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SPEAKERS

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Welcome. So in the past few lectures, we've been talking about how, okay, if I wanted to look at an average rate of change, or how does the output change when I change the input, right? Then we looked at two points, we looked at the line between them, and that was the slope, or we change, we took the change of output divided by the change of input. So we've done this in several contexts. And we said, well, maybe I actually want to know how much I'm changing at an exact instant. And if I want to know how I'm changing it at an exact instant, then I want to do this zooming in process, right, where I'm looking at values that are closer and closer. So I was looking at, I wanted to look at the instantaneous velocity at time equals 5, right? So how did I do that? I looked at a change in time, that was very, very close to 5, right? So I took 5.1 minus 5, and then, you know, 5.01 minus 5, and so on. And what was that little change? That was that little change was at H. And so we ended up taking this limit, right, the change of, you know, a position where I'm taking my times being 5 and something super close. So five plus H, and I divided that by this change in time, which was H. Okay, so well, so now I want to kind of connect all these up, and say, kind of formally what we're actually doing. Okay? So in this previous circumstance, we had that the instantaneous velocity.

Was actually, so that actually was, so this was the instantaneous velocity at time T equals 5, that was what we were looking at in particular. How did we get that? We took the limit, as my change in time got really, really small. So this H here is my change in time. And as this got really, really small, so I have the change in position. So I have 5, plus the change in time, right, so 5 plus a very small value. So I'm looking at the position when my inputs very close to 5, but not quite 5 dividing by H. So we already did that in the past few lectures. And then this is also the slope. So here's another interpretation. So this is also the slope of the tangent line to the function.

To the function, P of T, right at the point, right, where I'm inputting, okay, so if I, if I replace this 5 with an A, and I'm actually going to do that. So let's replace this for a second to be more general. So let's say T equals A, and so this could be an A, and an A, so I'm looking at a plus H and A. So let's instead look at, so we're going to look at this point where I have A and I'm inputting that into my function, so I have P of A. Okay, so I'm looking at a particular point, so let's kind of draw that picture.

And then oh, okay, well, it's just going to be P. So now we're going to draw, so who knows what this function looks like. But maybe let's just say that it looks like something like this. If my A is going to be here then this point here, right, is going to be A and then I'm going to take, so for this being the graph, so this is kind of like Y equals P of A, that's kind of the, or P of T, sorry. I want that to be P of T. So we're going to take Y equals P of T, so we're going to get A, so then I need to put this in to be P of A.

Okay, so this is what the point there would look like. And now what I actually want to look at, when I'm taking this instantaneous velocity here, what I'm actually looking at is I'm looking at the slope of the tangent line here. Okay? So my slope here of this tangent line is going to equal this instantaneous velocity, right? So this is actually going to equal the limit as H goes to zero of P of A plus H minus P of A over H. Okay, so that's the slope at the particular point. So we're doing something slightly different here, right? Because we're taking, instead of taking these secant lines, which are the lines that go through two points, we're looking at what is the limit, right? So if I took, as I go, right, so I have secant line, and then it goes, you know, if I go through points that are closer and closer, pretty soon, the secant lines are actually going to approach a slope, okay? And so I'm actually looking at the slope of this tangent line here. And so then what is this? So here's kind of this new interpretation.

Okay, and so the derivative of the function F at A. So F at a particular point A is, so I'm going to take, and this is the notation for it, so I'm taking the function F, I'm going to stick a little prime thing next to it, this means I'm taking the derivative, so I'm actually looking at the rate of change. And so this is at A. Okay, and that's going to equal just kind of like what we did over there, we're going to take the limit, as, and now we want this to get really, really small, okay, so we have this, this this H thing, which is going to get really, really small, so we have this H has to approach zero of, and then we're going to have here we're going to have F of A plus H. So that's kind of our first input is just to add a tiny little bit. So I'm looking at how the output changes when I look at going from the value at this important A to this very, very close value, this A plus H, and then I want to divide this by.

And what I want to divide it by is actually that H again, right? So I want to divide it by this tiny little H value that's going to get smaller and smaller and smaller and smaller and smaller. Okay, and what I end up getting then as I take this limit, right, so I'm looking at this A, I'm looking at the A plus H, which is closer and closer and closer, so I'm looking at those secant lines. But as I do that, those secant lines get closer and closer and closer to this actual slope of the tangent line. Okay? So these are just a few ways to connect up instantaneous rate of change, slope of tangent line, and the derivative is this new thing that looks like a crazy limit. But it's actually a very useful concept because we're going to be going from quantities to rate of change. So I hope that made some sense, and I'll see you in the next lecture.