Exploring Motor Learning Differences Between Elite and Non-Elite Athletes Using Nonlinear Dynamical Analysis

Sijad Ghani

A Thesis Submitted to the Faculty of Graduate Studies in Partial Fulfillment of the Requirements for the Degree of Master of Science

Graduate Program in Kinesiology & Health Science
York University
Toronto, Ontario

August 2021

ABSTRACT

The application of nonlinear analytical tools to motor control studies is a promising approach. Measuring the complexity of a time-series of kinematic variables to explore motor learning differences allows us to discriminate between groups. The aim of the current study was to test the efficacy of nonlinear analysis, such as approximate entropy (ApEn), to effectively discriminate between elite and non-elite athletes' data. Using approximate entropy, we were able to discriminate between elite and non-elite athletes by discerning the level of regularity present in each group's time-series data of kinematic variables. An extension of our entropy analysis in conjunction with other nonlinear analytical tools affords us the possibility to better explore underlying neuromotor effects that may still be present in elite athletes with prior concussion.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1: INTRODUCTION

1.1 Motor Learning

Basic concepts

Motor learning is a composite of multiple components, covering concepts in adaptation, skill acquisition and decision making.[1-3] Depending on the task, different anatomical structures are involved.[4] Overall, it can be divided into at least two categories: sequence learning and sensorimotor adaptation.[5] In sequence learning, isolated movements are combined into a single smooth action. Sensorimotor adaptation covers learning in which movements are modified in response to changes in sensory inputs or motor outputs.[6] In a broad sense, sensorimotor adaptation tasks help us develop a better understanding of how humans represent their environment, body mechanics, and the interplay between the two during movement planning and execution.

Most learning curves across various tasks conform to a model proposed by Fitts and Posner.[7] In this model, skill is acquired in three distinct stages: cognitive, associative, and autonomous. In the cognitive stage, task goals are established, and the appropriate sequence of action are chosen. In the associative stage, the selected sequence of action is executed, this is where the subject is looking to translate declarative knowledge into procedural knowledge. In the final stage, the learned skill is established into an automatized routine. This model is initially marked by rapid improvements, followed by a plateau phase.

A complementary model to Fitts and Posner's was proposed by Vereijken et al[8] describing another three-stage theory: novice, advanced, expert. This theory of motor learning accounts for reductions in body degrees of freedom[9] observed in the acquisition of new skill. The first stage is marked by freezing degrees of freedom through co-contraction of both agonists and antagonists to constrain a joint for facilitation of a movement. In the advanced and expert stages, degrees of freedom are progressively released allowing more sophisticated synergies among multiple joints to smoothen out movements.

In laboratory settings, motor learning is typically studied by having participants maneuver the handle of a joystick and make reaching movements. Learning is induced by the resistive force applied by the robotic arm on the hand of the participant.[10] In such a novel environment, the motor system is tasked with learning to correctly predict the perturbing forces in order to cancel them out during the performance of the movement. This demonstrates the ability of the motor system to create a map of the environment between the somatosensory input and the appropriate output required to counter the external perturbations. This model of the environment is constantly updated as the experiment progresses.[11,12]

Cortical mechanisms of motor learning

The underlying circuitry and neural mechanism of motor learning are not localized to a particular area of the brain, making it difficult to pinpoint an exact region. This complex circuitry encompasses the primary motor cortex, premotor and supplementary motor cortices, basal ganglia including the neostriatum, and cerebellum.[13-15] Given that motor learning comprises distinct phases, they are signalled by different brain activity patterns. Overall, an overactivation of the frontal and parietal regions is exhibited in the early phase due to the high attentional costs.[16,17] Following that, in the later phase of learning (i.e. autonomous phase) there is less dependence on attention-executive networks and more optimized activity of cortical and subcortical motor areas.[18,19]

There is strong evidence in the literature for the support of cerebellar and striatal thalamocortical pathways contributing to early learning in sensorimotor adaption.[5] The cerebellum contributes to sensorimotor adaptation by predicting sensory results of actions related to motor commands and an internal map of the body dynamics, these representations are then updated through error feedback.[11,20]

Further evidence for cerebellar involvement can be found in patients with cerebellar damage and its effects on sensorimotor control and adaptation. Attenuated adaptation is common in cerebellar ataxia patients, reflecting a compromised learning system.[21,22] Support for involvement of basal ganglia networks come from neuroimaging studies during both kinetic and kinematic tasks.[17,23] Furthermore, basal ganglia circuits along with prefrontal regions are implicated in studies looking to monitor reward. The basal ganglia are known to be involved with promoting movement associated with reward.[24] In the early phases of adaptation, basal ganglia circuitry provides support for important cognitive functions such as attention and working memory. It is possible that these structures contribute to sensorimotor adaptation by detecting and correcting motor errors, as well as storing new representations.

Concussion is a form of brain injury that induces an altered metabolic state in the brain.[25,26] Previous analysis shows that relative to their healthy peers, those with a history of concussion show significant differences in behavioural performance deficits in variables such as timing or trajectory, or both. These findings indicate that networks involved in novel visuomotor tasks can be affected by concussion.[27,28]

Relative to other brain injuries that cause focal damage, injury from a concussion is more diffuse.[29] Concussive trauma happens as a result of acceleration and deceleration forces applied to the head. The application of these forces triggers a metabolic cascade in the brain. The events of this cascade initiate an energy crisis that arise from both increased glucose requirement and decreased supply of glucose due to a reduction in cerebral blood flow.[30] Additionally, this sequence of events leads to altered membrane permeability, ionic disruption and cytoskeleton breakdown, which can lead to neurotransmission impairment. However, the recovery and relationship of these effects to concussion symptoms are still not fully understood.

Evidence suggests that athletes with prior concussion may have a reduction in synaptic plasticity related to reduced implicit motor learning on certain tasks.[31] Also there appears to be a reduced capacity for executive functions, slower cognitive processing speeds, and attention deficits, factors that are all important in the early stages of skill acquisition.[32,33] Given previous reports of altered brain networks resulting from concussion,[34-36] we speculate that prior concussion can affect skilled visuomotor performance.

