything (much as the letters of the alphabet are meaningless until taken in a certain context, i.e., from a certain point-of-view, as words in sentences).

Sounds, therefore, (and colors) cannot be considered apart from a *point-of-view* which determines them as functions of space-time, for to be considered at all is to be represented as ordered by a function, and to be represented as ordered by some function other than space-time is not to be represented as *sounds (or colors).* Hence, although I may in one circumstance refer to a notation of a work of art as representing sound-functions, and in another as representing space-time functions, the notions are equivalent: to represent sound functions is to represent space-time functions. (The original *energy-level* and *energy-completion* diagrams of sounds and colors, thus, must be understood in the same sense as representing functions of space-time.)

10. What sort of clocks, then, are works of art? As determined by sound-functions, or color-functions, they are sound or color clocks, in that their space-time metrics are so determined. The difference between works of art and other objects is that in the former the metrics are *intrinsic,* while in the latter extrinsically determined, in that the metric which is determined by the sound or color clocks of the former exists only within the work of art as a whole, and is relevant to measuring only those sound or color functions by which the work (the clock) is itself determined.

In this regard, the notion of a 'periodic function' deserves closer attention. To change periodically (i.e., to be a clock) is not only to change relative to something which remains unchanged, but necessarily to become over and over again something indistinguishable from what one was before *except for being later.* (Again, circular definitions are the order of my day.) For example, if I take the natural numbers as my space,

$$1, 2, 3, 4, 5, 6, \ldots$$

and determine that series which is ordered from the space by applying the function

$$n - 3k \qquad \text{(where k is any number of the space such that } 3k \leqslant n \leqslant 3\,(k+1))$$

to each number (n) of the space in turn, I obtain the ordered series

$$1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, \ldots$$

The numbers in this series are *periodically determined* in the sense that each term is indistinguishable from those terms which are 3-terms removed from it functionally, except for the fact of being 3-terms removed. (Every third term is said to be 'numerically identical'.) And the series itself, therefore, is a *periodic function,* for each group of three numbers in turn is indistinguishable from the 3-numbered groups immediately preceding and following it.

Suppose I call each 3-numbered group beginning with '1' a 'triad', and, beginning with first, distinguish them from one another in order by associating with each in turn a natural number in order:

$$\underbrace{1,\ 2,\ 3,}_{1}\ \underbrace{1,\ 2,\ 3,}_{2}\ \underbrace{1,\ 2,\ 3,}_{3}\ \underbrace{1,\ 2,\ 3,}_{4}\ \underbrace{1,\ 2,\ 3,}_{5}\ \ldots_{\ \ldots}$$

The ordinary rules of arithmetic can be defined for the new composite series, and if I call the triad associated with the natural number '1' the 'first triad', that with the number '2' the 'second triad', etc., it can easily be seen that the new series constitutes a *clock,* for with it I can measure any series of numbers, random or otherwise. For example, if I place the new series in functional relationship to (e.g.,next to) the series of even numbers thusly,

$$\frac{2,\ 4,\ 6,\ 8,\ 10,\ 12,\ 14,\ 16,\ 18,\ 20,\ 22,\ 24,\ 26,\ \ldots}{\underbrace{1,\ 2,\ 3,}_{1}\ \underbrace{1,\ \ 2,\ \ 3,}_{2}\ \underbrace{1,\ \ 2,\ \ 3,}_{3}\ \underbrace{1,\ \ 2,\ \ 3,}_{4}\ \underbrace{1,\ldots}_{\ldots}}$$

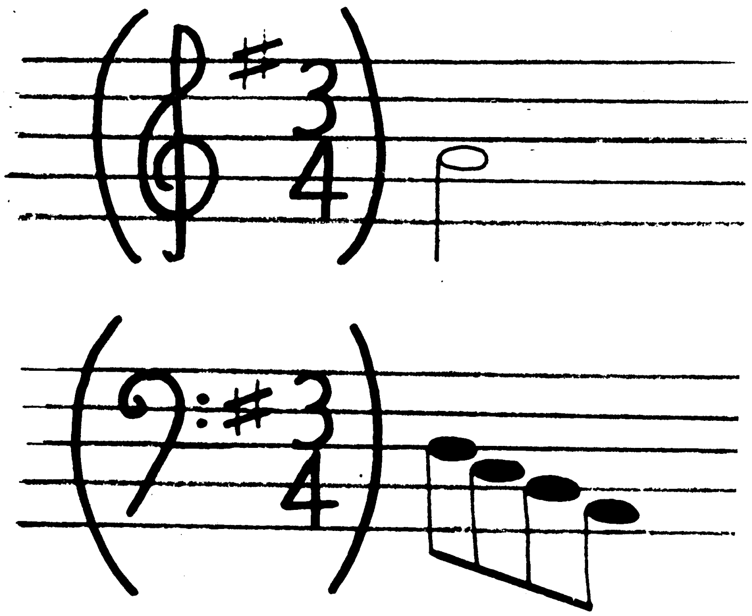
I can say correctly that the even number 8 occurs *two triads* earlier than the even number 20 (or 'to the left of', or anywhere else consistently intelligible).

A similar principle holds for all repeating functions, whether numerical or otherwise. Any repeating function determined from a given space, and capable of being functionally related to another space, constitutes a *clock* in virtue of which the latter can be measured. Thus, any periodic progression of colors or sounds, in particular those within works of art, constitute clocks in virtue of which the whole can be measured. It was pointed-out earlier that polyadic functions can only be sensed, not defined, at present, and hence *periodic* polyadic functions assuredly cannot be defined as were the numerical 'triads' discussed above, only sensed. Yet the notion of periodic progressions of colors and sounds (e.g.,shapes, lines, contours, themes, rhythms) as standards of evaluation and appreciation are present in every discussion of works of art. The point is that they are essential to an object being a work of art, in that they provide the only means by which the work can be measured, and they are intrinsic to it.

A clock, then, is not an object which measures either time or space independently of the other. Rather, it is itself a function of space-time (i.e. in the older Newtonian terms, it exists only within a particular *point-of-view)*, and measures other functions of space-time. Color-functions and sound-functions, then, as clocks, are functions of space-time which measure in turn other functions of space-time: color-functions and sound-functions. (Indeed, within a work of art which consists of both color-functions and sound-functions (e.g.,the film '8½'), it is incorrect to speak of them as distinct except as determined by a color-sound-function in general. In this thesis, however, I shall remain principally concerned with the cinema as a purely visual medium.) A working notation which would be sufficient to indicate color-functions, or sound-functions, therefore, would at the same time be sufficient to indicate the functions of space-time relevant to the work of art in progress, for the functions are equivalent.

(It is in this sense that many of the object-words which I use to speak of periodic color-functions in a film, eg. 'Cary Grant', refer to a certain perceptual pattern of colors which (recognizably) is repeated at intervals in the film, and against whose metric other elements in the film, and the film itself, are to be judged. The figure of Cary Grant in a film, as a periodic color-function, is a *clock,* and therefore a function of space-time. At any given moment, of course, it exists in this capacity vis a vis the other color-functions present. But it also exists as a function of a progressing series of color-functions (and, as was pointed-out earlier, an entire tradition of 'dramatic' terminology has been developed to enable filmmakers concerned only with color-functions such as 'Cary Grant' to notate their working notions in an abbreviated and hence more easily, though less exhaustively, workable fashion).)

11. Thusfar in this Section, color-functions and sound-functions have been shown to be determinable by the same two mathematical functions, and to be functions of perceptual space-time. And, indeed, as functions of space-time, certain sound-functions and color-functions can be formally equated. For instance, as discussed above, the sound-function diagrammed as follows



requires that a particular sound, A, be perceptually present while each of four other sounds, low D thru A', (each of which are determined by an energy-level and an energy-completion function to A and to each other) be made perceptually present in succession. In precisely the same sense, a particular color, β, could be made to be perceptually present while each of four other colors, L, α, π, and ϕ (each of which were determined by energy functions to β, and to each other) could be made perceptually present in succession. (Imagine a film in which a particular color is held constant in some area of the lower left side of the screen, while a succession of four related colors appears in some area of the upper right side of the screen.)

But the above example shows that a singular difference between color-functions and sound-functions, as functions of space-time, also exists. For what corresponds, in the space-time of perceptual sound, to the directions 'lower left' or 'upper right' in the space-time of perceptual color? Apparently there *is* a difference in the metric of the two space-time functions.

And yet one must be careful to decide whether or not this *prima facie* metrical difference is *actual,* for, as I have shown in discussing 'tonality' and 'dissonance' versus 'complementarity' above, at least one other *prima facie* metrical difference between the two functions is actually a difference of degree, not kind, and hence not metrical. In this instance, as with the other, the evidence seems to indicate that the difference is not metrical either.

Composers of music have traditionally been concerned to write for instruments which, in performance, were to remain at a given space-time position vis à vis the members of the audience. It simply would have made no practical sense, in Mozart's time, to write (e.g.,) a concerto for oboe and chamber orchestra in which the sound of the oboe was to move *spatially* relative to that of the orchestra, for neither oboist nor orchestra could have been expected to have much freedom of spatial movement relative to each other and the audience. (There were limited exceptions. Compositions were written to be sung 'antiphonally' by two choirs situated in opposite lofts of the church, for example; and (e.g.,) the Gabrieli's (c. 1510-1610 A.D.) composed works requiring similar spatial distinctness between brass and other instrumental choirs. But the exceptions only prove the rule.)

This is not to say, of course, that spatial sense was neglected. The seating of orchestras (strings grouped together on one side, etc.) was standardized after experimentation on precisely this point. But the possibilities of this sense were curtailed by the practical spatial restrictions on the means of producing sounds available. To have asked for the sound of an organ flying above the trumpets and ending beneath the floor would have been to speak nonsensically.

The advent of recording devices and multiple-speaker systems, however, has changed this picture almost overnight, though few composers for orchestral instruments have recognized the fact pragmatically. Multiple-speaker systems have come to be known as 'stereo' systems, for their ability to reproduce the perceptual space-time sounds of the concert hall. But, as a few composers are imagining, to limit the spatial possibilities of such systems to reproducing the sound relationships deriving from an orchestra of spatially fixed instruments is to stick one's head in the compositional sand. Given a room with a speaker in each of the eight corners; the perceptual sound of an organ playing C# emerging from each of two speakers at diametrically opposite corners of the room *is a different function* (i.e.,a different sound) from the sound of an organ playing C# emerging from either speaker in isolation, or from any other two speakers.

Hence, although the ear has been taken to be much less effective physiologically as a spatial discriminator than the eye (due to its lack of focusing ability), the difference is one of degree, not kind; and one need only recognize wherein lies the difference between hearing a monaural reproduction and a stereo reproduction to become clear on the matter for my purposes. It is apparent, I should think, that sound composers in the future will make intelligible notions like 'lower left' and 'upper right' with regard to their own works, though perhaps not as precisely as visual composers.

The point of the above discussion is that there is *no* metrical difference between the perceptual space-time functions of sound and color. (It should also be clear, however, why it is currently impossible to make a visual transcription of a musical composition in the traditional sense, for such compositions were largely unconcerned with that metrical aspect of their perceptual space-time functions which is of immediate concern to any visual composition. Due to historical accident, the musical composers of the past have had no need to concern themselves with the 'lower left'-ness or 'upper right'-ness of their sound-functions; but it is with precisely this aspect of his color-functions that a composer of colors must be immediately concerned.)

12. What, then, is the metric of perceptual space-time? There are several ways in which (as I have shown) it ought not to be conceived:

(a) It does not consist of four independent dimensions, three spatial and one temporal. For:

(1) no measurement of perceptual space-time can be taken without using clocks which presuppose measurements in *both* space and time (Newtonian sense); and

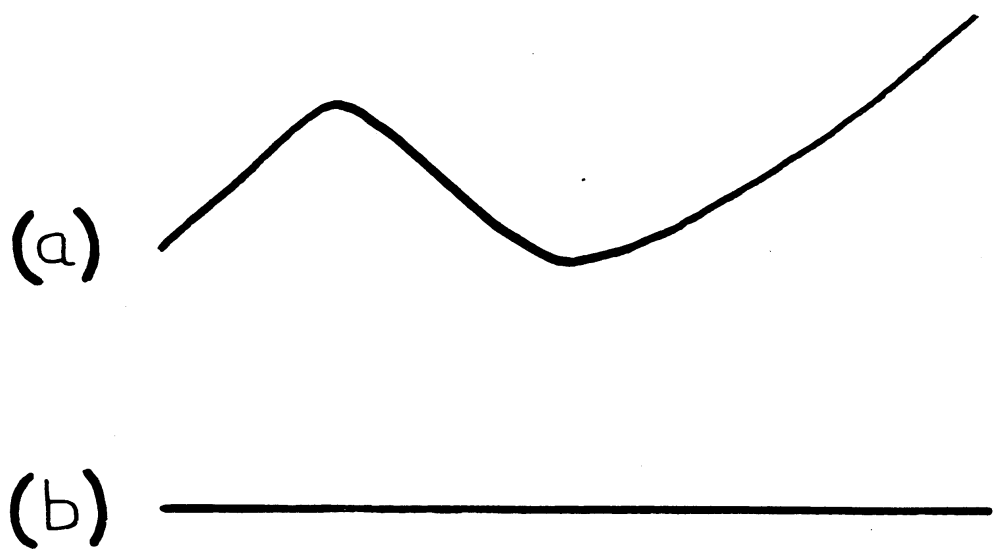
(2) *time* cannot be considered an independent dimension, for, unlike mathematically proper functions, it is irreversible in some curious sense (which I shall not discuss, though the sense of which I shall assume);

(b) It cannot be conceived apart from the existence of clocks which measure it;

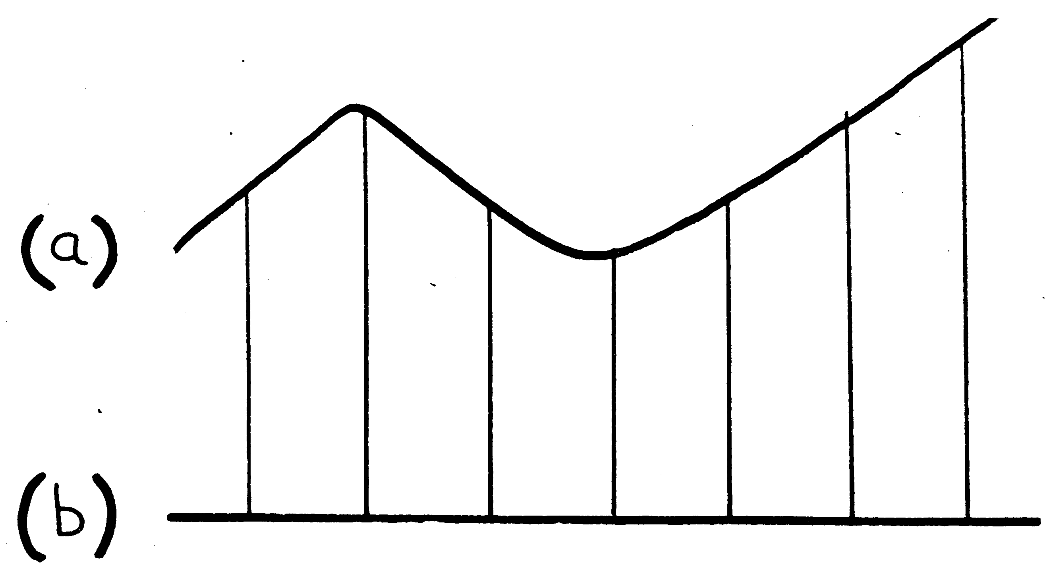
(c) It cannot be mathematically expressed, for it involves at the very least polyadic functions which, as I have shown in Section I, can only be sensed at present. (I have a hunch it depends upon complex polyadic functions akin to those which determine the growth-patterns found in nature, about which I know very little.)

