PfaffModule4L02

Sun, 1/30 1:47AM 11:25

SUMMARY KEYWORDS

secant line, function, equals, slope, points, interval, line, change, rate, average velocity, input, divided, values, approximate, x naught, naught, lecture, equalities, mx, velocity

SPEAKERS

Catherine Pfaff

Welcome. In this lecture, we're going to move from talking about just lines to actually kind of getting closer to talking about the rate of change, or at least the average rate of change for more general functions. So last time, we talked about the fact that when we have, right, we talked about that for a line Y equals MX plus B. So for a line, Y equals MX plus B, right? If we wanted to find how Y changed as X changed, or how Y changed with respect to X, then we would look at the slope, right? So for this line, the slope says, how Y changes as X changes, right? And this is kind of important that, you know, when I just say, if I just say the rate of change of Y, I don't know that I mean as X changes. So that's kind of actually important to say that. And we're going to move on from that this time, and we're going to talk about what we're going to do if the function is not actually of that form. So what if a function is not of the form, right, because that's not actually a function exactly as written there. But we want it to be not of the form F of X equals MX plus B or some kind of you know, variant of that function. Okay. So let's we're going to like in this lecture, we're going to kind of look at it first try for what we're going to do. So here's our first try. And we're going to look at something called secant lines, which are just the lines that connect two points on the curve for the function. Okay, so we're going to approximate the slope using secant lines, okay, so with secant lines.

So what we're going to get is going to be the average rate of change. So we're going to get the average rate of change of kind of the output function, so get the average rate of change. And I'll draw a picture that will help you understand what I mean of change over the interval. And then let's look at this kind of interval. So I'm going to kind of, right so this is the interval X naught, X one. Okay, so before I kind of jump into what you actually end up getting there, let's look at the picture. So I have my I said, I'm looking at the change of Y with respect to X generally here, so I have an X axis. So this is my X axis, and then I have a Y axis.

And then I have the graph of some kind of function. So maybe the graph of my function looks something like this. Okay, so this is the graph. This is the graph Y equals F of X. So just to remind you, what is the point on the graph of Y equals F of X look like? Right? The first point is always X, the second point is always F of X for each value X. Okay, so now we're going to have, we're going to look at two points. So we said we're looking on the interval X naught X one. So right, so these are input values. So these are going to come from the horizontal axis. So let's say I have X naught here, naught, and maybe X one's all the way somewhere up here. Okay, so these are two input values. And then what I'm going to be looking at, so what do I mean by the secant line here? So I'm going to go up, tick, tick, tick, I'm going to look at this point here. I'm going to go up, tick, tick, tick, tick, tick, I'm going tolook at this point here. Then I'm going to draw the line between them.

So we're going to attempt to do this. Not horrible, okay? So this is a line, or it should be a line. Okay, and this is going to be some kind of Y equals MX plus B because it's a line. We don't know what our M and our B are, but it's a line. And this is called, this is the secant line because I just kind of went through the, I took the line that went through the two points. It's not a tangent line. It's not just like touching, it's actually a secant line that goes through two points. Okay? So what are our points on here? So let's say that we have, right, so if we're looking here, and if we just think of these in terms of kind of, from the earlier picture, maybe this would be Y naught, and then this would be over here, some kind of Y one. Then if I wanted to know the slope of this line, right, how would I actually figure out what this M is? I would actually get here that, well, this slope, M is going to equal, so I would get M is going to equal, right, so this is just rise over run. So it's Y one minus Y zero, or delta Y, which is Y one minus Y zero. And then I'm going to divide this by what I get here for my change in my run, so my X one minus X zero, that distance down there. Right, and that would give me the slope of this line. Okay, but we're going to use that to approximate here okay? And something to kind of notice here is that, what is this Y one, actually? It's good to notice that this is actually F of X one, right? And this here is actually F of X zero. Okay, so we're actually taking a difference of F values, and that's actually going to give us our slope there. Okay. So what we're going to get is that the average rate of change over that intervals, so the average rate of change, we have change over this interval.

Over this interval where I have, so now I can kind of remember that these are, this is an input interval, so I can go ahead and already write it in this orange. So the average rate of change over this interval, well we have this from, it's actually just going to come from the slope, that's our rate of change. So it's actually going to be this delta Y, which is equal to right, so this is Y one minus Y zero, which is the same thing as F of X one minus F of X zero. I'm just kind of going through, and I'm going to divide each of these, these are all equal to the same thing. So we're just going to have some equalities here, and then we're going to have delta X, right, which over here, so this is just X one minus X zero. And that's just X one minus X zero. Okay? So if I'm just looking at so if I have an arbitrary function, and I want to kind of approximate what is the rate of change, or the average rate of change on that interval, I'm just going to take the slope of the line that goes through the two, right, I took my two endpoints of the interval, I looked at the points that I get by plugging, to get the Y values, I plug these X naught X one into the function, right, so I did that. And then that actually, when I took that slope, so that's the change in Y over the change in X, or the change in F really over the change in X, then that would actually give me the average rate of change over that interval, even if the interval isn't actually, even if the function isn't actually a line. Okay? Now, I want to connect this up with kind of a key example because we, it's one that our brains understand so well. That what do we, what if our function is a position function? Okay, so what if instead we're looking at position?

Okay, so what if this is instead, Y equals P of T? So then this is actually a T, and this would be actually a T one. And this would actually be a T, zero, okay? And then what's going to happen over here so now what we're looking at is, instead, so now we're looking at the interval, right, T zero, T one. Right? Because our inputs are time inputs, then this rate of change is to keep this straight in our mind is actually the velocity, okay? So if you want the average velocity on that interval, you would actually look through the secant line through those two points. Okay, and so the average velocity is going to equal so now, right we're looking at this this equation here because this is for a function, so we have P of T one minus P of T two divided by, and now this is, oops, sorry, this is a T zero. So this is T one minus T zero. Okay?

So what we've done is we've actually just used this formula, we just had to change around a few variables. But this is good to know this rate of change that we're looking for, you can always have in the back of your mind, this is something that you're used to, which is just your velocity, because you know this from driving or bicycling or walking, okay, so it's a good example to have in the back of your mind. And that circumstances rate of change, your average rate of change is the average velocity. And then you're looking at the interval, T zero T one, because usually your input is T because it's time. So you kind of want to change your labels on your axes and so on. But the exact same thing is going to work, we just have to actually use the formula that has the function which is now P. So that we're looking at a change in position divided by a change in time. Okay, so it's kind of like how far you went divided by how long? Like, you know, the miles per hour, right? Okay, so, hope that made some sense, and I'll see you in the next lecture.