1.2 Nonlinear approaches to analysing motor performance

Complex systems and non-linearity

One framework for studying the neural control of complex skill is the motor psychophysical one. In this approach, the aim is to indirectly understand the control mechanisms underlying movement control by observing patterns of, or limitations to, behavioural performance. In the current research, we are examining how well individuals control their limb while interacting with a robotic limb to perform skilled visually-guided movements. The multijointed limbs in humans can be considered complex systems, given that they have more degrees of freedom to move than are required to define a location and orientation in space.[37,38] Complex systems are a class of systems characterized by the emergence of self-organized behaviour produced by interaction among multiple components.[39] The primary source of complexity added to a system is that we cannot fully isolate one component. These components are highly interdependent in a complex system.[40,41] This interdependence is what gives rise to nonlinearity.

Nonlinearity arises from the nonadditive nature of the interactions between components and the effect of feedback loops acting on the system over time.[42] Due to feedback loops, nonlinear systems are subject to rapid change and could grow or decay at an exponential rate.[43] Therefore, complex systems are known to shift to new organizations or regimes within very brief time periods, known as phase transitions. Within these phase transitions, a small input from one component could trigger a large system effect, resulting in the emergence of new behaviour.

In the context of biological or biomechanical systems, the application of nonlinear dynamical analysis has proven to be a powerful analytical approach that will allow us to extract dynamical information regarding the evolution of values in a dataset.[44] The system of interest in our experiment, the visuomotor system, is an example of a complex system characterized by nonlinear interactions across multiple components such as mechanical, sensory and cognitive which produce the execution of the movement task. By applying nonlinear dynamical analysis to the dataset extracted from this task, we can study the dynamics of the system.

Variability

Within a complex system, evidence suggests that an optimal level of variability exists on a continuum ranging between complete randomness to complete regularity.[45] Trial-to-trial variability observed in movement tasks are widespread, and mostly considered as undesired 'noise'. Noise in the nervous system is an inherent feature that guides our actions.[46,47] Noise can be a result of random events found anywhere from the neuronal level to instabilities in the dynamics of neural networks.[48,49] This inherent attribute of the nervous system provides uncertainty and randomness to how the brain generates movements. Traditionally, minimizing variability is deemed desirable in relation to motor performance. Research in this area has not provided much focus to how motor variability can support motor learning. Alternatively, recent evidence suggests that variability is a mechanism of how sensorimotor systems operate and learn.[50]

When faced with a novel task, the variability demonstrated by the participant can be interpreted as exploring the motor space. Improvement in performance can stem from this exploration as the motor system develops new patterns of activity.[51] In this sense, motor variability can shape adaptive behaviours. This view also falls in line with the framework observed in reinforcement learning theory, the process where a system is updated by reinforcing conditions that lead to desirable outcomes.[52] A movement task executed by the limbs is a result of activity executed by the musculoskeletal system generated from a hierarchy of neural networks of the motor system.[53] In theory, motor variability can originate at any level of the motor pathway, whether it be from variation at the level of the motor areas in the cortex, to variability in execution at the level of the periphery.

The motor system is hard at work promoting variability in some areas over others. Studies suggest a reduction in task-relevant variability after repeated practice, and the contrary for task-irrelevant variability.[54-56] Providing specificity to motor variability allows the motor system to exploit areas established as task relevant. In line with this, evidence shows that the nervous system is able to alter the shape of motor variability as a function of learning in order to enhance the task relevant component.[57] Furthermore, this reshaping is not entirely random, but accomplished in sophisticated ways to meet task specific demands. The system is constantly updating and reshaping motor variability in order to serve learning.[53] Relative to traditional theories of movement control, where variability is defined using linear measurements, it can also be examined through temporal variations in the movement output. Evidence that supports variability as adaptive considers complexity of movement patterns to be representative of system stability.[45,58,59]

CHAPTER 2: FOCUS OF CURRENT STUDY

2.1 Nonlinear analysis of motion data

To help us study the dynamics of the system under analysis, we turn to entropy. There are multiple formulations of entropy, and many of their definitions cannot be related to one another.[60] KS (Kolmogorov-Sinai) entropy, first developed by Kolmogorov and later modified by Sinai, is used for classification of purely deterministic dynamical systems by determining the rate of information generation, making it ubiquitous in calculations within Chaos Theory.

In dynamical systems, entropy is the rate of information production.[61,62] The particular field of information theory known as Algorithmic Complexity Theory can allow us to study the complexity of a data series by analyzing its entropy. The idea linking complexity and information is that complex systems can be seen as non-trivial structures that are not completely ordered, and the order they do contain can sometimes be recognized as maintained by the internal processing of information.[63]

The amount of information contained in a data series is equal to the length of the shortest possible message that can represent the series.[64-66] The simplest way to look at it is to think in terms of binary strings and to associate every number or vector with the string that describes it. Kolmogorov and Sinai's work in this field explain that a binary string is considered random if the complexity of that string is at least the length of the string.

The application of entropy allows us to measure the degree of randomness in the system. Randomness can be defined by the existence of patterns within a series. We can establish a hierarchy of randomness based on the different patterns and their repetitions.[67]

Entropy serves as a function of the probabilities of the values in the series. The probabilistic interpretation of a random variable in this sense recognizes that each of the possible outcomes of the variable has a particular probability of occurrence, and that the value associated with the outcome is random.[62] Measuring the randomness of a data series using entropy is a matter of determining to what extent does the data behave stochastic and how much determinism exists.[68]

Despite its use in dynamical systems, KS entropy is flawed for statistical use. It is discontinuous to even the smallest amount of noise and is usually infinite for stochastic processes.[69] Given this shortcoming, a more suited analysis is approximate entropy (ApEn). Although, based on the same theme surrounding KS entropy, the focus of ApEn is to be widely applicable and serve as a valid alternative in distinguishing data sets.[60,69] Formulated as a quantification of the rate of regularity in a data series, ApEn serves as an appropriate algorithm for classifying systems and studying the progression of its complexity. ApEn allows us to classify a system without having to completely reconstruct the dynamics of the system. ApEn measures regularity by representing a more predictive (deterministic) system with a low value, whereas a high ApEn value represents a more random (stochastic) system. This matches the intuitive understanding that systems with more random probabilities will have higher entropy. In a binary string, ApEn values range from 0 (perfectly predictable series) to log2 for a completely random series.

Intuitively, ApEn measures the logarithmic frequency with which vectors of a particular length that are close together remain together in the next incremental comparison. Greater likelihood for runs of patterns to remain close imply regularity, thus representing smaller ApEn values and vice versa.[70] Using the algorithm established by Pincus, the idea is to divide the series into blocks and then compare them to one another to compare their similarity within a noise filter. Another way of looking at it is given a particular pattern, what is the probability that it will repeat itself in the next increment of observation.