Given (b), however, the working composer in color or sound can know that the metric of perceptual space-time depends essentially upon the existence of *clocks,* and hence that by clarifying his sensitivity to the presence of clocks he is clarifying his sense of the functions of perceptual space-time determining his work. For the last time, then,. what is a clock?

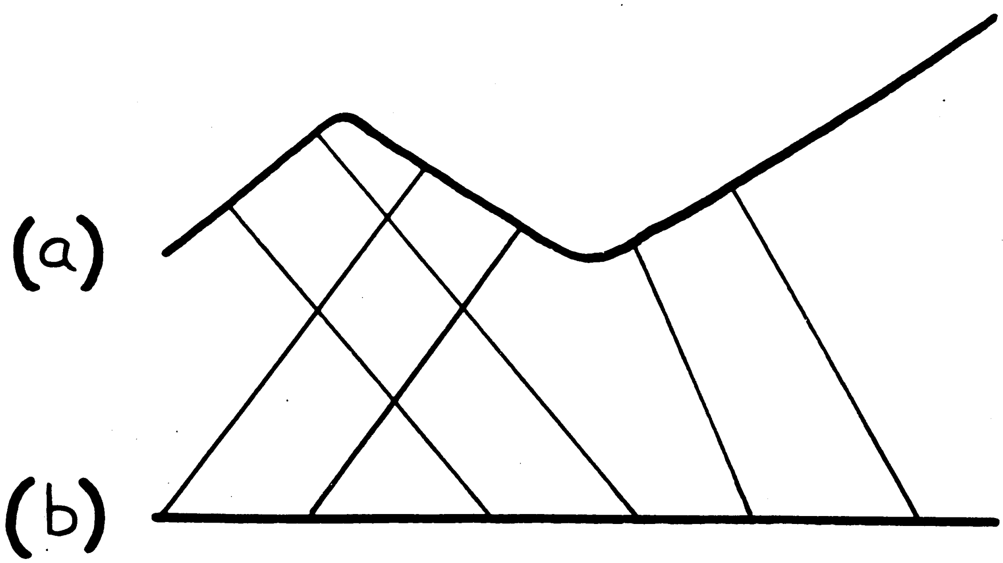
It was shown above that any periodic (i.e.,repeating) function can serve as a clock. But is there some perceptual feature, some earmark (or eyemark) by which a working composer can decide which color or sound functions are indeed periodic? The answer, although expressed earlier by saying that all elements of a work of art must measure and be measured by it, may yet seem surprising: *any continuous function may be considered periodic.* To ask the question is precisely analogous to asking 'When is a line straight?', and the answer can be seen as follows:



Of lines (a) and (b), which is straight? If one should answer 'b', and should then be asked 'But how can you tell?', one might answer that, if (b) were to be divided into equal segments and perpendiculars erected from the end of those segments to line (a), it would be seen that the segments of line (a) were not equal; hence (a) must be curved and (b) straight.



But, by reversing the same procedure, if one had begun with line (a), divided it into equal segments, and erected perpendiculars from it to (b), it would have been seen that the segments of (b) were not equal, hence (b) must be curved and (a) straight.



The point is that it makes sense to speak of one line being straight, or curved, only as it is determined by a function from another line *given as being straight or curved.* And since, for our purposes, a line may be considered straight if and only if it consists of contiguous segments of equal length, such a line is a periodic function. Hence, any continuous function may be taken as periodic (i.e.,as a clock).

The words 'contiguous' and 'continuous' were used above for the first time, though the notions of contiguity and continuity have been assumed thruout this paper. I shall not attempt to clarify their meaning here beyond simply saying that two things are *contiguous* if and only if nothing exists between them. If two things are contiguous, then the function determining them is said to be *continuous* with respect to them. To the working composer, however, the notions must be intuitively grasped with assurance, for they alone determine which of the color-functions (or sound-functions) available to him can be taken as periodic functions of perceptual space-time. That is, it seems a necessary condition of the metric of perceptual space-time, be it what it may, that its functions be continuous. Hence, although the composer is free to take any continuous color-function (or sound-function) as a clock against which to measure other functions, (a) the function he chooses *must be* a continuous function, and (b) it must be taken to be periodic (i.e. to be a function which permits him to measure other continuous functions; that is, it must permit him to operate in a fashion analogous to 'erecting perpendiculars' in the above example of the lines).

The working composer, of course, begins with single functions (e.g. 'themes' in music) and develops his work from these. It is, therefore, imperative that he learn to develop his sense of *periodicity* in these elemental themes, whether visual or aural, so that he may learn to develop other themes from them which, when related contiguously to the others, are determined by richer functions without loss of the elemental periodicity (i.e. rhythm).

12. To an extent, therefore, I think I may now summarize the answer to the question with which this Section began, 'What are the limits on the functional possibilities of cinematic space?', as follows:

All functions of cinematic space:

(1) must be functions of color-functions and sound-functions, which, in turn, can be expressed in terms of two functions:

(a) an energy-level function; and

(b) an energy-completion function;

(2) must be functions of perceptual space-time; hence continuous, and thus measurable by clocks (i.e. other continuous periodic functions).

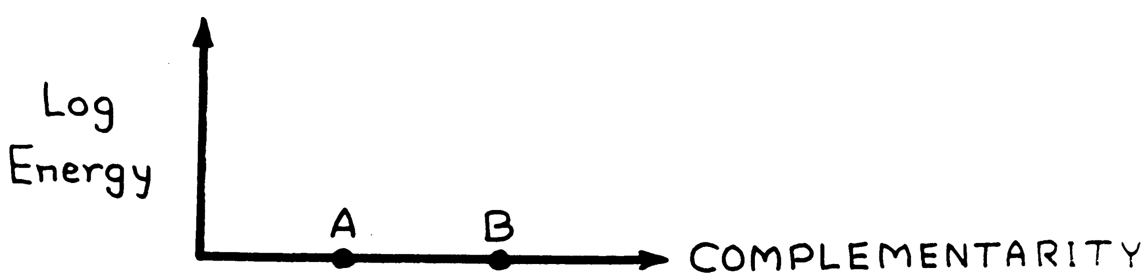
(All other functions of cinematic space (e.g.,dramatic, documentary, etc.) must be expressible as convenient working *abbreviations* of color and sound functions.)

What is needed, therefore, to provide the working composer with a notational means of expressing and developing his elemental ideas according to his finest functional sense, is a procedure which would enable him to combine the maximum degree of clarity in identifying these energy functions with which he begins, with the greatest flexibility in comparing and contrasting (i.e.,measuring) such functions with each other once they were identified. (In the latter sense, it ought also to include a means of easily and accurately abbreviating functions of such functions, so that composers would be free to work systematically on less-finely controlled levels of expression.)

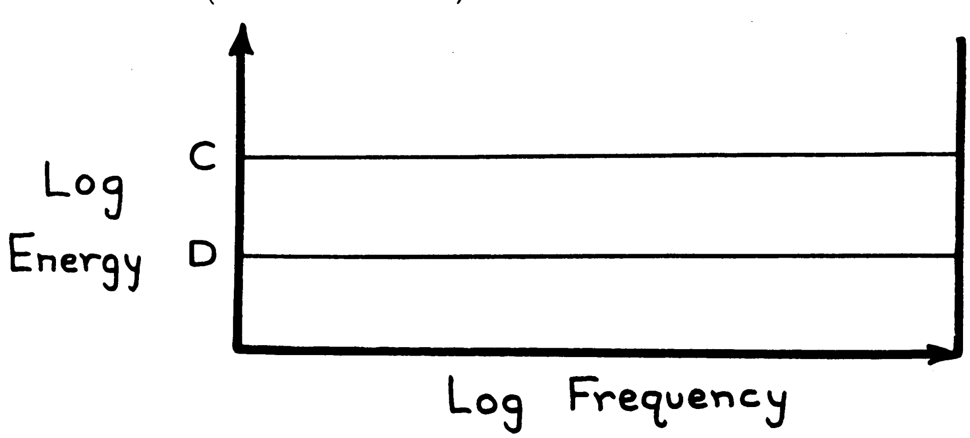
I now turn to the task of describing such a procedure.

1. In this Section of the paper I shall be concerned with describing a workable set of procedures to be used in the composing of a film. I shall *not* be concerned with describing a means whereby a film, so composed, may then be expressed in a notation easily reproducible on the printed page. (This is not to say that the former may not lend itself easily to such a transcription, but only that my concern is with the former as such.)

As indicated in Section II, color and sound functions can each be expressed using identical energy functions. Having used the word 'noisier' to designate that element in an energy function which is closest to the *white* end of the complementarity (i.e.,dissonance) scale for either color or sound, I shall now extend the meaning of the word to designate that white noise which has a higher energy level than another. Thus, not only is color (or sound) A noisier than B,

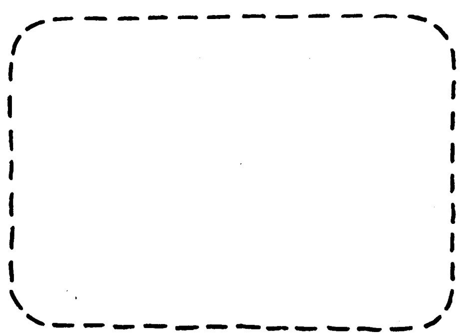


but white noise (sound or color) C is noisier, i.e. more intense, than D.



(That the above two graphs are not incompatible will be evident when it is remembered that the complementarity axis of the former is but an abbreviation for a complex of functions expressible on the latter graph. The latter graph, then, is logically prior.)

2. Consider now a certain space which I shall call the *perceptual field,* corresponding (roughly) to the area-in-time which is contained within the *frame* of a projected film. For purposes of analysis, I shall represent the field as it exists at any given moment by

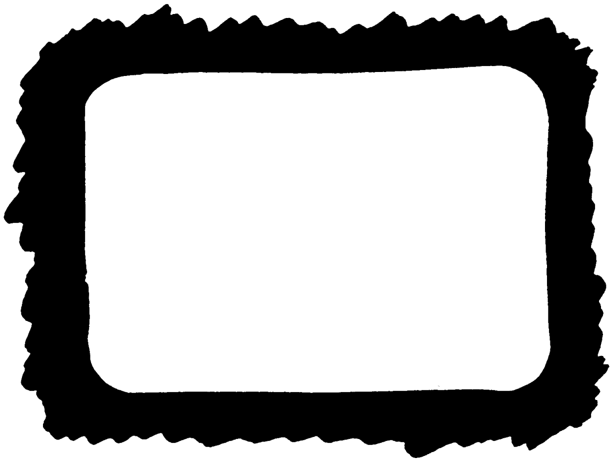


assuming that there is *no* energy present either beyond the boundaries of the frame or within the frame, unless otherwise specified. The frame-line, thus, is only an abstraction which must be filled-in with energy-specifications to become indicative of any perceptual situation whatever.

I shall assume for the remainder of this paper that there is *no* energy forthcoming from *beyond* the boundaries of the frame. As any filmmaker knows, this description does not even approximate normal projection situations; but the exercise of assuming that it does will simplify my discussion, and the means of extending the discussion to situations where energy is present beyond the boundaries should become apparent to anyone interested in pursuing it further.

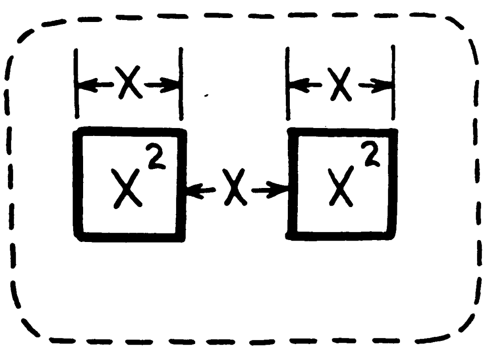
Given the above assumption, the *noisiest* possible situation available to the film-composer is the situation when the projector lamp is simply turned-on and the screen is evenly illuminated. No greater level of energy can fall onto any area of the screen (for the bulb is assumed to be operating at constant and peak efficiency), nor can a greater level of energy fall onto the whole of the screen.

