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SPEAKERS

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Welcome. In this lecture, I'm going to talk about differentiability, which is what it means to be able to take a derivative. Okay? So we're going to start out I would say, right, so what is kind of the, why would we lecture about this is because not every function has a derivative on its domain, okay? So not every function has a derivative on its whole domain. Okay, so that's kind of the whole point of this, but let's going to talk about it a little bit. Okay, so here's kind of the formal definition of differentiability. So this is the formal definition for differentiability or what it means to be differentiable. Okay, so we're going to say that F is differentiable.

Okay, at A, so at this particular value A, when we have that this derivative does not exist. So this derivative, oh, so it's differentiable when this derivative does exist, okay, so when this exists, i.e., when we're going to have that, so what am I saying, this is that the limit as H approaches zero of, and then we have F of A plus H minus F of A, so we can go ahead and put this in, A plus H minus A and this is all going to be over H right? So this limit exists, i.e., so another way of saying this is that we have the limit right as H approaches zero from the minus side of, right, and of F of A plus H minus F of A over H is going to equal the same thing as if I approach it from the positive side of F of A plus H minus F of A, all over H okay? And so kind of the way that you can kind of picture that actually of what's happening is that A, I'm looking at the limit as I approach from this side versus as I approach from this side. So do my slopes match up on either side? Okay, so that's what's going on there. Now let's do kind of first examples

Okay, so this is of differentiable functions. So functions that are differentiable. Right? I guess I didn't kind of say what that means. So let me kind of write that, which is that if F is differentiable for all A, then it's called differentiable, okay? So, okay that does fit on here. So, if my function F is differentiable at each A, right, so, at each kind of value A, then F is differentiable. I'm writing a little crooked, so I'll go down to make it smoother.

Then we actually say that F is differentiable. Right, versus saying it's just differentiable at A, is that I can actually take this limit and figure out the slope at every single value A. Okay? So let's go ahead and do some examples of differentiable functions. So all polynomials are differentiable. There'll be and do some examples of differentiable functions. So all polynomials are differentiable. There'll be more examples as we go along. The functions like E to the X, the natural log of X, are differentiable. And same thing with sine of X and cosine of X. So sine of X and cosine of X. Okay? So now let's kind of look at our fun, I'm going to call these kind of like our fun facts. So the first kind of facts to know. So these are kind of our first examples, but let's also look at some kind of fun facts. So.

Okay, so the first one is, if we know that F is differentiable at A, if F is differentiable at A, then F has to be continuous at A. Then we know that F is actually continuous at A. Now, why is that important? Because it actually also goes the other way, which is that if F is not differentiable at A, then it's not, or sorry, if it's not continuous at A, then it's not differentiable. Okay? So that's kind of going the other way. So, and then the last thing is that, but it's kind of good and important to know that not every continuous function is differentiable. Not every continuous function is differentiable.

Okay, so let's kind of recap what we have here. Okay, a function is going to be called differentiable at A, if the derivative exists there. What does that mean? It means that this limit exists. What does that mean? It means that you get the same thing if you try to take the limit from approaching A from this side or this side, which is the same thing as H approaching from like the zero minus side or the zero plus side. Okay? And so there are some functions. We already know our differentiable, polynomials are, E to the X, a natural log of X, sine of X, cosine of X. These are all differentiable. We know that if a function is differentiable, then it's continuous. Logically then what that tells us is, if a function is not continuous, then it's not differentiable, okay? But it's really good to know that being continuous is not enough. Being continuous does not necessarily mean that you're derivative and we'll go through some examples where you'll see that. Okay, so I hope that made some sense, and I'll see you in the next lecture.