Despite making up for the shortcomings of earlier entropy formulations (KS entropy) ApEn has also received its share of scrutiny. The ApEn algorithm carries an inherent bias towards regularity, given that it counts each sequence as matching itself. To counteract this bias, a modified algorithm was developed, Sample Entropy (SampEn).[60,71] SampEn does not count self-matching sequences, thus eliminating the inherent bias towards regularity.[60] Regardless of this slight difference, both formulations are sensitive to their input parameters.[72] So, a careful selection of input parameters is crucial in avoiding findings that are parameter choice artifacts.

The use of entropy algorithms has gained momentum in the last two decades in human movement research.[73-75] Nonlinear analysis has provided new and important insight to a variety of clinical problems.[76,77] Quantification of entropic (ApEn/SampEn) values of a signal have served as useful techniques in various biological signals.[72,78,79] Based on clinical data looking at various sensorimotor systems, higher entropy values are often correlated with healthier states,[80-83] implying enhanced adaptability and flexibility to changing environments.[84,85]

2.2 Hypothesis

The intent of our investigation is to find differences in variability in the visuomotor task with the application of ApEn and SampEn. Therefore, exploring differences in learning trends between the elite and non-elite groups. We hypothesize that the data from the elite athletes will show higher entropy values, indicating a more stochastic system compared to the non-elite athletes. This would imply that their neuromotor system has greater adaptability to novel, complex, and coordination-requiring skills. Furthermore, we are interested to see if concussion history affects the performance of the elite athletes on a novel visuomotor task. Concussion history refers to individuals who have suffered one or more incidents of concussion in the past. Although, no differences can be found between the healthy elite athletes and elite athletes with concussion history using previous linear analyses,[86] we hypothesize that the nonlinear analysis will be able to discriminate between the two groups.

2.3 Methods

Participants

The present study is a secondary analysis based on data collected from 36 male athletes (12 elite, 12 non-elite, and 12 elite with concussion history). The elite group were NHL draft prospects (mean age = 17), who performed the task as part of the NHL combine testing in 2014. Participants in the non-elite group consisted of male Kinesiology and Health Science students at York University (mean age = 21.7). Informed consent was collected from all participants at the time of testing. Individuals in the two groups without a history of concussion had no neurological disease or injury, and had normal or corrected-to-normal vision.

Procedure

The experiment was conducted using the Phantom™ 3.0 haptic 6D robotic arm (Sensable/Geomagic, Inc., USA). The robotic arm had a sampling frequency of 1000 Hz, and a position resolution of 0.02 mm. The robotic equipment was connected to a PC with a custom-made gaming software titled PlayBall. PlayBall provided the participants with the three-dimensional virtual force environment to be navigated. During the experiment, the participant would stand in front of the PC monitor while holding and maneuvering the Haptic robotic arm with their dominant hand (Figure 1).

Each participant completed a total of ten trials. The task required participants to navigate a virtual force environment by controlling the arm of the Haptic robotic device, which allowed motion in the x, y, and z dimensions. They were tasked with controlling a ball in the 3-D virtual force environment designed as a standard slalom course consisting of six pylons, a starting and ending location, and a dividing barrier splitting the course in half (Figure 2A). The pylons would jump slightly as the ball drew closer, forcing the participant to react quickly in order to avoid hitting the pylon or the wall surrounding the course, which would add a time penalty of 500 milliseconds. The penalty would be added to the raw time, in order to obtain the overall corrected movement time for each trial. The force and virtual ball x, y, and z position for each trial were collected every millisecond as the kinetic and kinematic dependent variables to be analyzed (Figure 2B).

A visual display along with verbal instructions about the virtual environment was provided before one complete trial demonstration was performed by the researcher. The goals of the task were provided as performing ten consecutive trials as smoothly as possible from starting location, around each of the six pylons, to the end location. The task required finesse instead of force when maneuvering around the pylons. Minimal verbal feedback was provided, in order to observe if participants could learn from the feedback provided by the Haptic robotic equipment as they navigate through the virtual environment. The robotic setup provided sensory feedback in the form of visual display and resistive force to explore the integration of information and decision-making abilities in the performance of motor skill.

A

B

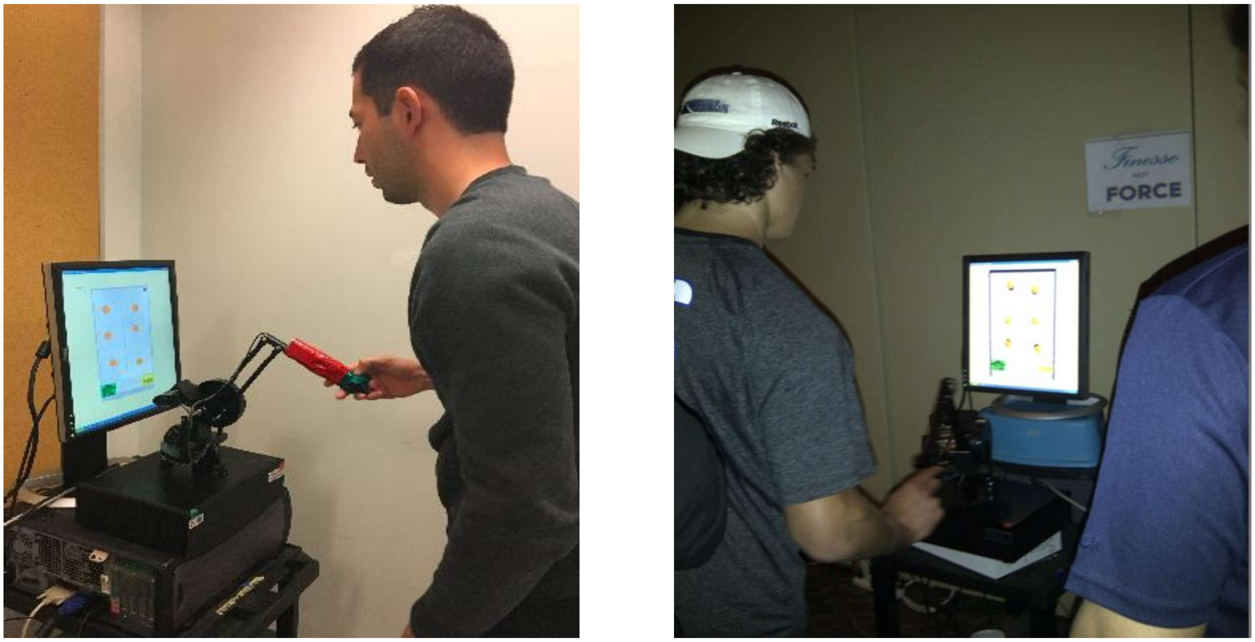

Figure 1: Task procedure using the Phantom 3.0 haptic 6D robotic arm **A.** One trial demonstration from the experimenter. **B.** NHL prospect performing the experiment task using the robotic setup.