The noisiest possible state of the perceptual field, thus, is when all of it is as white as possible. (That is, whenever the intensity of the light at any point in the field is as high as it can be , and the *sum* of the intensities of the light at all points in the field is as high as it can be.) Diagrammatically, the situation can be described as follows:

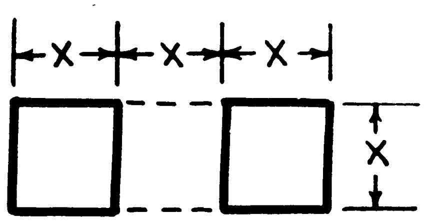


(Although I shall not, in general, shade the area surrounding the field, it ought always to be taken as shaded, i.e.,as providing no energy stimulation whatever, unless otherwise specified.)

Consider now, two areas of equal size x^2 of maximum-intensity white noise situated within the visual field at a distance x from each other.

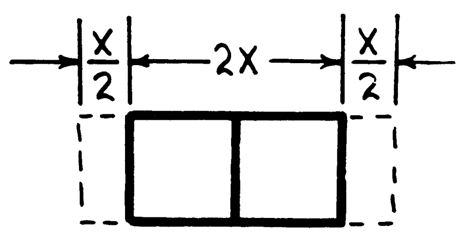


Consider next that rectangle of the visual space bounded by the two areas.



Its area is $3x^2$. The area of the rectangle in which no energy is present is x^2. If I take x to be the fundamental unit of measurement of length in the visual field, and let the maximum intensity of energy available per any unit area x^2 be I, it is apparent that the above situation differs from the noisiest possible situation of the rectangle (i.e.,the situation in which maximum energy is present at all points in the rectangle) in that only 2/3's of the total possible energy is present.

But notice, it differs in yet another way, which can be seen as follows: suppose that the two areas were to move so that they become *adjacent* rather than being separated by a distance x.

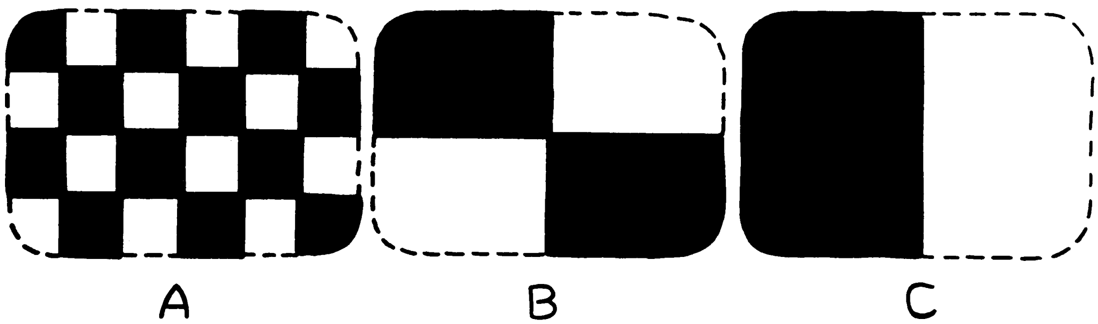


The resulting situation still presents only 2/3's of the total possible energy vis-a-vis the original rectangle. But, whereas in the former situation it would have been impossible to find a *contiguous* area greater than x^2 in size in which the intensity of energy per unit area was identical to that of the rectangle in its noisiest possible state, now there exists a contiguous area $2x^2$ in size which has this property (and an unlimited number of contiguous areas less than $2x^2$ but greater than x^2 in size which have this property). That is, a greater *contiguous* area of the rectangle in the second situation matches in average intensity the noisiest possible situation of the rectangle.

Thus, although the total energy present in the visual field is identical in each instance, the latter situation is *noisier* than the former, for a greater contiguous area of the visual field of the latter is identical in quality (i.e. average intensity of energy present) to that of the noisiest possible state of the entire visual field.

From the above, it should be apparent that, if two objects of maximum intensity white noise were such that the first was greater in *area* than the second, the first would be *noisier* than the second. For suppose the area of the first to be $2x^2$, that of the second to be x^2, and the maximum intensity of energy per unit area x^2 to be I: the first object would contain *twice* the contiguous area identical in quality to that of the noisiest possible state of the visual field.

To re-emphasize again, then, *noise* is not just a measure of the average intensity of energy per unit area of the visual field, but also a measure of the *concentration* of that average intensity of energy. For example, of the following three functions of the visual field,

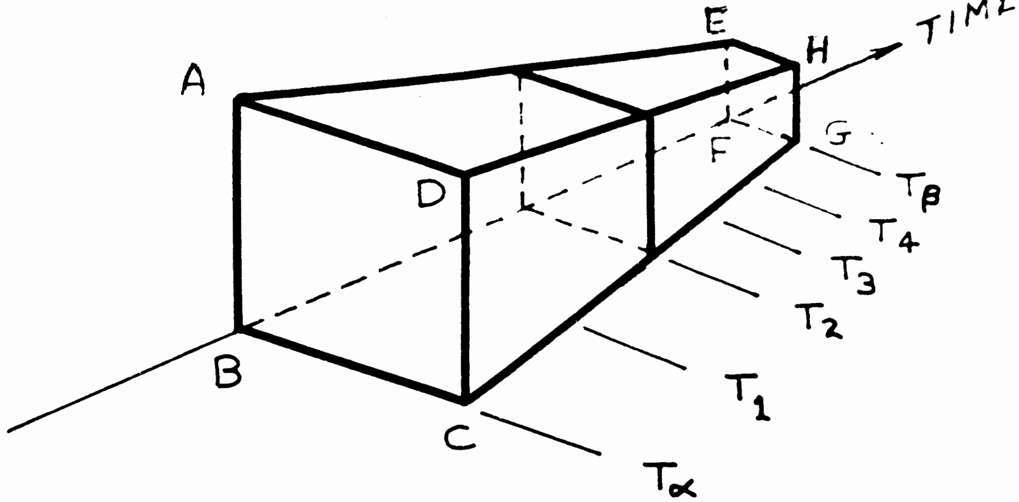


A is less noisy than B, which is less noisy than C, although the average intensity of energy per unit area of the visual field is the same in each instance. The point is that these unit areas of C having the highest average intensities of energy are *concentrated* together, giving C the largest contiguous area identical in quality to that of the noisiest possible state of the visual field.

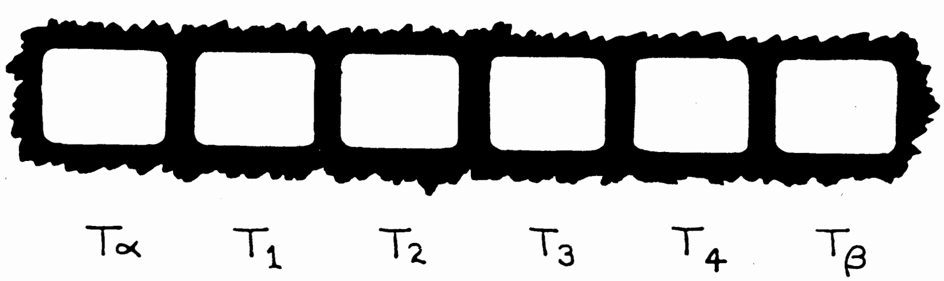
If the visual field contains just two objects of uniform maximum intensity per unit area, therefore, it is in general the case that the larger object is noisier than the smaller, and that the closer the two objects come to each other, the noisier the visual situation becomes. And, although the mathematics is not available to functionally determine accurately cases of more than 3 objects in general, (except under limiting circumstances), a similar conclusion holds for 3 or more objects: (a) the larger the object, the noisier it is; (b) the closer together objects come, the noisier the situation becomes; and (c), if all objects are of equal size, the more objects there are, the noisier the situation is.

3. The above discussion has been concerned with the effect of the visual field at any moment of time. But, of course, the visual field does not exist instantaneously. Rather, it consists of a bounded portion of perceptual space-time. If the restrictions noted in Section II on considering time as distinct from space are kept in mind, it will do no harm to consider time as a unique axis for my present purposes. The noisiest possible state of the visual field for any interval of time $T_\beta - T_\alpha$, then , would be that situation in which the

maximum amount of energy is present at all points in the visual field at all moments from T_α to T_β , as represented by the solid ABCDEFGH in the following figure:



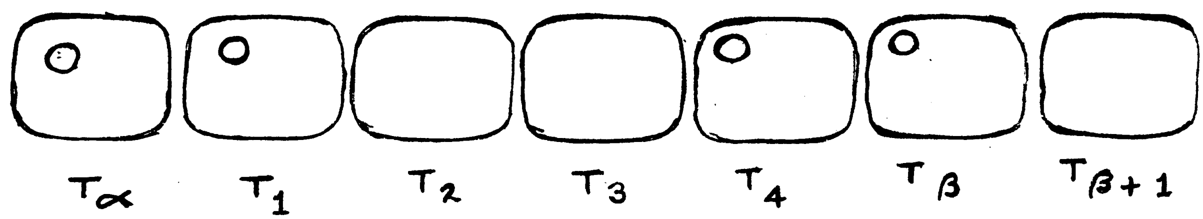
If I consider the interval $T_\beta - T_\alpha$ as being divided into 5 segments of equal duration, and represent the visual field (the area perpendicular to the Time axis) as it exists as a limit at each of the six moments in order, the above representation can be abbreviated in a fashion more suitable to my purposes as follows:



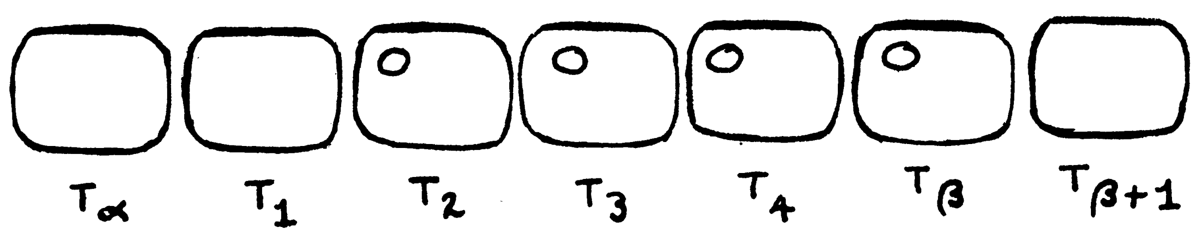
(Again, I shall not bother to shade around the frames shown below, though they ought to be considered as so shaded.)

An analysis similar to that given above for perceptual objects at any moment of time can now be given for perceptual objects existing thru intervals of time. The above set of six time-shots of the visual field represents the noisiest possible situation of the field, for it expresses the situation in which the maximum amount of energy is present at all points in the visual field (i.e.,at all times).

Consider two objects of equal volume (and of uniform maximum intensity of energy) separated by a time interval as follows:



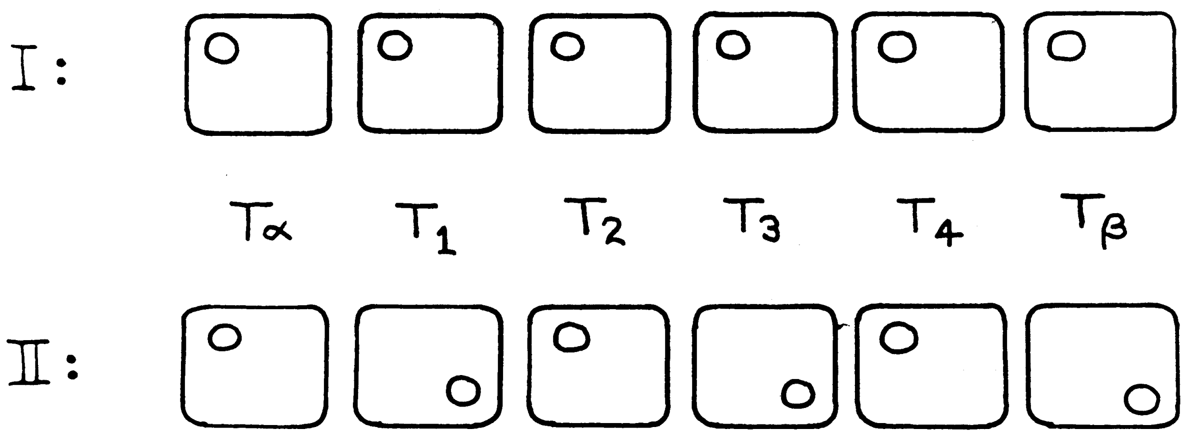
(The objects are to be taken as existing continuously thru the intervals $T_2 - T_\alpha$ and $T_{\beta+1} - T_4$). Consider now the same two objects presented so that they become temporally *contiguous*, as follows:



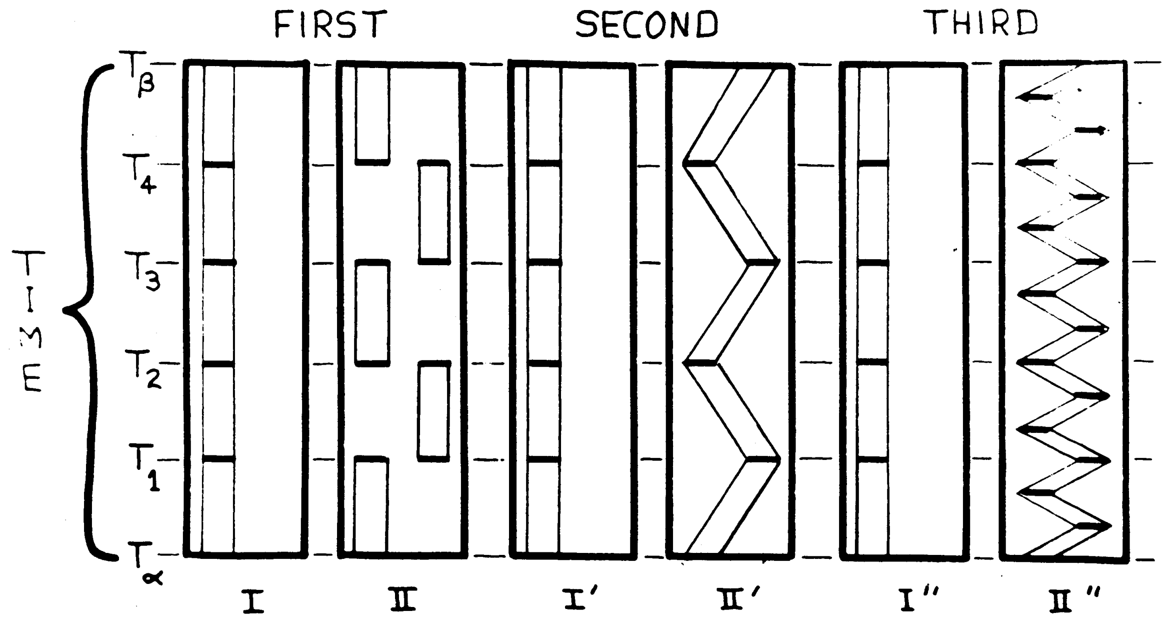
(The objects are now to be taken as *one* object existing continuously thru the interval $T_{\beta+1} - T_2$). By an argument precisely analogous to that given above for the instantaneous visual field, it is evident that the latter situation is *noisier* than the former, for a greater *contiguous* area (i.e.,volume) of space-time matches the noisiest possible state of the entire visual field in average intensity of energy per unit area.

