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SUMMARY KEYWORDS

equal, tangent line, problem, vertical tangent, differentiable, limit, continuous, side, approaching, function, slope, vertical, converges, left hand side, derivative, called, picture, happening, formally, circumstance

SPEAKERS

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Welcome. In this lecture, I want to give you some pictures where, of what it can look like when a function is not differentiable at a particular value A. So the first one that we're going to do is we're going to look at a corner, so called a corner, it's just the terminology for it. Okay, so in this circumstance, I have my axes, I have my X, I have my Y, put my Y over here. And then I have my function. So maybe this is so this is a Y equals F of X function. So this function is Y equals F of X. Okay, and I'm looking at this particular value A. And so what's happening here, just pictorially understanding what's happening here is that my slope here, you're like, oh, this is definitely a positive slope. But on this side, this is definitely a negative slope. So there's no slope that it converges in on at A. Okay? So that's kind of like the hand wave, as a mathematician would say, way of understanding that. But let's kind of write out the problem a little bit more formally. So what is the problem here? And the problem is, is that, you know, formally with what I'm saying, is that, well, if I take the limit from the left hand side, right, or yeah, so if I take the limit from the left hand side, right, so if I'm coming in from this side, right, this is what I mean, when I say I'm taking the limit from the left hand side, at this particular value A, so as H approaches A from the left hand, sorry, H is approaching zero, that was actually the correct thing to say.

Right, so it's a little funny from there, because what I'm really talking about is zero, but that would be the left hand side. That, that will ultimately be what happens if I approach zero from the left hand side. Okay, so I'm adding negative numbers instead of positive numbers, which is why I have things that are less than A, okay? If I do that of my F of A plus H, minus my F of A, so this is my A plus H, got H there, and my A. And A divided by this H, right, that this does not equal. Actually, let's keep on adjusting how I want to do things here, but I'm going to do this in red. So here's that H. H, because I'm not using red for anything else, so I can use it there. Okay, then this does not equal, what happens if I approach from the other side. So it's the same thing as approaching A from this side. When I say that I'm approaching, right, when I'm approaching zero from that side, it means that I'm adding very small positive numbers. So it's like coming in on A from that side, so it's the same thing. Okay? So this is going to give me F of A plus H minus F of A, so this is like my A plus H, right, and this is all going to be over H. Right? But the problem is, is that these are not equal, right, which is the formal way of saying that coming in from this side, I get a different value from my slope than coming in from that side, but those have to be equal. Or, you know, another way to say this is the limit doesn't exist, because coming in from the different sides gives me different values.

Okay, great. So the next type of problem that we could have is discont, a discontinuity. So I'll write it up here. I have two more to fit in here, so it helps if I can write that up higher. Okay, so what is kind of the picture of what's going on here? I'm just going to draw a function for you that is not continuous at some particular value A. And I have some value A here, and my function, so maybe I'll set my, maybe it goes like this. Right? So maybe this is my Y equals, now I'll call it G of X, just to emphasize it, it doesn't always have the same name. Okay? But G is not continuous. So it's not differentiable. So, oops, so that should be an A. Okay? So the problem here is, and I'll write the problem in red, so G is not continuous, right? And we talked about last time, if I'm differentiable, then I'm continuous. So if I'm not continuous, then I'm not differentiable. So, or so not continuous at A, that's kind of a key point here. So it's not continuous at A. Okay? So G is not differentiable at A, so being not continuous is enough to be not differentiable. Okay? And then, the last one is like a vertical, is called the vertical tangent. Okay, so now we're going to look at a vertical tangent.

Okay, and so for this one, let's look at a picture. So I have a circumstance. So this is like my X, or my T, here let's go ahead and make this my T this time to remind us that things aren't always F and X. So maybe this is my T. Okay, and then I have Y here, so this is my Y, and then I have P of T, maybe this is my P of T, looks like this. So this is my Y equals P of T, okay? And then if you tried to look at the tangent line here, so at, so here, this is T equals A. And if I was actually to kind of look at the tangent line here, then it would actually be vertical. Okay? And so the problem is that in kind of writing this formally, so the problem, in a mathematical sense, but you could just identify it as being that the tangent line is vertical, is that this limit if I was, as you know, I approach, so now it's the limit as H goes to zero of F of A plus H minus F of A divided by H. So the same thing, this limit that should give me the derivative. The problem is, is that this actually equals, you know, plus or minus infinity. Okay? Oops, I'm running off the board.

So this is actually going to equal plus or minus infinity. Okay, so let's just kind of go through. So these are kind of the pictures that go with some of the ways that something can fail to be differentiable at a particular value. And so for this function here, what happened was, as I approached A from this side, or I approached A from this side, my slopes look different, they're not converging in on something, okay? And so that's like a corner. You can have, if your function is not continuous, it's definitely not differentiable. So this is an example of a function that's not continuous, so that it's not differentiable. And then the last one is of having a vertical tangent, right, which is like an up and down tangent line there, which is the same thing as remember, we compute the derivative by this limit, but if that limit is infinite, as in what is, which is what happens when you have a vertical tangent, then you don't have a derivative. Okay, so I hope that made some sense, and I'll see you in the next lecture.