A B

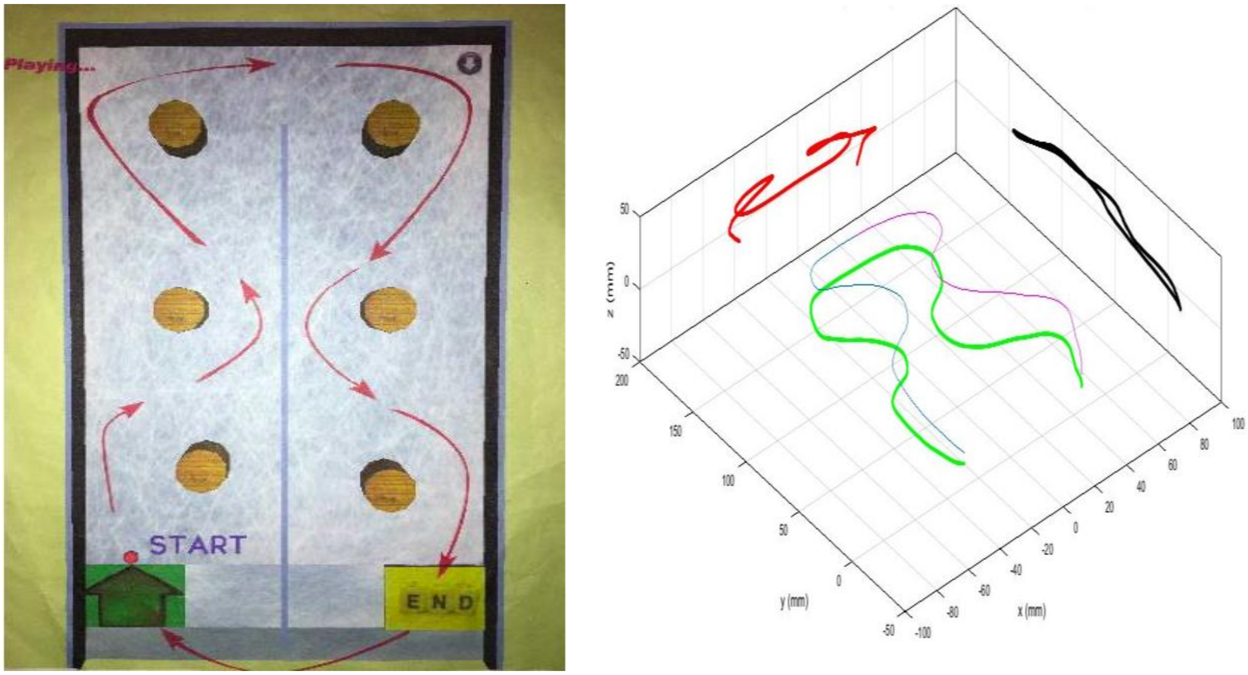

Figure 2: A. Visual display of the virtual force environment outlining the required movement path for the motor learning task. **B**. MATLAB plot displaying the movement tracer of the Haptic robotic device during the demonstration trial by the experimenter. The plot shows movements of the joystick in the x, y, and z directions. The blue and magenta represents the cursor path in the x, y, and z direction, the blue being the first half of the path and the magenta being second half of the path. The green tracer represents movement in x and y directions, the black tracer represents y and z directions, and red tracer represents x and z directions.

Data Analysis

Initially the variable of interest for our approximate entropy analysis of movement kinematics during the learning of a novel dynamic visuomotor task was jerk. Jerk represents the change in the rate of acceleration, making it an important parameter that allows us to assess the smoothness of the movement trajectory during motor learning. A second variable that we looked at in our approximate entropy analysis was velocity, and the assumption that this was another kinematic parameter that an athlete might want to maximally control when learning a new skill. Further, because an aspect to our study was the exploration of the non-linear analytic method itself (given its infrequent use in the motor control field), the dataset was analyzed in two different formats. One analysis had the length of each dataset as the full length of a trial, meaning the different datasets were not of equal length with each other. In the second format, we explored using a middle portion of each trial to generate datasets of equal length for every trial and participant. In summary, we analyzed two different movement performance variables, using two different data formats to represent the kinematics. Each point in the dataset was a value representing the chosen variable, spaced out one millisecond from each other. MATLAB programs were used to perform the analyses on the kinematic variables extracted from the Haptic robotic interface used to assess visuomotor skill learning in a three-dimensional force environment.

Approximate Entropy

Two important parameters must be selected in order to perform the approximate entropy analysis: the embedding dimension and similarity criterion. The embedding dimension, *m*, serves as a value for the size of the vector that will be used as the template to be compared. The similarity criterion, *r*, effectively serves as a noise filter that is used to identify a particular range where fluctuations in data are considered similar. Based on the typical recommendations, we set the embedding dimension at 2, and the similarity criterion at 0.2 times the standard deviation[70,71]. Originally, a combination of input parameters were tested for the jerk variable (Table 1). In the end, they all produced the same difference in mean values. So, we decided to carry our analysis with the recommended input parameters of $m = 2$ and $r = 0.2$ times standard deviation. The dataset contained values of either velocity or jerk spaced a millisecond apart.

Statistical Analysis

Since each dataset represented one trial, we calculated an entropy value for each trial that a participant completed. As a result, we generated ten ApEn values per participant. We then calculated an average ApEn value for each participant. Also, a mean ApEn value representing an entire group was calculated (Tables 2 & 3).

Based on the ApEn values generated, both a between groups and a within groups analysis was performed. The two variables of jerk and velocity were separately analyzed. A t-test was used to explore differences in mean ApEn values between groups. As part of our within groups analysis we observed differences between the first, middle, and last trial. This would allow us to evaluate the evolution of jerk and velocity ApEn values throughout the experiment. A repeated measures ANOVA was performed based on the ApEn values of each group at trials 1, 5 and 10 (Tables 6 & 7) . The t-test and ANOVA were performed using SAS 9.4 for statistical analysis.

