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SPEAKERS

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Welcome. In this lecture, we're going to go through an example of showing a function's not differentiable. This is one of these circumstances where, will you not be able to do further mathematics if you haven't seen this or don't know how to do this? Well, no, it's not going to stop you from doing further mathematics. So why did I choose to go through this example? I mean, one reason is just, you're going to feel really accomplished, or hopefully, when you can do something like this, and you you're able to kind of put all these things together and do this. Well, the other reason I like to do this is because it involves a lot of different pieces of things that I'd like you to get comfortable with. A lot of mathematics is just getting comfortable with how things work. And so this one involves a lot of them in kind of nice ways, so that you have to kind of think about how these things really work and what they really mean. And so I feel like it's, you know, sometimes it's good to do some examples like that, to kind of imprint these things in your mind, or maybe emphasize for you that oh, wait, I don't quite understand how this thing works. So maybe I'd better go back and look at it. And this is one of these examples. The last thing is that I would say about it is that it's important not to just understand how to take a derivative, but also to understand when you can't. Because otherwise, you're going to have weird things that go on, and you're not going to understand why things aren't working out. So, without further ado, hopefully I've emphasized this enough, so you're excited about this. So here's my example.

And the example is I'm going to take the function, which is going to be F of X equals the absolute value of X, I'm going to take F of X equals the absolute value of X. And we're going to look at the derivative of this function, right? So I'm going to look at the derivative, and I'm going to evaluate it at zero. And I want to know what I get out. Okay, so let's kind of look at the picture. And we have that there. Okay, so we have our X axis and we have our Y axis. Now, let's look at Y equals F of X. Now I'm going to graph Y equals F of X.

And in here, you're going to see something, right? So Y equals F of X. So this is actually the absolute value of X. And what do we actually know about the absolute value of X. So you can actually write the absolute value of X, like as a piecewise defined function, right? We know that the absolute value of X

is actually equal to, so it's going to equal X, if X is greater than or equal to zero. It's going to equal minus X if X is less than or equal to zero. So looking at this here, right, we actually have, so here we have that F of X is going to equal, so it's the absolute value of X, we're greater than zero here. So this is just actually equal to X. And over here, this is actually equal to minus X. Okay? So looking at it, I say oh, wait, over here, the slope is, right, this slope is 1, right? And you can even tell that from the fact this is, is X over here, and here, the slope is negative 1. Hmm, well this is interesting, because I, I can get infinitely close, as close as I want to the right, this is where we have zero. So if I'm looking at this, I can, on this side, I can get infinitely close and say the slope should be 1. On this side I can get infinitely close and say the slope should be minus 1. So you're kind of confused. Well, what do I, what's going on here? And we kind of remember that the derivative, this is exactly why the limit, the right limit and the left limit should have to be the same. Okay? So we know that.

We know that a function F of X is differentiable, right? So what do we know? So we know that F of X is going to be differentiable where, right, this is something we know. So it has to be differentiable where we have that this limit, right, as H approaches zero, I don't know what color I'm going to use here, I'm going to use orange, I hope that's not confusing, where H approaches zero of, right? Well, in general, it's going to be of F of, right, in general is going to be, you know, you're going to have here like X plus H, this is that H, and you're going to have X and then you're going to divide this by H. I know like, my, my color scheme is falling apart a little bit here. But right, so in this circumstance, it would be, right, so in our circumstance, so here. Right, so it's where this exists, right? But what is that here? So here, this is going to be the limit. And then we have as, you know, this is the limit. Okay, where we have as H goes to zero. And then we have right of the stuff we have X plus H, right, minus my, right, minus this and then we divided by H exists.

Okay, well, so this is a, we know that we need to understand this. Okay, so let's, let's look at understanding this for a moment, right? And particularly, we're looking at this at zero, right? Particularly at zero, zero. So particularly for X equals zero, right? Because that's what in the end, we're looking for the derivative at zero, okay? So let's kind of evaluate that. So we want to understand, so for, right, I want to understand what this looks like to kind of simplify it. Okay? So for X equals zero, so for X equals zero, what is this here? Okay, I'm going to know, right, so and maybe I'll point out here. So need to understand. So need to understand or simplify, okay? So that's what we're going to do first. So let's go ahead and do that. Okay, so for X equals zero, we're going to have that what is this thing here? We have that X plus H, like the absolute value of that, minus the absolute value of X over H? Well, what are these, you know, these, these are these X's, right? This is the same thing as I'm saying X equals zero, I'm actually looking at these X's being zero. Okay? So this is actually going to equal, well, I'm just going to put zero in there. So I've got zero, oops, I've got zero and then I'm adding H. And then I've got a minus, I'm just putting zero in for each of these X's and I'm going to minus zero and divide by H.