By a precisely analogous argument, also, it could be shown that, of two areas of space-time presenting uniform maximum intensity of energy, the larger is the noisier. And the arguments can be extended to three or more objects. In general, then, for objects of uniform maximum intensity of energy in perceptual space-time, (a) the larger the object, the noisier it is; and (b), the closer together objects come, the noisier the situation becomes; and (c), if all objects are of equal size, the more objects there are, the noisier the situation is.

4. Perhaps one aspect of the above deserves closer attention. Consider the following representations of objects of uniform maximum intensity of energy per unit area in perceptual space-time.



Which of the two represents the noisiest situation? At any *instant* of time, both have the same concentration of average intensities per unit area; hence, one might expect that the summation of such concentration would be identical. But such is not the case, and, indeed, from the information given above, one cannot determine which is noisier. For, consider the following three top-views of the continuum of perceptual space-time each of which is compatible with the situations as described above:



In the first case, object I is noisier than object II, for indeed there is *no* object II; rather, *five* distinct objects were represented and erroneously taken to be a contiguous object II. In the second case, object II' is noisier than object I', for the contiguous area of space-time of object II' is greater than that of object I'. In the third case, object II'' is *much* noisier than object I'' (and also noisier than object II' of the second case), for the contiguous area of its space-time is much greater than that of object I''.

Three things should be obvious from the above. Firstly, an instanteous area of the visual field which remains unmoved thru time, all things being equal, is less *noisy* than an equal area which moves continuously thru time. Secondly, the *faster* an area moves, the noisier it becomes in comparison with other areas. And thirdly, the story-board method of representing functions of perceptual space-time, without modification, is *necessarily* ambiguous to the detriment of compositional precision.

5. The above analysis of perceptual noise in general does not hold only for areas of uniform maximum intensity of energy. As was shown in Section II, the relative complementarity of colors (and sounds) is but a subtle noise function. Hence, one ought to expect that the above analysis could be extended to encompass uniform areas of non-maximum intensity without further ado. And, indeed, it can be. For example, given two areas of equal size and identical color at a given distance from each other, if the first should change to a more pastel (whiter, less complementary) color, the resulting visual situation would become *more noisy*. Similarly, if the resulting two areas should now come closer together, the resulting visual situation would become even more *noisy*. (Similar arguments hold precisely for sound functions.) And the above sorts of situations could be extended to include three or more color-areas (or sound areas) by arguments analogous to those given above for multiple areas of uniform *maximum* intensities of energy.

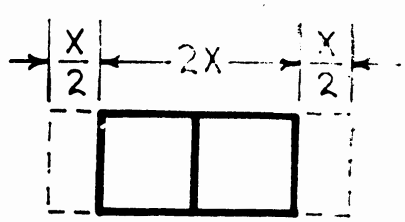
6. Thruout the above discussion, I have assumed constancy of *shape* on the part of the color areas under discussion, overlooking until now the fact that continuous areas, like continuous functions, can be conceived in a limiting sense as consisting of infinitesimals (areas) integrated in a certain manner, and that consequently a change in the *shape* of an area would necessarily change the visual relationship between many of these infinitesimal areas and thus (by my earlier argument) necessarily effect incremental changes in the noise-level of the visual field.

I must now indicate in a workable manner how changes in the *shape* of a single area of maximum intensity energy can be expected to modify the noise-level of the visual field. (As in the previous discussions, I shall use mathematical notation only when necessary, hoping to avoid superficialities while remaining intelligible to the working composer.)

It was noted earlier that a noise-function is really a *completion* function, indicating how much a particular perceptual situation would have to be modified for it to become the *densest* energy situation possible. A noise-function, thus, could be conceived as an energy *density* function (e.g.,the closer together two unit areas come, the noisier the situation, etc.).

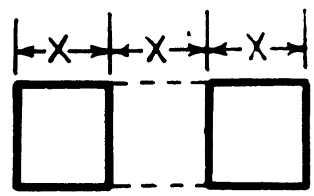
But a question immediately arises: If two unit areas effect a noisier perceptual situation when two units apart than when four units apart, how *much* noisier is it? That is, if Fig. 1

Fig. 1



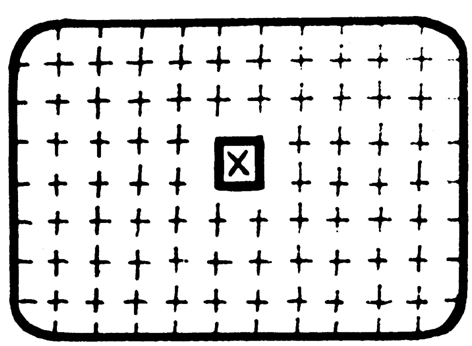
indicates a noisier perceptual situation than Fig. 2,

Fig. 2



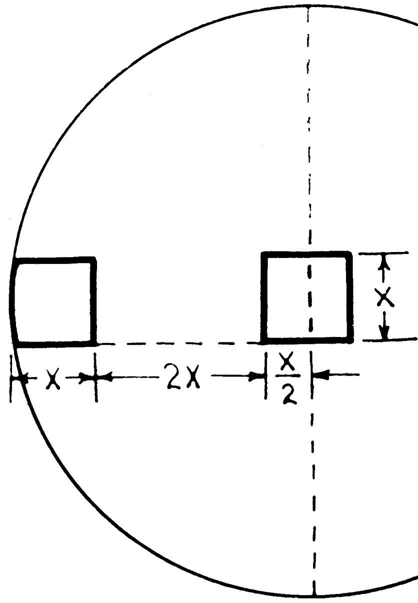
(for the *contiguous* area of maximum intensity energy is greater in Fig. 1 than Fig. 2, even though the percentage of the visual field at maximum energy is the same in either case), how are we to *measure* the difference in noisiness? Even if we accept Fig. 1 as momentarily representing our unit standard of measurement (as noise-level = 1.00), how are we to determine how much less continguous are the unit areas in Fig. 2? What are we to take as the standard of *contiguity?*

For purposes of analysis, if we consider the visual field (eg., a motion picture screen) as it exists instantaneously, it is apparent that a single unit area of maximum intensity white light would be surrounded by many unit areas of darkness.



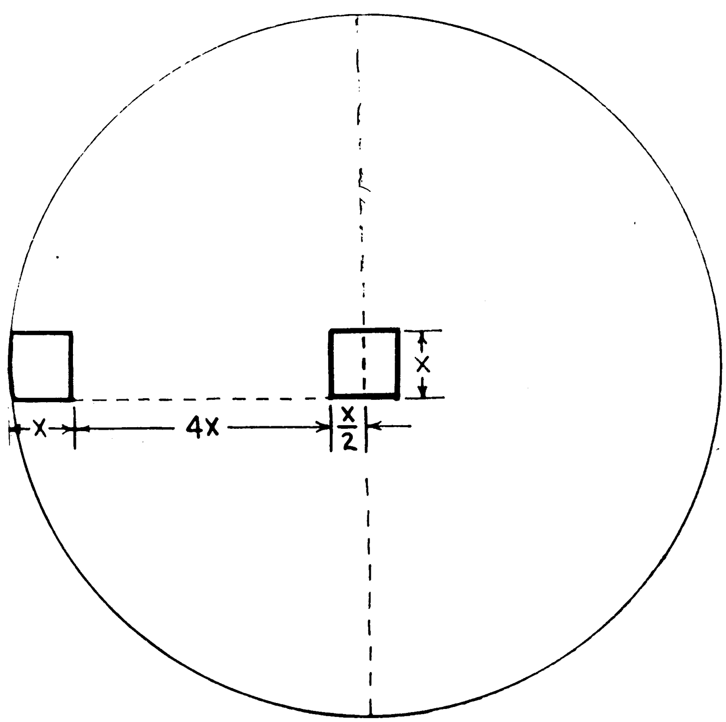
How noisy the perceptual situation would be would depend upon how large the unit area is, relative to the rest of the visual field. The larger the visual field (i.e.,the more unit areas of darkness), the less noisy the perceptual situation given the single unit area of maximum intensity white light.

Suppose now I introduce another unit area of maximum intensity white light, two linear units away.



Considering the circular area about the first unit area having a radius of $\frac{7x}{2}$, it is apparent by simple computation that there exists at least an area of $\frac{49\pi x^2}{4} - 2x^2$ of darkness (the area of the circle minus the two unit areas), i.e. at least $\frac{49\pi - 8}{4}$ or 38.47 unit areas of darkness, having equal or greater claim to our perceptual attention vis a vis the first unit area than the second unit area, for they also exist at distances equal or less than 2x from the first unit area.

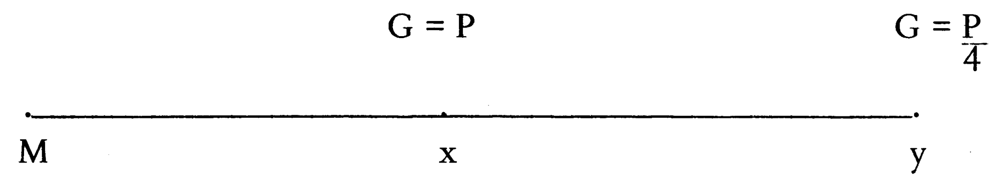
Suppose the second unit area were now to be moved twice as far away.



Considering the circular area about the first unit area having a radius of $\frac{11x}{2}$, it is apparent by a similar computation to the above that there are at least $\frac{121\pi - 8}{4}$ or 92.99 unit areas of darkness having equal or greater claim to our perceptual attention vis à vis the first unit area than the second unit area, for they also exist at distances equal or less than 4x from the first unit area.

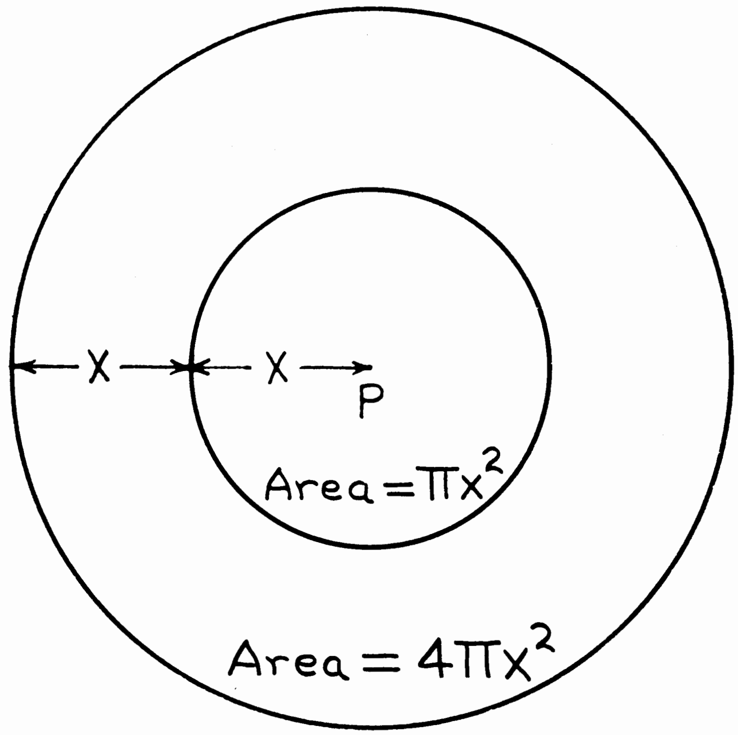
The above is meant simply to indicate in detail that the result of *doubling* the distance between two unit areas of maximum intensity energy is *much more than to double* the number of unit areas of darkness which must now be considered as being in the perceptual vicinity of the first unit area. That is, the contiguity relationship is not *linear.*

As gravitational analogies have often been of frequent assistance in the thesis, it ought now to be recalled that Newton demonstrated that all gravitational problems involving two spherical masses could be treated mathematically as if each mass were concentrated at a point. In such a situation, the effect of the gravitational field about either point falls off as the square of the distance. That is, at a point y located twice as far as point x from mass M, the effect of the gravitational field of mass M (G) is one-fourth as great.