A sample entropy analysis using the same input parameters was conducted. We wanted to see if SampEn results would point us to a different direction relative to the ApEn analysis. Although, the values generated using SampEn were different than ApEn, the difference between groups were similar. Therefore, moving forward our results and discussion only focus on the findings for approximate entropy.

2.4 Prior results from linear analysis

An initial, linear analysis was previously performed on these data.[86] The between groups analysis enabled us to investigate performance measures and trends in motor learning hypothesized between elite and non-elite athletes. Using a one-way ANOVA between groups, significant differences were found for variables calculated using custom-written software (Matlab, Inc. USA) such as movement time and mean jerk. Mean jerk, as previously mentioned is obtained as the derivative of acceleration, therefore providing information on the smoothness of the movement trajectory of the participants. Given the two key variable measures of movement time and number of obstacles hit, the elite group performed better. The elite group were superior in completing the trials as quickly and smoothly as possible. They were also in a highly competitive environment given their belief that the results of the motor learning task could influence their draft potential. However, this was not the case, nor the purpose of the experiment. The main aim of this initial analysis was to explore learning trends when presented with a novel motor learning task by providing minimal verbal instructions and feedback and asking participants to perform ten consecutive trials without breaks. Given the superior visuomotor abilities of the elite athletes, the non-elite group performed significantly slower than the elite group. In order to overcome this difference in visuomotor skill between the two groups and observe their learning trends and skill acquisition during the novel task, a normalized measure of performance was applied. Each participant served as their own control trial by taking the fastest trial and assigning it a value of 100%. The other nine trials were normalized to the participant's fastest time in order to obtain self-control performance outcomes. After comparing mean performance percentage between the groups, the elite and non-elite groups show very similar learning progression during the experiment.

Thus, in the present study, we expand on previous analyses. Our analysis looked at results from three different groups: Elite athletes, non-elite athletes, and elite athletes with a history of concussion (asymptomatic). We were interested in characterizing motor learning differences (if any) between elite versus non-elite, and the effect of concussion history on motor learning in elite performers. We expanded our previous linear analyses by using non-linear metrics to explore these data.

CHAPTER 3: RESULTS

3.1 Between Groups Differences In Overall Approximate Entropy During Motor Learning: Elite vs Non-Elite

Movement Jerk

An independent samples t-test was performed on the resultant jerk, as a measure of smoothness (or lack thereof) of visually-guided movement control in three dimensions. A significant difference was found between the ApEn values for the elite group and non-elite group for both methodological approaches, where the length of the full trial was used as the full, unequal lengths of the dataset (t_{22}= 7.95, p<0.01, *d* = 3.25), and also when trials were cut down to be of equal length across every trial and participant (t_{22}= 9.34, p<0.01, *d*= 3.81). The elite group displayed significantly lower approximate entropy values compared to the non-elite group (Table 4), representing a more deterministic system (Figures 3 and 4), meaning the generated jerk values in the dataset showed greater repeatability. Originally, we did predict a significant difference between the two groups. However, this particular finding falls contrary to our original hypothesis where we predicted a higher ApEn value for the elite group and a lower ApEn value for the non-elite group.

Movement Velocity

Based on the t-test analysis, a significant difference was found between the elite group and the non-elite group in the unequal length dataset condition (t_{22}= 4.73, p<0.01, *d*= 1.93), and in the equal length dataset condition (t_{22}= 3.37, p<0.01, *d*= 1.37), (Figures 5 and 6). Contrary to the jerk analysis, in this case the elite group displayed significantly higher approximate entropy values compared to the non-elite group (Table 4) So, when analyzing resultant velocity independently, we find that it falls in line with our initial hypothesis.

3.2 Between Groups in Overall Approximate Entropy During Motor Learning: Elite vs Elite with Concussion History

Movement Jerk

We investigated significant differences between the elite athletes and elite athletes with a history of concussion (asymptomatic) by performing an independent samples t-test on the resultant jerk variable. Although, the healthy elite group did have higher mean ApEn values, there was no significant difference found between the two groups for either methodological approach, where the full length of the trial was used as the length of the dataset (t_{22}= 0.44, p= 0.66), and when trials were cut down to be of equal length across every trial and participant (t_{22}= 0.22, p= 0.83). This shows that the incidence of concussion in the past did not affect their movement dynamics.

Movement Velocity

Significant differences were investigated by performing an independent samples t-test of the resultant velocity. No significant difference was found between the two groups in either methodological approach, in both unequal length dataset condition (t_{22}= 0.62, p= 0.54), and in the equal length dataset condition (t_{22}= 0.05, p= 0.96). Concluding that elite athletes with prior concussion are able to maintain their elite performance to match their healthy peers.

3.3 Within groups Differences In Approximate Entropy Throughout Motor Learning Phases

Movement Jerk

Looking at the elite group, no significant difference in ApEn values was observed between the trials, both in the unequal length dataset condition ($F_{2,21}$= 0.72, p= 0.7232), and in the equal length dataset condition ($F_{2,21}$= 1.66, p= 0.1455). Similarly, no significant difference was found in between trials for the elite group with concussion history in both the unequal ($F_{2,22}$ = 1.44, p= 0.2196) and equal length condition ($F_{2,22}$= 1.97, p= 0.0779), shown in figures 11 and 12. However, there was a significant difference between trials within the non-elite group. The significant difference was observed in both the unequal condition ($F_{2,22}$= 4.46, p = 0.001), and the equal length dataset condition ($F_{2,22}$= 3.43, p= 0.0055).

Movement Velocity

Performing an ANOVA based on the ApEn values derived from velocity, there was no significant difference between the trials for the two elite groups. This was the case with the ApEn values extracted from both the unequal length and equal length dataset condition (Figures 13 and 14). Although, the ANOVA shows a significant difference in the unequal length condition for the non-elite group (Table 7), a post-hoc analysis failed to show a significant difference between the different trials.