Okay, but what is that this is just equal to the absolute value of H over H, okay? So let's kind of look at that. Well, what is this? And looking at the number line, okay, so we have, right look at, remembering this here. Okay? So we have that the absolute value of H over H is going to equal, well, what do we have if H is greater than zero? Then we just get H. So we get so for, so if H is greater than zero, what is this look like here? Then we're going to get, right, this is just going to be H over H, which

is going to equal 1. Okay, but what about if H is less than zero? Okay, well, now we get this minus H, just looking again at this formula. So we get minus H over H which is equal to minus 1, okay? Now we know, so for a limit to exist.

So for the limit as H goes to zero to exist, we're going to need that, right, we have to, we need that if I approach from both sides, I get the same thing, right? So we need that this is going to equal the limit, like this, right? We know that's what it means for that limit to exist. So let's kind of draw this out here, okay, a little bit. So what do we have kind of putting together what we have so far with this? Okay? And here's zero. And I'm going to approach from this, right, so if I'm approaching from this side, right, then I have positive, my H is greater than zero, I'm like adding something greater than zero. And so I'm in this circumstance, okay? So I'm going to have that the absolute value of H over H is going to equal 1, right? But if I'm approaching from this side, then I have that the absolute value of H over H is going to equal minus 1. Okay, so what does that give me? So now let's look at those limits. So I have the limit as H approaches zero, from the plus side, so here, so we have H is approaching zero from the plus side.

Right, what do I get there? So I'm going to get H over H, like looking at this here is going to give me just 1. Right? Because I'm looking at these limits, right? I've just simplified this down, right? So maybe I can emphasize this somewhere here. So this here, right, is the same thing. From over here I know that this here is the same thing, right, as that there. Okay? So this is equal to, right, from over there, I know that this is equal to X plus H, right? X over H. Okay, so when I'm taking this limit, I'm actually taking this limit. Okay. So I'm looking at this, I'm getting 1. And if I come from the other side, so my limit, and I go ahead and take this 1, I'm getting minus 1, because I'm just taking, right if I'm coming from this side, I'm always 1, but if I'm coming from this side, I'm always minus 1. So I'm going to get out minus 1. Okay? But these are not equal. Right? So these are not equal. So.

So what do I know? I know that this limit that I was looking at from the start. So this limit as H approaches zero, and then we had this, right, so this X plus H minus X over H does not exist. Okay? So, the function is not differentiable. Okay? So the output of all of this is that, so we have that F of X equals the absolute value of X is not differentiable.

At X equals zero, okay, so at X equals zero. Okay, let's go through this. There's a lot going on here. Right, we want to know what happens at X equals zero, we're pretty sure the slope over here is 1, the slope over here is minus 1. So probably we're going to run into a problem as we take the limit, because we have to have that the limits on both sides are the same. But let's actually kind of look at the limit. So we're looking, we need that this limit exists, right? In our circumstance, that's the same thing as this. Okay, and so we go ahead and we try to kind of simplify what this looks like. And we realize like at X equals zero, because we're looking for the derivative at X equals zero. So we go ahead and put X equals zero in and we see that we actually have a much simpler form which is like this. And then we remember how the absolute value works, right, and are able to reduce this down to being 1 if H is greater than zero or minus 1 if H is less than zero. And this is actually starting to correspond with how we think that the derivative should go be 1 here and minus 1 there. Okay? Now for this derivative to exist, the limit has to exist, which means the left hand and right hand limits have

to be the same. And you can kind of see this function, right, which is this function, which is this function, right, is actually going to end up being 1 for H greater than zero and minus 1 for H less than zero. Okay?

Well, that would tell me that this limit, right, is 1 and that limit is minus 1. But those aren't equal. Okay? So this limit actually doesn't exist, or not differentiable at X equals zero. Okay? I know this was like kind of a lot. It's not that you shouldn't try to understand it, because you actually should. Even though it's confusing, you should try to understand it, because there's a lot of different pieces in it, and you'd have to get all those pieces straight in your head. And in some sense, kind of understanding all the pieces of it, it's going to help to make sure that you actually kind of understand everything that we've been doing. So it's actually kind of worth looking through it. And then you can also feel like, oh, I know that the derivative shouldn't exist there. Because there's a problem there, like there's a corner and I know that's a problem. But you want more than that, like you want to, now you can actually show that this is true, like this intuition you have is actually, works and you can show it and that's kind of exciting. Okay, so I hope that made some sense, and I'll see you in the next lecture.