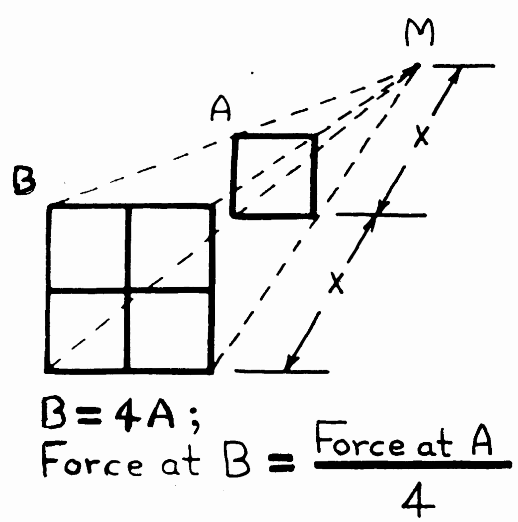


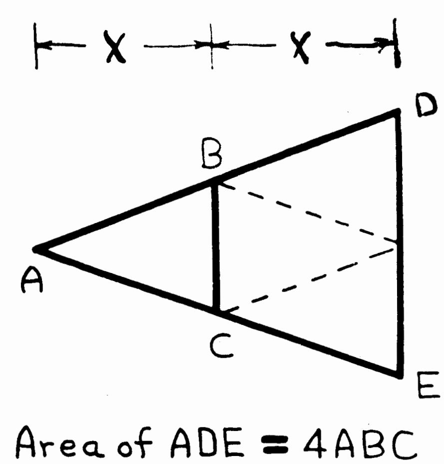
In general, the effect of the gravitational field about a mass decreases as the *square* of the distance from the mass. The gravitational effect of a mass, therefore, is an *inverse square* function of the increasing linear distance of a point in the field from the mass.

But would we be justified in assuming that the mathematical treatment of noise-functions is also an inverse square, or is the gravitational analogy simply happenstance? Earlier I indicated how doubling the distance between two areas of maximum intensity white noise more than doubles the amount of darkness involved in the perceptual relationship, hence more than halving the noise-level of the perceptual situation in general. The resulting noise-level was not ¼ of the original, but it must be remembered that I was not speaking of point-sources of light, either. Had I chosen to speak of a point-source of light, the resulting drop in the noise level would have been ¼, for areas of circles increase as the *square* of their radii. That is, to move twice as far from a point-source of light is to lessen by ¼ its effect as a noise-argument, for the area of darkness has been increased four times, not merely doubled.



Newton, of course, was integrating in a 3-dimensional space, while I have been concerned with the instantaneous 2-dimensional surface of a projection screen. Newton was concerned with the falling-off of a *force* as it decreased with distance, while I, by nature of the definition of a noise function, havebeen concerned with the increase in the *areas* of darkness as distance increases. But mathematically the results are identical: in each instance, the effect decreases as the square of the distance. For, as the usual Newtonian gravitational diagrams illustrate, Newton *was* in effect concerned with increasing *areas* at a distance from a point-source (although, unlike my screen, these areas do not occur in the same plane, but rather in parallel planes).





Put in simple terms, areas of maximum intensity white noise don't *attract* each other in any sense. But the noise-function is such that one is concerned with relative *areas* of light and darkness, and the relative squared increase in areas of darkness vis à vis a given point of light as one increases one's distance from it. Superficially, this appears different from Newton's concerns. But although Newton was dealing with attractive forces, he was interested in the relative squared increase in the *area* of the surfaces perpendicular to a point over which a given force from the point had to spread its effect, for as the areas of the perpendicular surfaces increase as the square of the distance from the point, the effect of the gravitational force decreases inversely as the square of the distance. Thus, mathematically, the situations of a gravitational force in 3-dimensional physical space and a noise-function in 2-dimensional visual space are identical.

(The transition from the instantaneous 2-dimensional visual space to areas of light and darkness changing in time, therefore, is made in a mathematically similar fashion to making the transition from the instantaneous 3-dimensional gravitational space to the effect of masses changing position in time.)

7. Before proceeding to apply the results of the above analysis to some typical noise-function, I wish to comment upon a further parenthetical question: Do we remember perceptual noise-functions logarithmically? That is, do our memories of perceptual noise-functions fade logarithmically?

What I am going to say in this section is strictly conjectural, for the psychological experiments which have been conducted on time-perception are either irrelevant to this point or inconclusive. But the conjecture is intended to be carefully made, and to be based upon observation of my own ways of remembering. I submit that the perceptual effect of a given noise-function (as opposed to our *idea* of the perceptual effect) ceases to exist logarithmically. As I am convinced (for independent reasons which I shall not discuss here) that art is structurally dependent upon perceptual *effects*, and not upon the *ideas* arising from them, I am therefore inclined to think that the formal structure of the peculiarly temporal arts like music and the cinema, with their emphasis on thermatic development, must be dependent upon this logarithmic memory fade.

But how can I argue for this conjecture? I know of no way, except to say that it strikes me as most certainly not obviously incorrect, and from my own experience quite likely to *be* correct. To take a simple example, the conjecture is that the bell I heard ten seconds ago will have, not one-half, but more nearly one-fourth the perceptual effect it now has ten seconds hence. Perhaps, like most perceptual matters, questions of the degree of concentration of the perceiver are so entwined with strictly physiological reactions that no precise figure can be given for perception and perceivers in general. Since composers are hardly perceivers in general, however, possessing uniquely-developed powers of concentration, the field is narrowed somewhat. And I think most composers would agree that one *senses* the fact that an event perceived 20 seconds ago is not one-half as fresh as it was 10 seconds ago, but much less than that. (The immediacy of the moment falls-off so quickly.) Given the pervasive influence of logarithmic scales in perceptual studies, I suggest that it is not unlikely that the effect-fade of which I speak is also logarithmic in nature. (And notice: experiments claiming to have shown that *time* is not perceived logarithmically because we can *judge* linear intervals accurately are irrelevant to this suggestion. I'm not here speaking of the correctness of our *ideas* about our perceptions, but about our fading perceptions themselves.)

Needless to say, if the conjecture is correct it has great relevancy for the formal structures of the temporal arts in particular. But such an analysis lies beyond the scope of this paper, in the remainder of which I shall be concerned solely with noise-functions of the instantaneous 2-dimensional visual space.

8. Now that the relevance of the gravitational analogue has been made clear, let me clarify one other lesson we can learn from it. The gravitational attraction between two masses, M_1 and M_2, is given by the formula

$$F = G \frac{M_1 M_2}{d^2}$$

where d is the distance between the masses, and G is a constant (which for my purposes can be neglected, since it can always be made equal to one by choosing suitable units of measurement).

If one were to concern himself only with equal areas (i.e.,analogous to equal masses), the relative noise-difference (i.e.,difference in attraction, translated into *area* terms) between the situation in which two areas (a) are at a distance d_1 apart, and the situation in which they are at distance d_2 apart, would be indicated by the difference between the values of the two ratios

$$\frac{a^2}{d_1^2} \quad \text{and} \quad \frac{a^2}{d_2^2}$$

Since I am not interested in establishing *absolute* units of noise-value, but only in determining the *relative* noise-value of one visual situation versus another (with which it may be formally structured in a work of art), the relative noise ratio of the above two situations (taking the first as standard) is given by the ratio of the noise value of the first situation to that of the second.

$$\frac{\dfrac{a^2}{d_1^{\,2}}}{\dfrac{a^2}{d_2^{\,2}}}$$

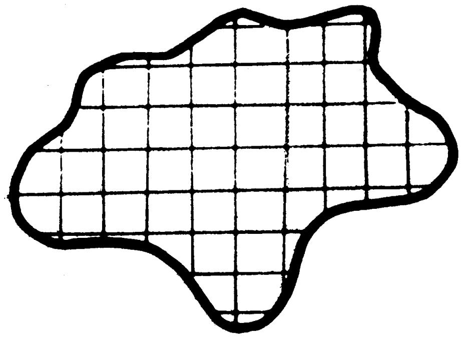
Clearly, the numerators are irrelevant, cancelling each other out. Hence, the relative noise ratio between situation one (with masses at distance d_1 apart) and situation two with masses at distance d_2 apart reduces to the ratio

$$\frac{d_2^{\,2}}{d_1^{\,2}}, \quad \text{or} \quad \left(\frac{d_2}{d_1}\right)^2$$

In simple terms, given two equal areas, the noise-value of the visual situation is an *inverse* function of the *square* of their distances apart.

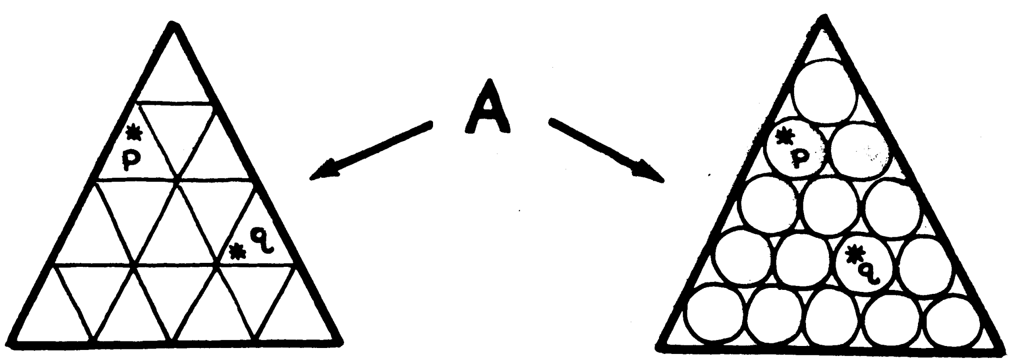
9. It may seem as if I have simply formalized more precisely what I have already said in the thesis concerning two areas, yet remain as far as ever from saying anything about the noise-value of a single area having a given *shape*. But if it is remembered that the integral calculus (the first general approach to determining areas) was developed using the concept of the real infinitesimal, and indeed that many engineers today get along quite well without a mathematical sophistication much beyond this, it will be noticed that I have been establishing in the latter sections the basis for a conceptual treatment of the noise-value of variously shaped areas using a system of *identical areas* having much in common with the traditional theory of the real infinitesimal. Areas of any shape, after all, can be construed as precisely as one wishes as consisting of many smaller regular identical sub-areas grouped together (the smaller the sub-areas, the better the approximation).

E.g.,

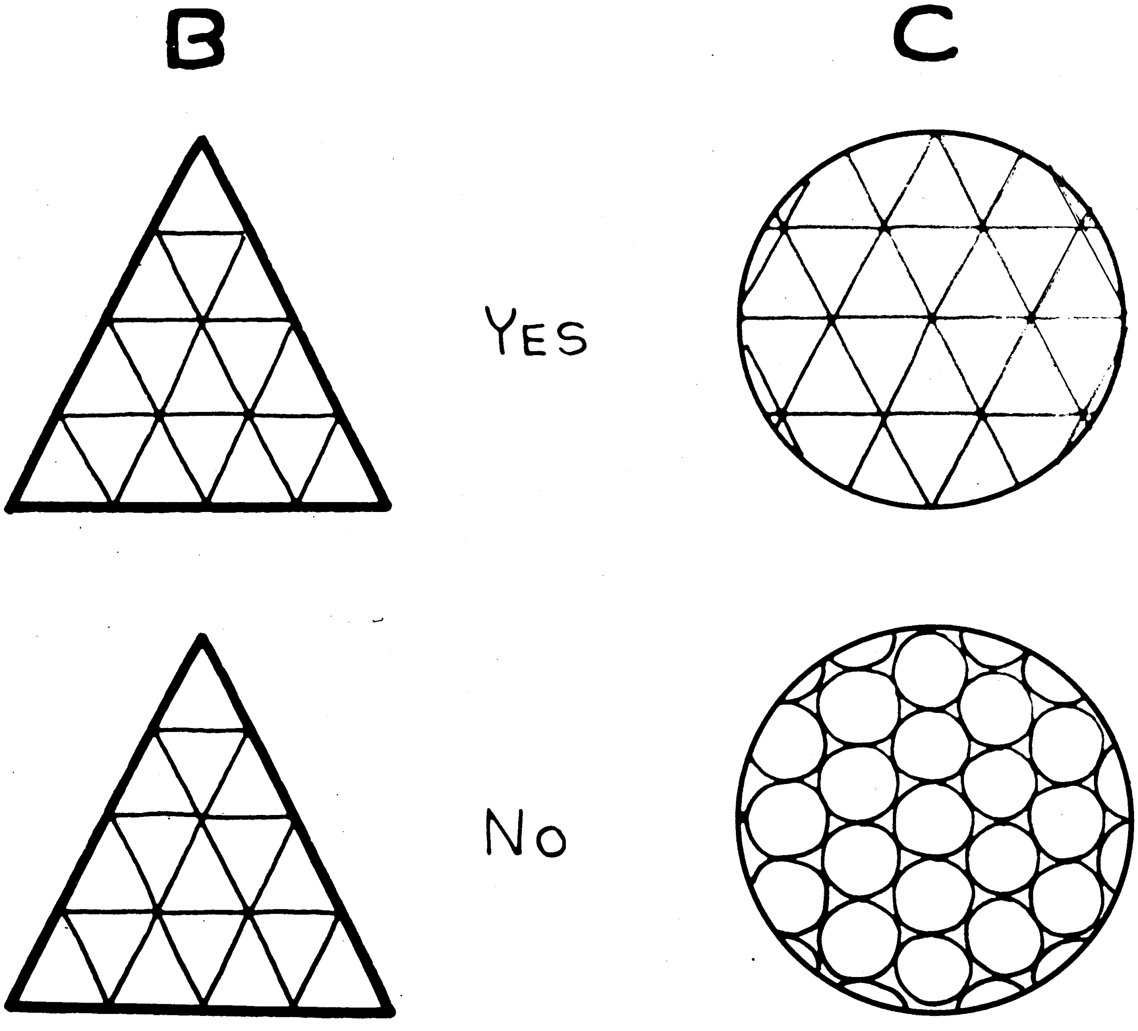


The sum of the smaller areas approximates to the area of the figure itself. And if each of the smaller areas should happen to exert a force on every other smaller area, the sum of these partial forces would approximate to the internal forces existing in the figure itself. Construing the noise-relationship between two areas as a force (as we did in the last section), the sum of the noise-values between each sub-area in a figure would approximate to the noise-value of the figure itself, as closely as we care to determine it.