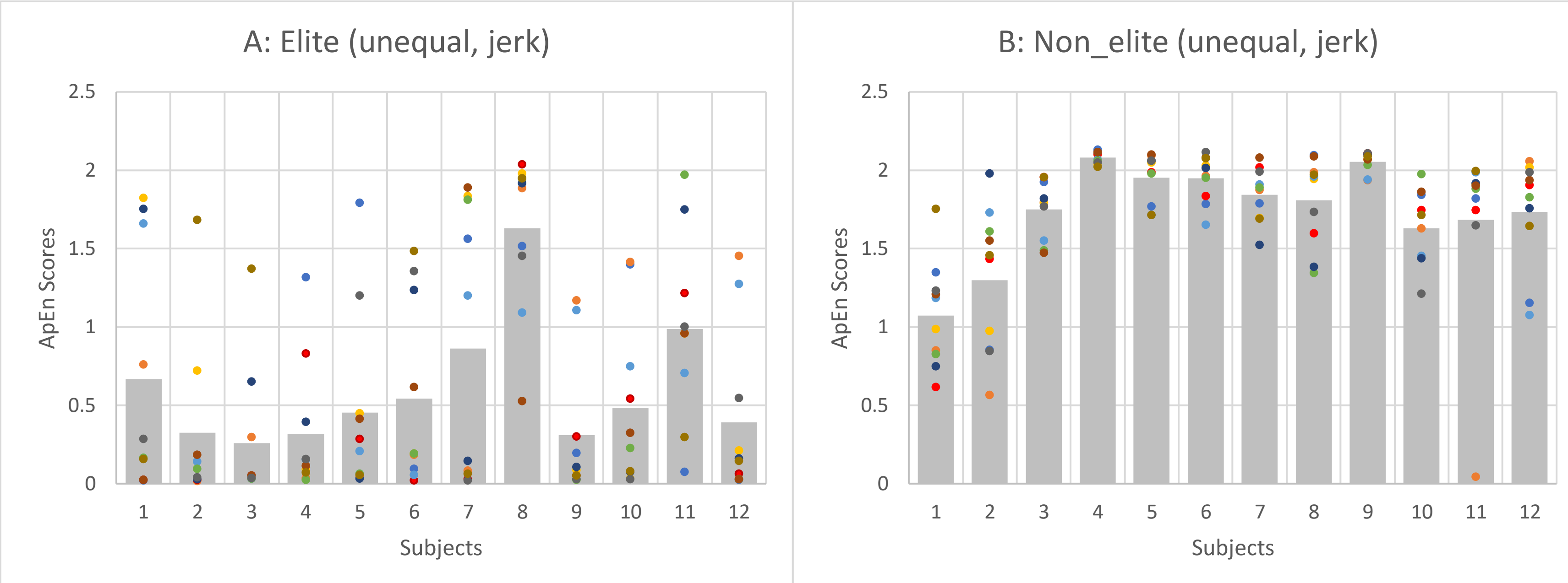

Figure 3. Side by side comparison of ApEn scores for the elite and non-elite group for the variable jerk. Bars represent the average ApEn score for each subject over the course of ten trials. Dots represent ApEn scores on individual trials. Length of dataset for each ApEn score differed to match the full trial length. A) Data representing ApEn scores for the elite group. B) Data representing ApEn scores for the non-elite group.

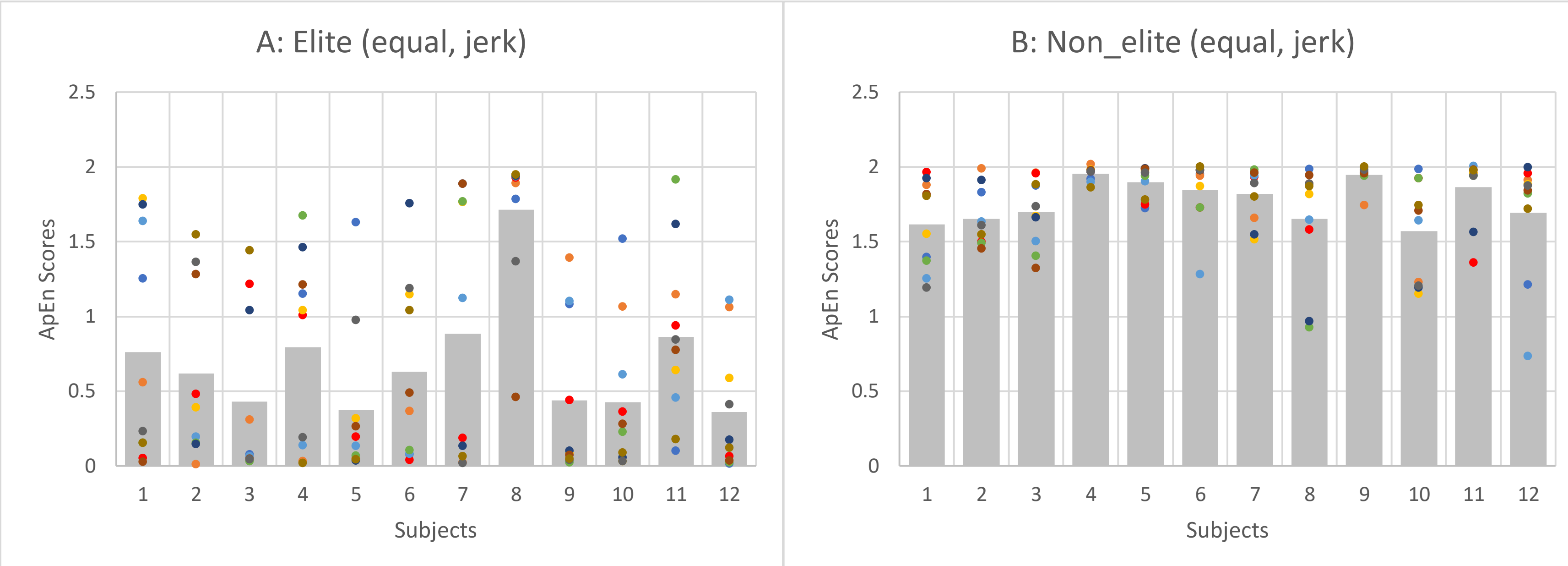

Figure 4. Side by side comparison of ApEn scores for the elite and non-elite group for the variable jerk. Bars represent the average ApEn score for each subject over the course of ten trials. Dots represent ApEn scores on individual trials. Length of dataset for each ApEn score was of equal length. A) Data representing ApEn scores for the elite group. B) Data representing ApEn scores for the non-elite group.

