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SPEAKERS

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Hello, everyone, welcome to our video on systems of equations, not just any equations, but systems of linear equations. Linear equations have a nice property. If we have three unknown variables and three linear equations, we can solve for all the unknown variables. Similarly, similarly, if we have six unknown variables and six linear equations, we can solve for all six unknown variables. And this is a useful property of linear equations. And therefore, it actually shows up in quite a number of topics and has useful applications that you're going to need to know to do well, in business and economics. So I hope you're ready. And if you are, just keep watching and we'll get started.

As I mentioned in the introduction, if we had six unknown variables and six linear equations, we could use the linear equations to solve for the six unknown variables. Let me write this out more formally. If we have, so not too formally, if we have N, so N, some positive integer, we have N unknown variables. Sometimes we just say unknowns, but we'll say unknown variables. And we have N linear equations, we can only be sure this is going to work if they're linear equations, so they can't be quadratic equations. We're not sure if that would work. But if we have N unknown variables and N linear equations, we can use the N equations, and we have to use each of the N equations to solve for the unknown variables.

Now, in this video, we're just going to stick with two unknown variables and two linear equations. The solution methods that we use for two linear equations with two unknown variables is the same as we would for a system of equations with 20 unknown variables and 20 linear equations, it would just take us a really long time to solve such a big system. Or we might even need to bring in a computer to help us solve such a large system. In fact, I think we would. So we're just going to stick with the two unknowns, two linear equations for now. And I want to introduce two methods to solve a system of linear equations by hand. So these are just by hand and algebraically. And the first one is called the substitution method, which you you really, as you'll see, you already, almost certainly know how to do that. I think we've done that already in this course. And then number two is the elimination method, which students tend to shy away from, but is really a powerful way of solving systems of equations, and actually ends up once you get comfortable with it, it takes a lot less work than the substitution method in many situations.

Here is the substitution method, you can see we've given some instructions that you can read yourself, essentially pick one equation and solve for one of the unknown variables, its unknown. And then take that expression that you have and substitute it into the other equation and solve for the one remaining unknown variable. So that sounds a little bit complicated. Let me show you what I mean, and I imagine you'll you'll get a much better understanding. So let's take this first expression here at the top. And I'm going to solve for Y. So I could minus 5X on both sides of the equation. I can rewrite this has to Y is equal to three minus 5X. And now I'm going to divide both sides of the equation by two. So I've got Y must be equal to three minus 5X divided by two. Now notice that I use the top equation, I'll call it, equation one. Now I want to take advantage of this second equation. And the fact that I have an expression for Y right here. And what I'm going to do is I'm going to sub this Y expression into what we'll call the second equation, and then solve for X. So let's take a look at what that does.

Replacing the, whoops that's not right, I'm going to replace, I'm going to replace the Y right here. So we're going to have 2X minus X plus three times three minus 5X over two is equal to negative one. Now notice that we eliminated all the Y variables, so we have no Y variable in this expression. And so next, we're going to solve for X. So let's go ahead and solve for X. Well, first I'll write down solve for X. So now let's solve for X. How am I going to solve for X? Well, I want to collect like terms. So I'm going to just have one term with an X in it, that's what I want. Looking at this expression, I don't really like this denominator down here. So I think what I'll do is I'll multiply both sides of this algebraic expression by two.

And when I multiply both sides by two, I get 4X. Right, the X doesn't change, but the coefficient in front of it is multiplied by two plus, I'll write it like this, minus 5X is equal to negative two, right, I'm multiplying two by both sides of the equation, and then that's going to have a two divided by two here so that this thing here is gone. Now I didn't multiply into the bracket. So why don't I do that next. We've got 4X plus nine, minus 15X is equal to negative two. Well, I won't go too quickly, I'm going to end up, if I collect the like terms, and I move and I subtract nine on both sides of the expression, I'm going to get negative 11X is equal to negative 11. Or X is equal to one.

Now I'm not sure that it's correct. If I've done the algebra correctly, X is going to be equal to one, but I can't be sure yet. I also don't know what the Y value is yet. So Y still remains unknown. So why don't we use the fact that X is equal to one and solve for Y? I can use either equation, and I should get the same answer for Y. But I'm going to use equation one, the top one, so we have 5X plus two, Y is equal to three, I'll replace this with five multiplied by one plus two, Y is equal to three. And I end up with 2Y is equal to three minus five. And it looks like we've got Y is equal to negative one.

So that's the answer that I've come up with. Now, how can I be sure that this is truly the answer? I can be sure this is the answer if these values correspond with the equations up here, so they give me the correct answer. And at least one of those equations that I know that these are going to be correct.

the correct answer. And at least one of these equations that I know that these are going to be correct. So we have 5X plus 2Y is equal to three and 2X plus 3Y is equal to negative one, and we had X is equal to one and Y is equal to negative one. While looking at equation one, I have five times one plus two times negative one, that is equal to, well it's supposed to be equal to three? Is it? Yes, it is. It's equal to three. We've got five minus two on the left hand side. Similarly with the second expression, the second linear equation, we have 2X. Well, I'll write that out again. I'll say we've got two minus one, plus three times minus one is equal to minus one, and yes, that's true. We've got two minus three on the left hand side, so we get negative one is equal to negative one. And that's how we make sure that we got the right answer.