Let me now take several figures of different shape and evaluate their relative noise-values in terms of the above. Notice: what is important is not the *shape* of the smaller units into which I divide the figure, but rather their *identity* to each other. For as indicated above, as long as the units are identical to each other, I need only consider the *distance* between them to discuss their relative contribution to the noise-value of the whole. For example, given area A subdivided as follows,



the important fact is the *distance* p – q between the sub-areas p and q. It is irrelevant that the sub-areas are triangles in the first case, and circles in the second. We must only be careful that, in comparing the noise-value of two areas, eg..B and C, we use *identical* sub-units in evaluating each.


B
C
YES
NO

10. For convenience, I shall use circles as my sub-areas in the remainder of this paper.

It will be recalled that the noisiest state of the visual field was defined to be that state in which maximum intensity white-noise was present at all points of the field. Thus, if I consider a field as consisting of many small circles of maximum intensity white-noise, the noisiest state of the field ought to occur whenever all of the circles making up that field are grouped together as closely as possible. Let me consider several possible configurations of *seven* such circles of maximum intensity white-noise, evaluating their respective noise-values in terms of the sum of the *inverse* of the *squares* of the approximate distances between each of the seven circles in each case – in accordance with the formulae discussed earlier. I shall number each circle in each configuration for reference, listing in the table which follows the value of the distance, square, and inverse for each pair in each configuration, and then giving the *sum* of the inverse squares of the distances apart for each configuration. The higher the sum, the noisier the configuration. (For uniformity of measurement, all circles will be drawn with diameters of 1"; and all measurements and computations are *very* approximate.)