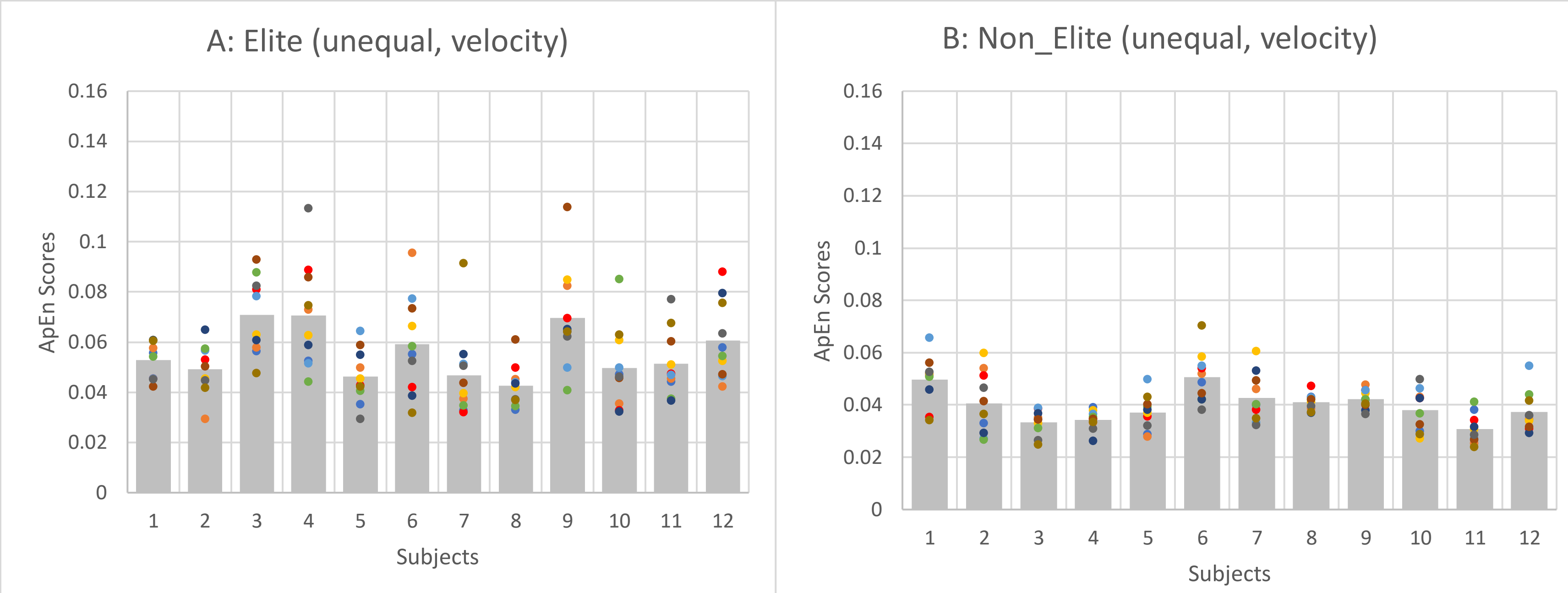

Figure 5. Side by side comparison of ApEn scores for the elite and non-elite group for the variable velocity. Bars represent the average ApEn score for each subject over the course of ten trials. Dots represent ApEn scores on individual trials. Length of dataset for each ApEn score differed to match full length of the trial. A) Data representing ApEn scores for the elite group. B) Data representing ApEn scores for the non-elite group.

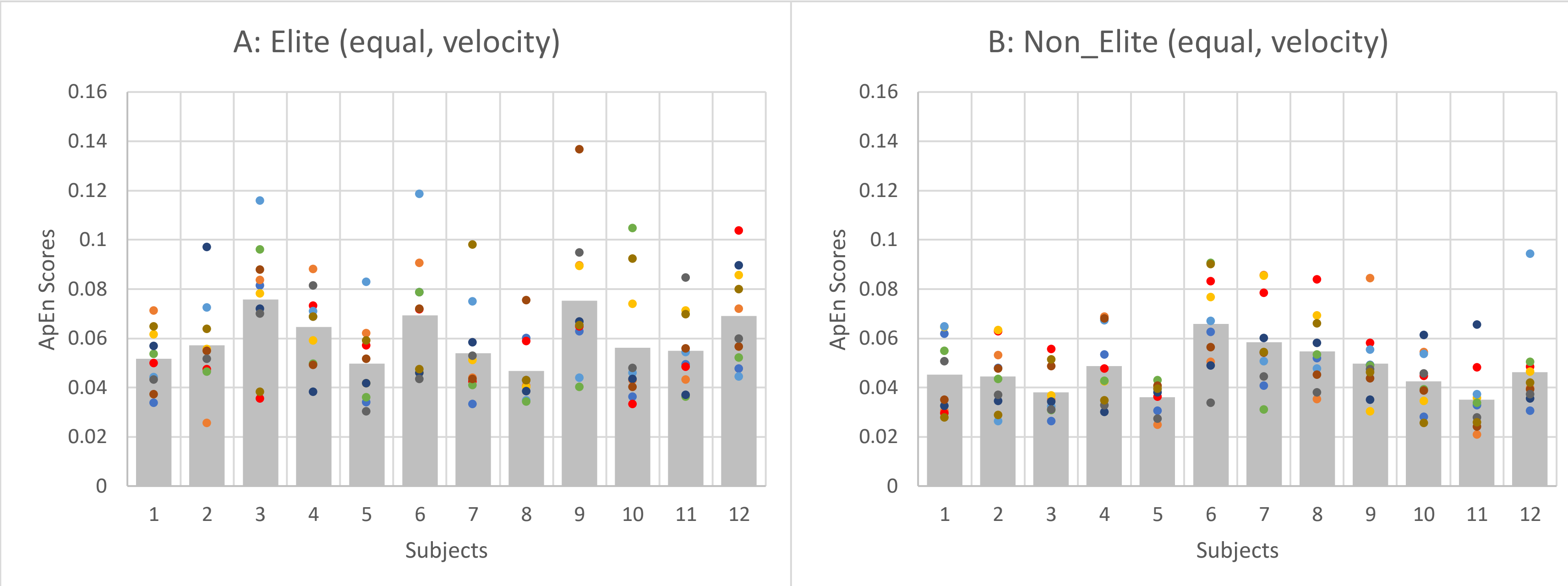

Figure 6. Side by side comparison of ApEn scores for the elite and non-elite group for the variable velocity. Bars represent the average ApEn score for each subject over the course of ten trials. Dots represent ApEn scores on individual trials. Length of dataset for each ApEn score was of equal length. A) Data representing ApEn scores for the elite group. B) Data representing ApEn scores for the non-elite group.

