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SPEAKERS

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Welcome. In this lecture, I want to give a bit of an introduction to tangent lines. This is, you know, I thought for a while about the word I wanted to use there and decided introduction, partly because I'm going to introduce you to a friend. Why is this a friend? because tangent lines are good for approximation. Approximation is super important, because in most circumstances, you can't actually compute any, okay? So it's very good to get used to approximation. The other thing is that tangent lines give you good visualizations so that you can get a good feel for what's going on in different circumstances. If, you know everybody thinks differently, but for many people in order to feel like they understand what's going on with a function or derivative or something, they want a good visualization in their mind. Okay, so last time, you saw a particular kind of picture. Picturing I'm very fond of, okay, and you have, so you have this, maybe you have T here I think was how I did this. So I have T, and then I have, this could be like P or Y. Let's do this over here, I think. Okay. So we want this could be, you know, maybe P, or Y depending on who you are, and how you like to write these things. Okay, and then you have your your graph of your function.

Okay, so this was, if we're on here we have, so Y is going to equal, and maybe we'll kind of write it like this to kind of make it clear. P is our outputs, input T I get an output P of T. Okay? And then if I have some kind of point A here, then I have this picture here. Okay, so I have, okay, so, and the slope of this is equal to or this is actually the slope of this curve is actually kind of you, you take this line that just kind of, just touches it, and it has the same slope there, which is equal to the limit as H goes to zero, and then you'd have P of, and then you would have A plus H minus P of A, all divided by H. And I usually like to put in this point because people often forget kind of, what are we, what is actually a point in a graph of a function, this is going to be A and then this is P of A. So that's kind of where they touch, or where that line gives you the slope, is there. Okay, so what's going to be new? So new is what is this line? So new is what was this line?

Okay? The answer, and then we'll kind of say formally, but the answer here is, so this is the tangent line. So this line is the tangent line. Okay, go over it in red just to make sure you can see this is the tangent line, and then put a nice box around it. So this is the tangent line to, so maybe the whole thing should be kind of red. So to the tangent line to P at T equals A. The whole thing is kind of what it is. I'll just put a nice box around this, so that we can then talk about what this is, okay? So formally, so what do we have going on? So, formally

okay, um, what is this? So this is the line where we have, okay, so the first thing that we know is that its slope is equal to with slope, and this, this this function P, so we have slope P prime, and then this is evaluated at A. Okay, so this is the first thing that's true about this line. And the second thing we know about this line is that it's contains a point so, and containing the point, right, it has, it touches there. So it contains that point there. Okay. So this is the point, it's going to be where your first coordinate is A and your second coordinate is sticking that into the function P. Okay? So, so this is what we know about what this line is. But that tells me that I know the slope, I know the points, so I can use a point slope form, okay? So we can use this the, so can use the point slope form. So point slope form for a line, okay, to find the equation. And this is where, you know, when I was taught the point slope form, I was taught it is like some kind of random formula. And I could never remember it and found it very, very confusing. And so I like to do this a little bit differently. Because what you're actually looking at, you know, what is this kind of point slope form actually look like? Well, so this is P prime of, I guess there was a T here, okay, is going to equal?

Well, I know this is my slope. So this is my rise over run. Okay? So if I had another point on this line, and maybe I'll just actually kind of draw it on the line, I have another point here. Okay, and this is an any other point on this line. Okay? Then this is going to just look like, you know, with my variables here, this is just going to look like T and then Y. Okay. Well, what do we know, if we have two points on the line, like what is the slope between them, that's going to be rise over run. Okay, so let's just look at the rise over run here. So I'm going to take the rise, so Y minus P of A.

And then I'm going to divide this by, and then on the bottom, I just have my run, right, so I take T minus A, I did Y minus P of A, so now I do T minus A. So I have T minus A. And I'm just going to emphasize for you, right, this was my, this, I really am just doing my rise over run for the slope. Okay? So now I just kind of rearrange this, okay, so this kind of gives me that,

that actually should be an A, A in my notes, and then I was not believing myself for a second. So you can see why, because this is the slope, right? This slope is a derivative at A, which is this, so that's why I want that to be P prime of A, okay? So then what I get in the next step, and I'm just going to rearrange this a little bit by multiplying both sides by T minus A, okay? So when I multiply both sides by T minus A, that brings T minus A over here. So I get P prime times P prime evaluated at A times, and I'm just going to multiply that by that T minus A, which just came from the bottom because I'm multiplying both sides by T minus A. Okay. And then this is just going to equal Y minus and then I have P of A. Okay, that's just what's left on the top because I multiplied both sides by T minus A, and that got rid of what was on the bottom. Okay, great. So now usually we like to solve for Y, just we do. So this gives me Y equals, and then I have this part and then I'm going to add this P of A to both sides. So I'm going to get P prime of A times T plus this is my input variable. So this is like my slope times my input variable. And then I have like my intercept, which is actually going to be right, so what do I get? I've got to, you know, there's some kinds of distributing that goes on here, so I'm kind of doing a lot of steps at once. So maybe try to work through those on your own to make sure that you kind of are following along. And then I have minus, and then I've got A times P prime of A, okay? And maybe actually, I can put a little bit of steps in here. Let's distribute this out. So I got P prime of A times T, minus P prime of A times A, the minus coming from there. P prime of A, there we go. Okay? So when I'm trying to isolate Y, which is what I'm doing, this stuff stays on the same side. So that's like my P prime, right? This goes here, this ends up in here, but I had to add this to both sides, which is how we got that there. Okay? So this is actually the equation that we get for the tangent line. But sometimes, right, this is with a funny kind of variable situation. So let's give kind of a more general one. Okay, so if the function, so if the function is instead, is so now we have F of X, right? Then what do we get the equation is?

Okay, so here's the equation, when, this is more typical circumstance where my input is X and my function is F. So here, I'm going to get that the equation is Y equals F prime of A times T plus F of A minus A times F prime of A. Okay? Did I close all my parentheses? No, I have one more to close, alright, I might not have closed all my parentheses here either. There we go. Okay. 1, 2, 3, okay. Okay, so this is the equation that we get.

So let's just kind of go through all this. So what is the tangent line at the point? It's kind of the line that touches at that point, gives you the slope at, you know, has slope at that point. It contains that point, because that's where they touch. It is actually like a first order approximation for the function at that, at that point, okay. But the, the wording is the tangent line to P at T equals A. And then there's going to be a warning, and I'm kind of going to go through this in another lecture, which is that the tangent line depends on the point that you're taking it at. Okay? But once you do have like that, you know, you have that slope, you have that point, we can just use the point slope form. I like to do it this way. I actually, honestly, every time, I don't memorize the point slope formula, it's a mess. What I do is, I always think of it as the rise over run, and maybe even draw the picture to keep it all straight. Because the one thing that you might do is swap, if I'm taking this minus this, I have to take this minus this, I can't go this minus this, right, you got to keep the order the same. But then you're doing the rise over run. And then you just kind of manipulate it until you isolate Y if that makes you happier. And then that's just the equation for the tangent line. Okay, so I hope that made some sense, and I'll see you in the next lecture.