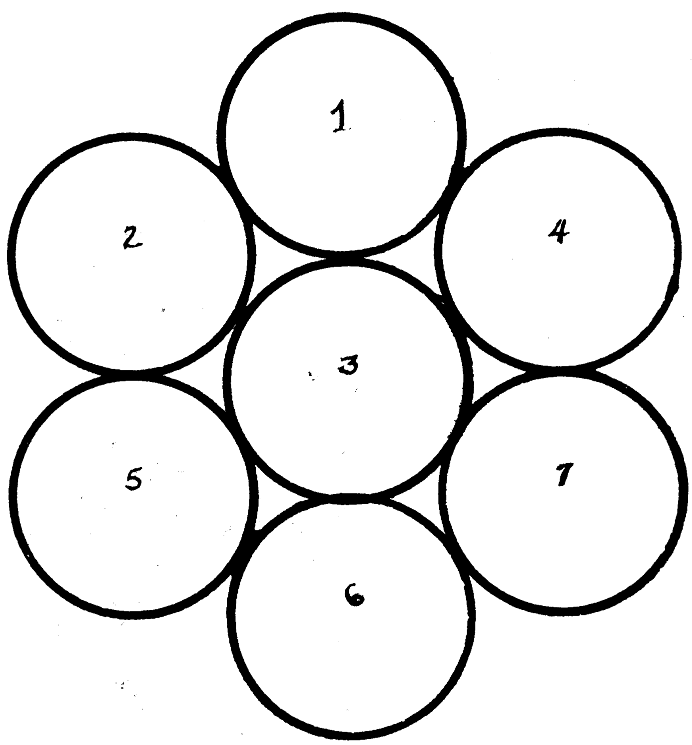


Fig. 1

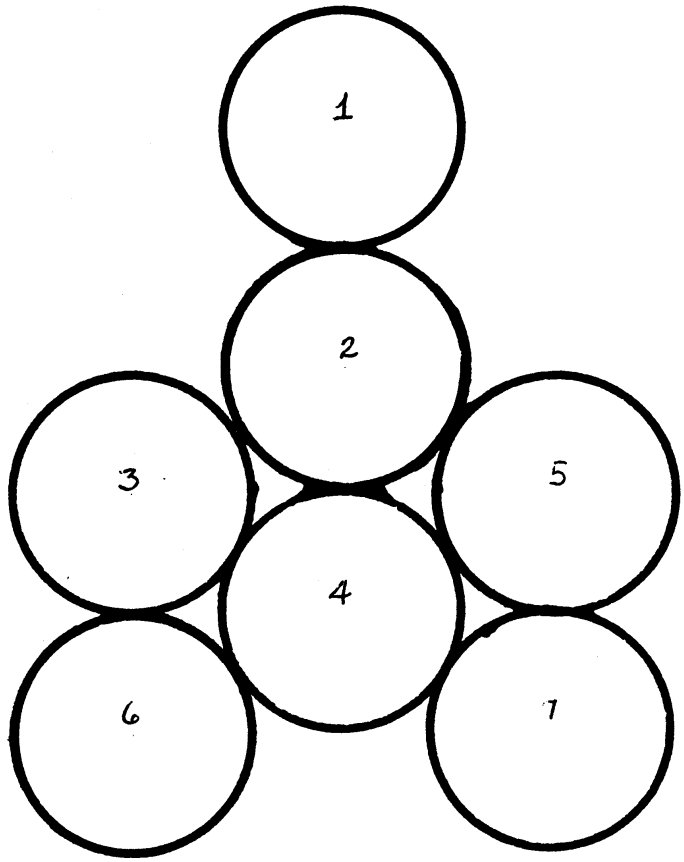


Fig. 2

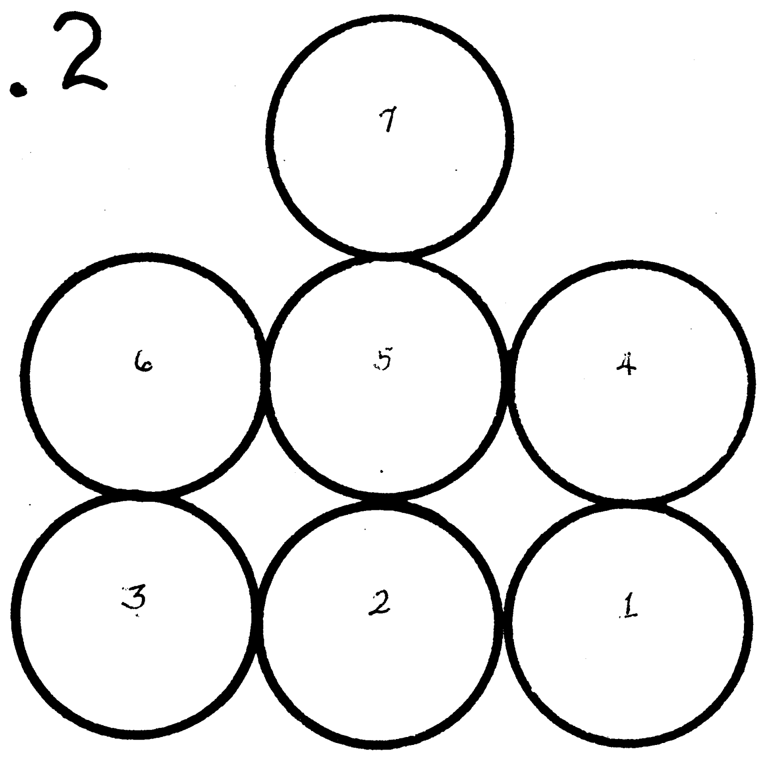


Fig. 3

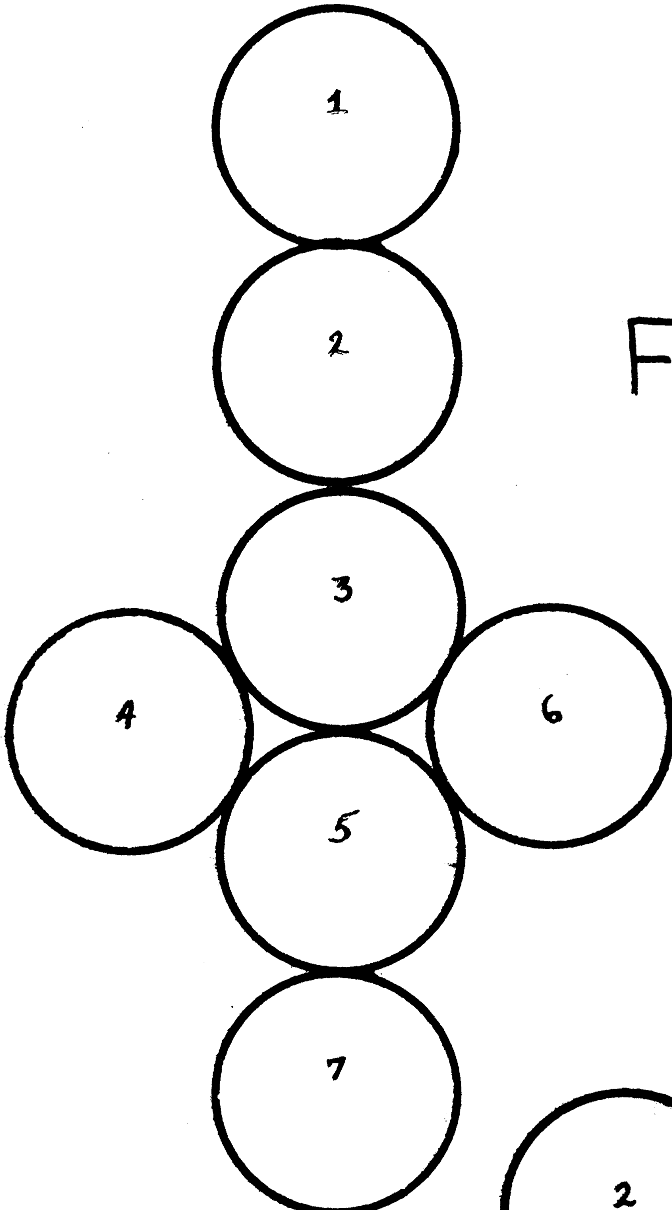


Fig. 4

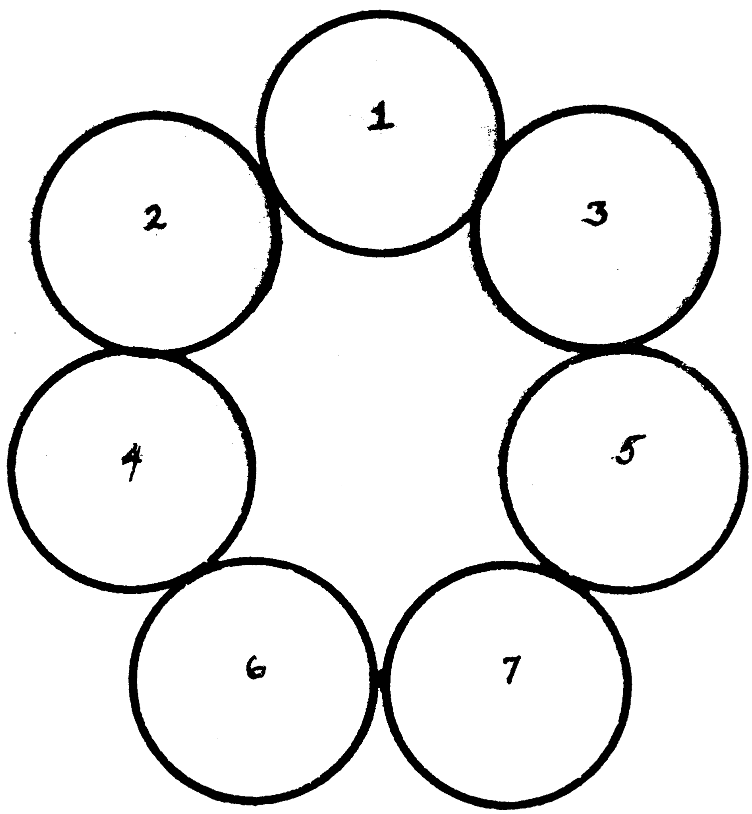


Fig. 5

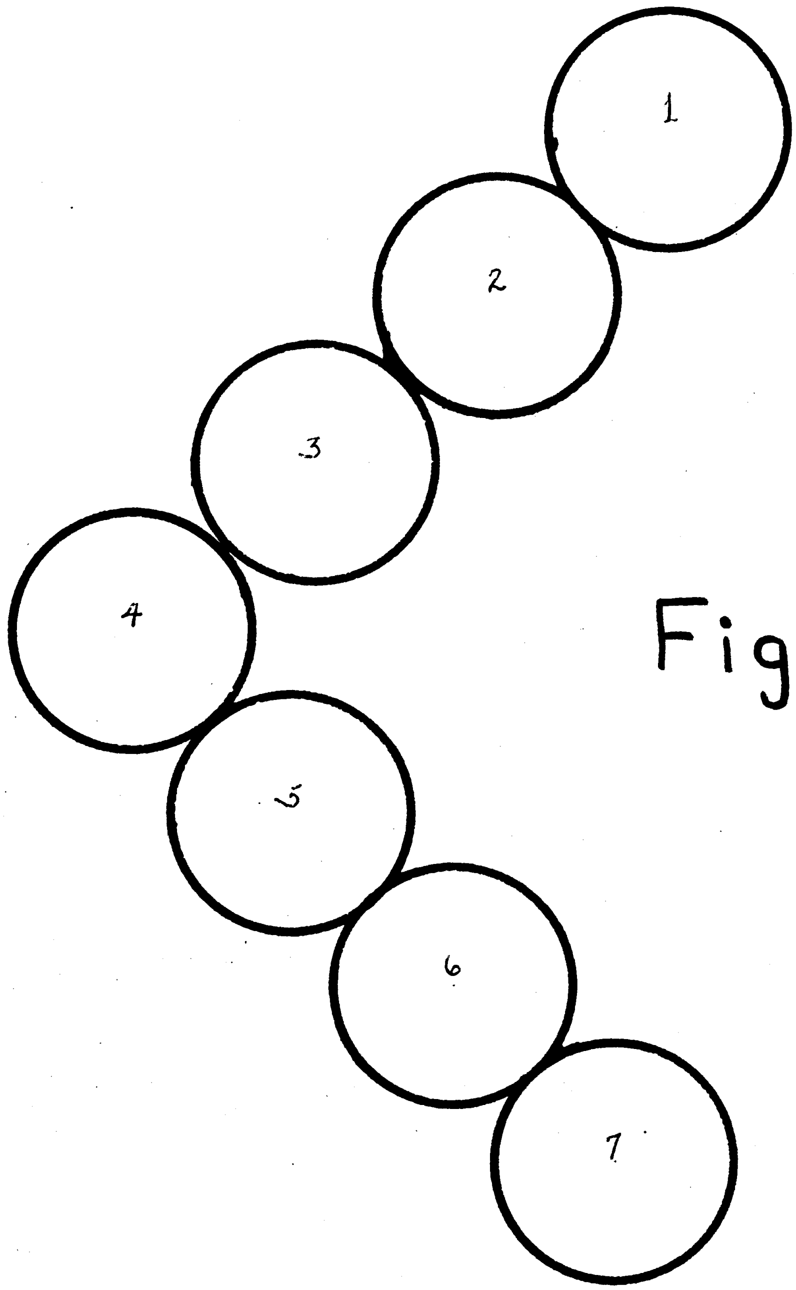


Fig. 6

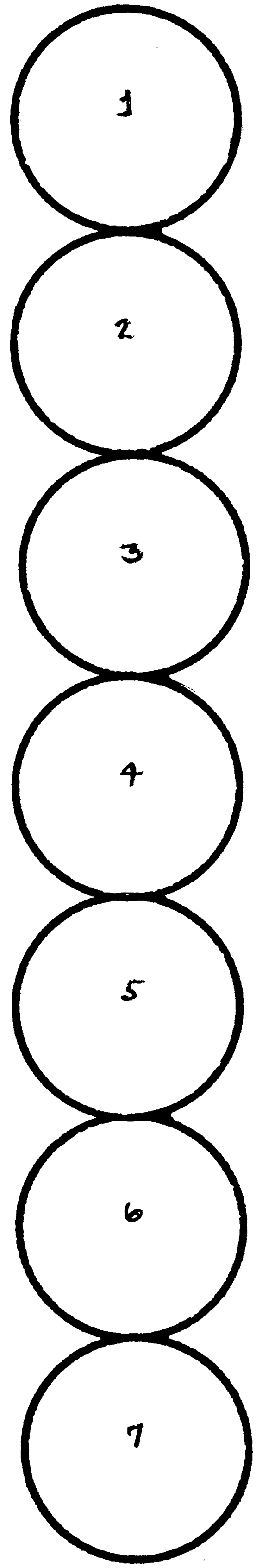


Fig. 7

TABLE OF NOISE – VALUES

Between circles	Fig. 1 D	Fig. 1 S	Fig. 1 I	Fig. 2 D	Fig. 2 S	Fig. 2 I	Fig. 3 D	Fig. 3 S	Fig. 3 I	Fig. 4 D	Fig. 4 S	Fig. 4 I	Fig. 5 D	Fig. 5 S	Fig. 5 I	Fig. 6 D	Fig. 6 S	Fig. 6 I	Fig. 7 D	Fig. 7 S	Fig. 7 I
1 - 2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1 - 3	1	1	1	1.75	3.06	.33	2	4	.25	2	4	.25	1	1	1	2	4	.25	2	4	.25
1 - 4	1	1	1	2	4	.25	1	1	1	2.63	6.89	.145	1.75	3.06	.33	3	9	.111	3	9	.111
1 - 5	1.75	3.06	.33	1.75	3.06	.33	1.50	2.25	.445	3	9	.111	1.75	3.06	.33	3.25	10.6	.095	4	16	.0625
1 - 6	2	4	.25	2.63	6.89	.145	2.25	5.06	.198	2.63	6.89	.145	2.25	5.06	.198	3.75	14.0	.071	5	25	.04
1 - 7	1.75	3.06	.33	2.63	6.89	.145	2.25	5.06	.198	4	16	.63	2.25	5.06	.198	4.25	18.0	.054	6	36	.028
2 - 3	1	1	1	1	1	1	1	1	1	1	1	1	1.75	3.06	.33	1	1	1	1	1	1
2 - 4	1.75	3.06	.33	1	1	1	1.50	2.25	.445	1.75	3.06	.33	1	1	1	2	4	.25	2	4	.25
2 - 5	1	1	1	1	1	1	1	1	1	2	4	.25	2.13	4.52	.22	2.25	5.06	.198	3	9	.111
2 - 6	1.75	3.06	.33	1.75	3.06	.33	1.50	2.25	.445	1.75	3.06	.33	1.88	3.52	.284	2.88	8.28	.121	4	16	.0625
2 - 7	2	4	.25	1.75	3.06	.33	2	4	.25	3	9	.111	2.25	5.06	.198	3.62	13.1	.076	5	25	.04
3 - 4	1	1	1	1	1	1	2.25	5.06	.198	1	1	1	2.13	4.52	.22	1	1	1	1	1	1
3 - 5	1	1	1	1.75	3.06	.33	1.50	2.25	.445	1	1	1	1	1	1	1.50	2.25	.445	2	4	.25
3 - 6	1	1	1	1	1	1	1	1	1	1	1	1	2.25	5.06	.198	2.25	5.06	.198	3	9	.111
3 - 7	1	1	1	2	4	.25	2.25	5.06	.198	2	4	.25	1.88	3.52	.284	3.25	10.6	.095	4	16	.0625
4 - 5	2	4	.25	1	1	1	1	1	1	1	1	1	2	4	.25	1	1	1	1	1	1
4 - 6	1.75	3.06	.33	1	1	1	2	4	.25	1.75	3.06	.33	1	1	1	2	4	.25	2	4	.25
4 - 7	1	1	1	1	1	1	1.50	2.25	.445	1.75	3.06	.33	1.75	3.06	.33	3	9	.111	3	9	.111
5 - 6	1	1	1	2	4	.25	1	1	1	1	1	1	1.75	3.06	.33	1	1	1	1	1	1
5 - 7	1.75	3.06	.33	1	1	1	1	1	1	1	1	1	1	1	1	2	4	.25	2	4	.25
6 - 7	1	1	1	1.75	3.06	.33	1.50	2.25	.445	1.75	3.06	.33	1	1	1	1	1	1	1	1	1
Approx. sums of inverse squares			14.73			13.02			12.21			11.54			10.70			8.58			7.99

With due allowance for the inaccuracies of quick computation with unwieldly fractions and gross measurements, a pattern clearly emerges. The noisiest combination of seven circles is Fig. 1, the least noisy Fig. 7. Remembering that these circles in combination were meant to represent sub-areas of closed *figures,* if one imagines a smooth curve drawn about the respective combinations, one can make the following conclusion: *for a given contiguous area of uniformly intense energy, the noisiest possible configuration of that area is a CIRCLE, which exists on a continuum thru increasingly angular configurations to the least noisy configuration of that area, the shape approaching a STRAIGHT LINE.*

The above is meant only as a rough guide to composition and further analysis. Like any other subject in art, if one is interested in the noise-functions of particular shapes, careful and precise measurement and control would be required.

Let me remind you that I have nowhere suggested that, because circles are noisier than squares, one *ought* (or ought not) to use circles rather than squares. The above is not intended as a *program* for composition with shapes, but rather as a means of unambiguously specifying the relationships between, and qualities of, whatever shapes one wishes to use. It adds to a *method*, not a *theory*, of visual composition.

11. But the purpose of this Section of the paper was not to *catalogue* possibilities of noise functions, for they are unlimited. Rather, it was to describe a set of procedures adequate to the needs of the film composer. The above discussion suffices, I shall assume, to show that functions of perceptual space can be reduced to noise functions. The problem then becomes: what procedures are adequate to specifying, with clarity and flexibility, noise functions, and functions of noise functions, etc.? (It was noted above that the story-board technique is inadequate to the task, for it is unable to specify continuous noise functions unambiguously. In general, any technique which does not make evident continuous noise functions must be rejected.)

One must first remember that a film-composer does not begin with an overall functional sense of his work in mind. He begins with bits and snatches of noise functions (visual themes), pregnant with functional possibilities but undetermined by any systematic higher-order noise functions. The procedure to be suggested, therefore, must not only be adequate to describing noise functions once they are systematically determined by other noise functions, but also to describing the initial fragmentary ideas of the composer in such a way that their possibilities as functions of functions, etc., becomes most clearly and flexibly evident.

12. It was shown above that a single continuous object of perceptual space-time can increase the noise-level of the perceptual situation in five ways:

(a) By increasing in *size* (i.e., by increasing the contiguous area of the perceptual field in which a given average uniform intensity of energy is being maintained);

(b) By increasing its *rate of change of position* in the perceptual field (i.e., by increasing the contiguous area of the perceptual field in which a given average uniform intensity of energy is being maintained);

(c) By increasing (shape-wise) in *circularity;*

(d) By increasing the achromaticity (whiteness) of its color (or sound);

(e) By increasing the *brightness* of its color, or sound (i.e., by increasing the average intensity of the energy presented).

As is evident from the 'i.e.' clauses, however, (a) and (b) can be considered identical. (But this only becomes pragmatically obvious when considering a second object. Given a second object, how can one change the noise-level of the perceptual situation by modifying the *first* object? One could change it in accordance with either (c), (d), or (e), of course; but one could also change it in accordance with either (a) or (b). Yet, as a direct analogue from gravitational theory again shows, to change the first object according to (a) is to change it according to (b), and conversely. Consider two masses, M_1 and M_2. How could one increase the strength of the gravitational field around M_1 and M_2 by changing M_2? Clearly, either by increasing the mass of M_2, or by moving it closer to M_1. The point, however, is that the change effected is the *same*. And if our *only* means of measuring the changing relative characteristics of two bodies is to measure the gravitational field between them, a change of gravitational field can mean only *one* thing (though, to any observer who could independently measure changes in distance and mass, it would mean one of two things, or some combination of the latter two). Thus, the noise-function *seems* identical to the gravitational function, and *contiguity* seems dependent upon the notion of field (though I shall assuredly not discuss this here).) That is, to change in size is identical to changing one's rate of position in terms of the effect on the energy (i.e.,noise) field in the vicinity.

Consequently, if I choose to say that an object increasing in either size or rate of change of position is *changing in extension* (i.e.,is being positionally accelerated), the upshot of the above is that the noise-effect of a single continuous object of perceptual space-time can be determined by specifying four parameters:

(1) a *change in extension* parameter;

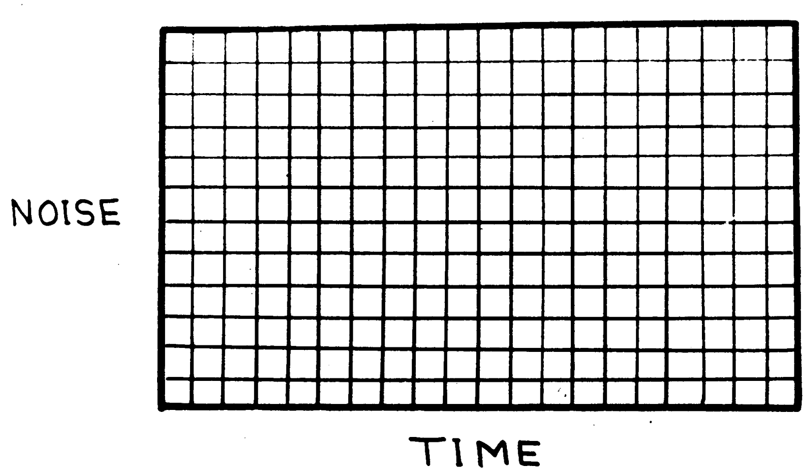
(2) a *change in shape* parameter;

(3) a *change in achromaticity* parameter;

(4) a *change in brightness* parameter.

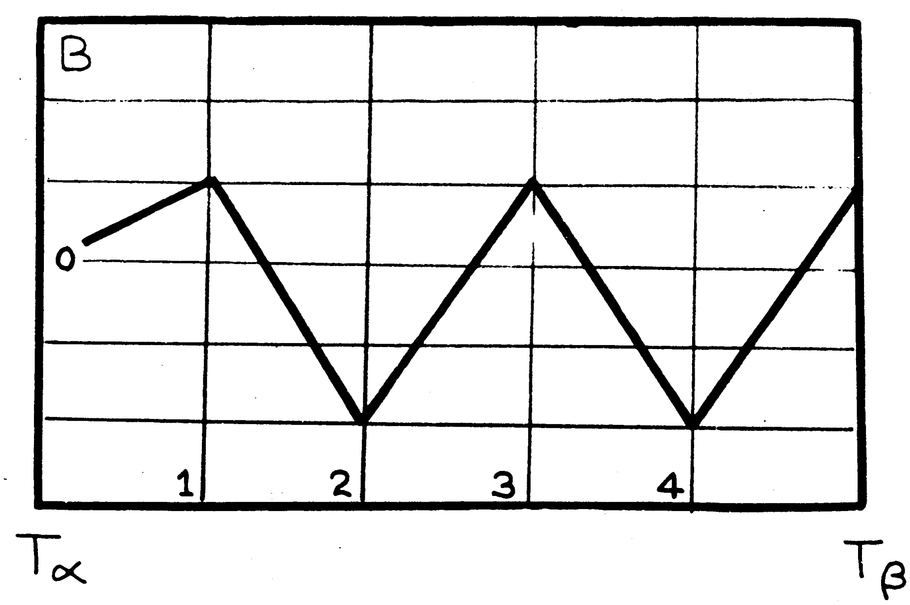
13. I shall now describe what I take to be a fundamental set of procedures for composing in film (or music, for that matter). For increased ease of exposition, I shall limit myself to discussing these procedures in terms of only three of the above four parameters, omitting a discussion of the procedure for specifying the *change in shape* parameter. (The means of extending the procedures discussed below to include this latter parameter, however, should be apparent.)

The basic tools of the composer are to be an unlimited supply of 3 x 5" white cards tightly ruled, a pencil, and many erasers.

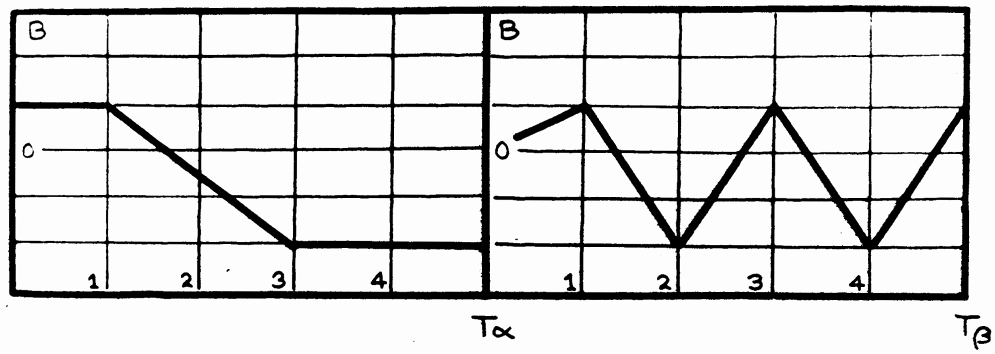


The 5'' edge of each card is to be considered a temporal axis. There are to be three basic types of cards, corresponding to the three parameters of the perceptual object: *change in extension, change in achromaticity,* and *change in brightness* cards. In addition, there are to be three additional types of cards which are to serve as the means of increasingly abbreviating the information given by combinations of cards: *contiguity* (i.e.,noise-level summation) cards, and *rate of change* (i.e.,acceleration) cards. The 3'' edge of each card is to be considered as indicating the relevant noise metric.

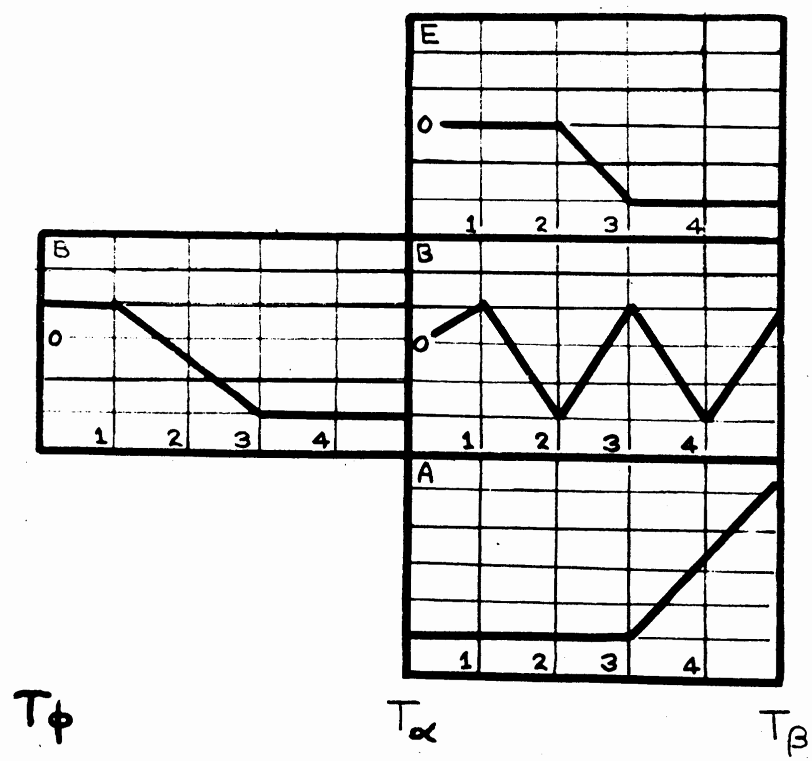
Having chosen a ruled line on a card as 'zero', (and the composer is free to choose any perceptual value as equal to 'zero', so long as he remains consistent), having taken the other lines and spaces in order as representing equal perceptual differences, and having considered the 5'' edge as representing a unit of time divisible in order as he wishes, the composer possesses a tool with which to express unambiguously a fundamental noise-function, whether continuous or otherwise. For example, the following card might represent an object changing in *brightness* from low to high (pegged at certain levels vis à vis *zero* level) every two seconds as indicated.



If this card were to be placed *after* another brightness card as follows,



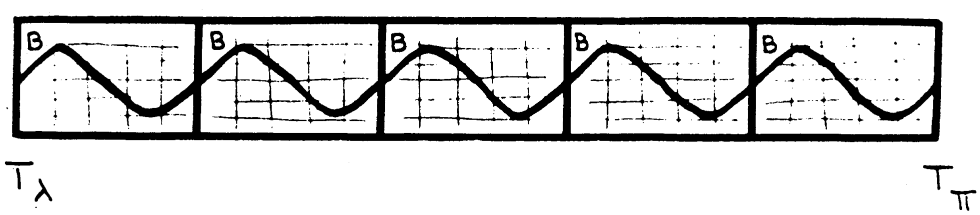
the composer would have indicated an object which, beginning at level two of brightness for one second, fell continuously during two seconds to level -2 where it remained for two seconds, then jumped *discontinuously* to level zero to begin the periodic fluctuation described above. Similarly, if the composer were to indicate for the interval $T_\beta - T_\alpha$ not only the *brightness* parameter of the object, but also its *extension* and *chromaticity* parameters, by placing an *extension* card and an *achromaticity* card directly above or below the *brightness* card, he would have provided all the information necessary to determine the effect of the object alone on the noise-level of the perceptual field during the interval $T_\beta - T_\alpha$, though not during the preceding interval $T_\alpha - T_\phi$.



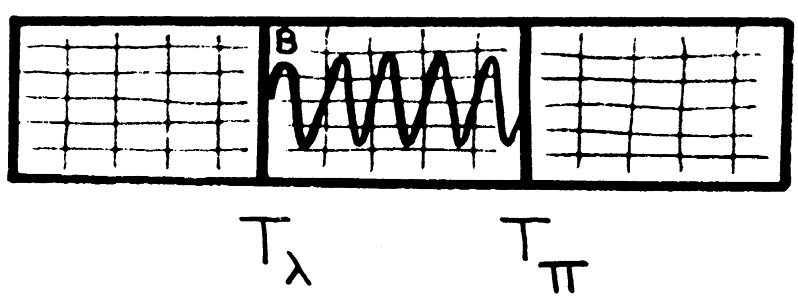
From the three vertical cards, the composer could then determine a *contiguity* card showing the cumulative effect of the information given on the three cards (i.e. the total fluctuation over the interval $T_\beta - T_\alpha$ of the noise level of the perceptual field due to the object by itself). If this object were to be the only object in the visual field, the composer could now work backward from this card by deriving from the pattern of *contiguity* evidenced during the interval $T_\beta - T_\alpha$ suggested functional possibilities of *extension* and *achromaticity* for the preceding interval $T_\alpha - T_\phi$.

Similarly, if the composer were interested not only in the change of extension, brightness, achromaticity, and (cumulatively) contiguity during the interval $T_\beta - T_\alpha$, but also in the *rate* of such changes (i.e. the pattern of *acceleration* of extension, or brightness, or contiguity), he could substitute *rate-of-change* cards for any or all of the above, and work with these, substituting *contiguity* cards for cumulative rate-of-change effects, etc.

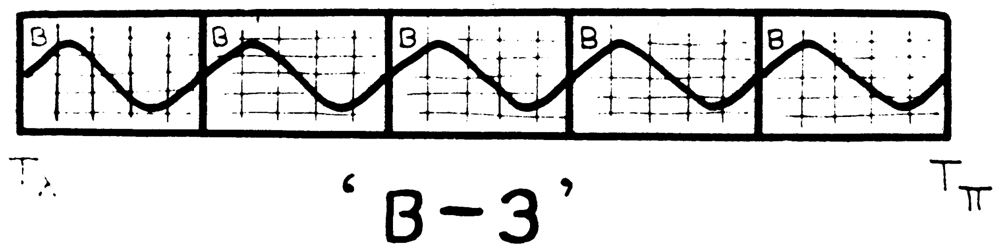
Given a series of cards determining a noise-effect over a certain interval $T_\pi - T_\lambda$,



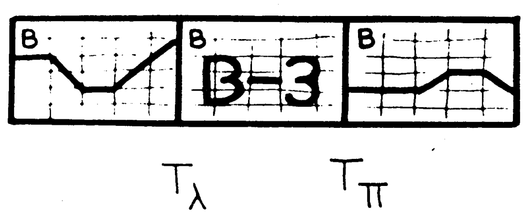
The composer is free to condense all the information onto fewer cards if he wishes simply by reconstruing the metric of the temporal axis (the 5" margin) of the cards being substituted for the others.



The 5" margin, that is, can represent whatever time interval the composer wishes, and substitutions made accordingly. Similarly, if in a series of cards the composer notices a pattern of noise fluctuation over some interval which he wishes to employ (e.g.,) thematically,

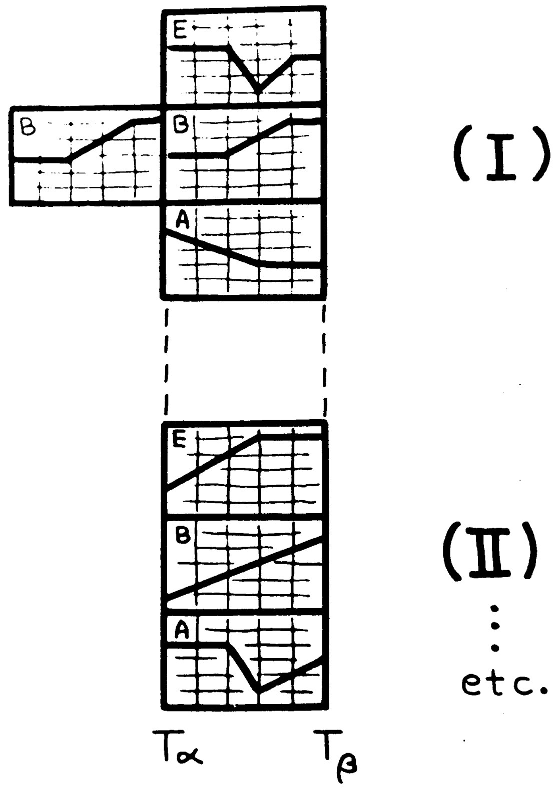


he is free to abbreviate the pattern (e.g.,by calling it pattern 'B-3') and substitute cards bearing the name of the pattern wherever appropriate.



It is the responsibility of the composer, of course, to be clear and consistent in his substitutuions. For example, if the name 'B-3' were to mean a certain pattern over an interval of 10 seconds, it would be incorrect to substitute a card bearing the 'B-3' into a context of an interval of only 5 seconds. (The composer could, of course, substitute a card bearing the name of a pattern like B-3 except for being temporally half as long: say 'B-3/2'. But this is the name of a different, though related, pattern. It is not the same pattern as B-3.)

When two or more objects are to be determined, the above tools and techniques need only be extended. For any interval $T_\beta - T_\alpha$, individual objects could be represented as below,



and *contiguity* cards or *rate-of-change* cards substituted for either object I or object II, individually or collectively. The process becomes much more complex, but the principle remains the same.

14. It ought also to be apparent that the composer who wishes to work on a less finely-controlled level need only use his cards to represent whatever objects he wishes, so long as he remains consistent and has a fine-enough functional sense not to violate the fundamentals in unifying the coarser elements. The point is that the above system provides the composer with the means of working with perceptual color and sound functions on any level of discrimination. If he wishes to be unconcerned with minute changes in time, or wishes to develop by substitution an overall temporal view of this work, he need only reconstrue the intervals specified by the 5" margins of his cards; the length of a 5" card can as easily represent 5 hours as 5 seconds.

The above set of procedures, therefore, provides unlimited flexibility of substitution with unlimited accuracy of representation. The composer can try different combinations as easily as shuffling his cards with a purpose. And, by the process of successive substitution, he can abbreviate entire sections of his work, enabling him to get a functional sense of the whole, without losing accuracy of representation. (The advantages of such flexibility over the notebook techniques of composition ought to be obvious.)

As suggested in passing earlier, the above set of procedures is also of advantage to the composer in that it provides him immediately with a tool wherewith to represent, compare, and contrast those initial fragmentary ideas with which he begins with the greatest possibility of sensing functional similarities and differences. Once further ideas are forthcoming, the extreme flexibility of substitution, etc., provides him with the means of functionally testing the ideas in an unlimited number of contexts quickly and precisely.

15. In conclusion, let me remark that I have not given a 'notation' for film which would be of much use as a production script or as a readable summary of the completed film. From the pragmatic character of the above procedures, it ought to be apparent that a work of art cannot be transcribed discursively in such a manner that all, or indeed its essential, *compositional functions* are made simultaneously evident. (It is no accident that the inverse task of learning to understand the compositional functions of a work of art from its production script, as any major in musical composition will attest, is a long, and tedious process of creative *reconstruction).* What I have tried to suggest is a set of procedures by which the composer may compose his work with maximum clarity and flexibility of purpose. The task of translating a work of art composed according to the above procedures into a production script is neither unimportant nor difficult in principle. But it is a task distinct from composition, the subject of this thesis.

Evan Cameron

FOOTNOTES

1. Quine [4], op. cit., p. 81.

2. Johnson and Kiokemeister, op. cit., p. 15, for a similar formulation.

3. *Ibid,* pp. 149-150.

4. Sciama, op. cit.

5. vide footnote in Austin, op. cit., p. 86.

6. vide Land, op. cit.

7. Bach, op. cit., p. 66.

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