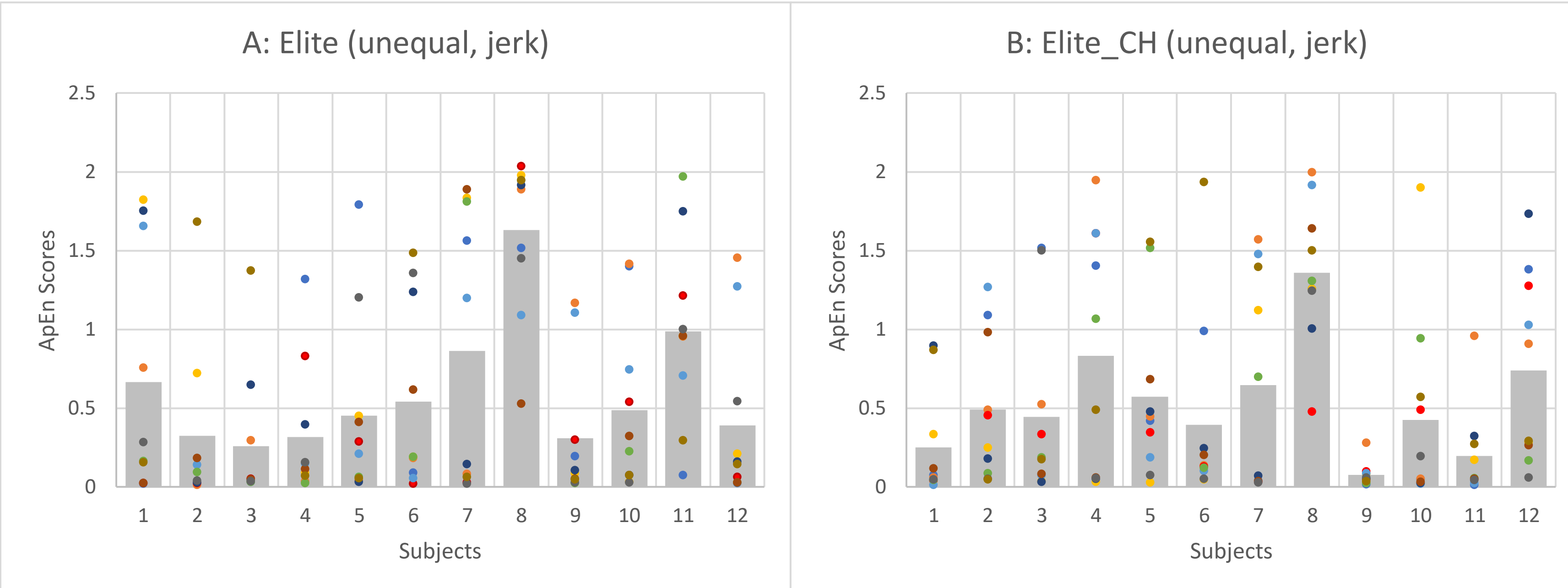

Figure 7. Side by side comparison of ApEn scores for the elite and elite group with concussion history for the variable jerk. Bars represent the average ApEn score for each subject over the course of ten trials. Dots represent ApEn scores on individual trials. Length of dataset for each ApEn score differed to match the full trial length. A) Data representing ApEn scores for the elite group. B) Data representing ApEn scores for the elite group with concussion history.

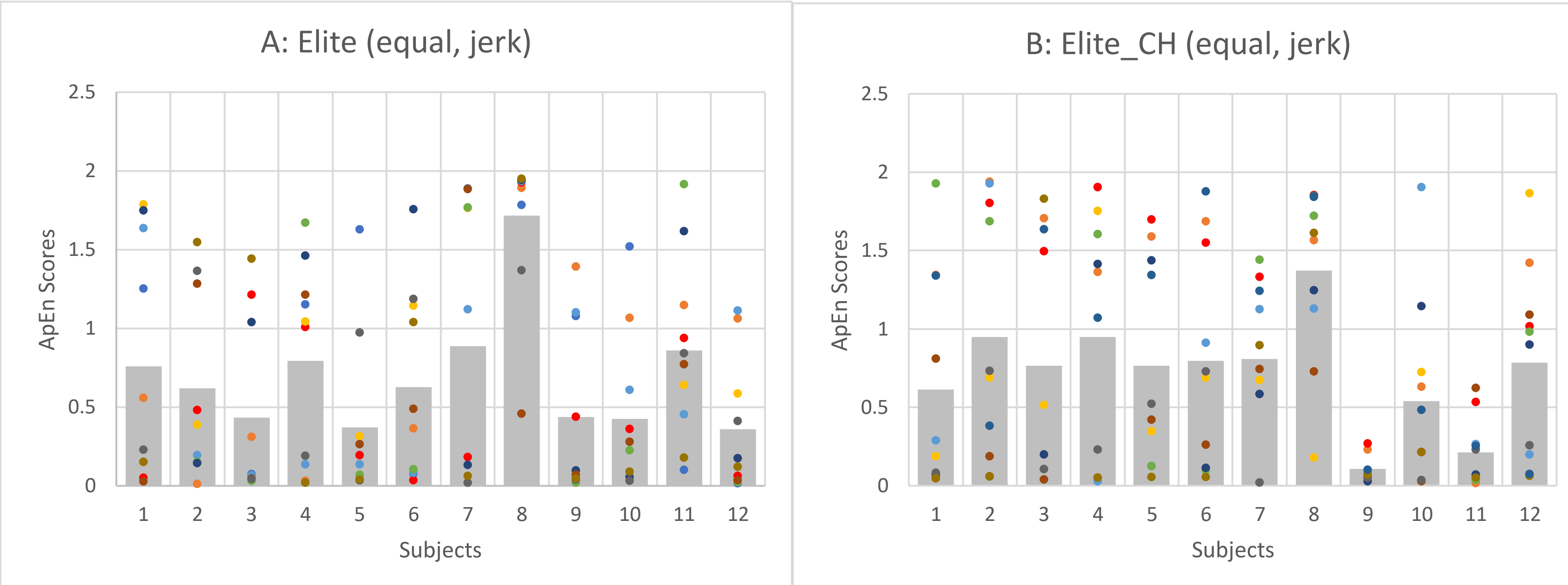

Figure 8. Side by side comparison of ApEn scores for the elite and elite group with concussion history for the variable jerk. Bars represent the average ApEn score for each subject over the course of ten trials. Dots represent ApEn scores on individual trials. Length of dataset for each ApEn score was of equal length. A) Data representing ApEn scores for the elite group. B) Data representing ApEn scores for the elite group with concussion history.

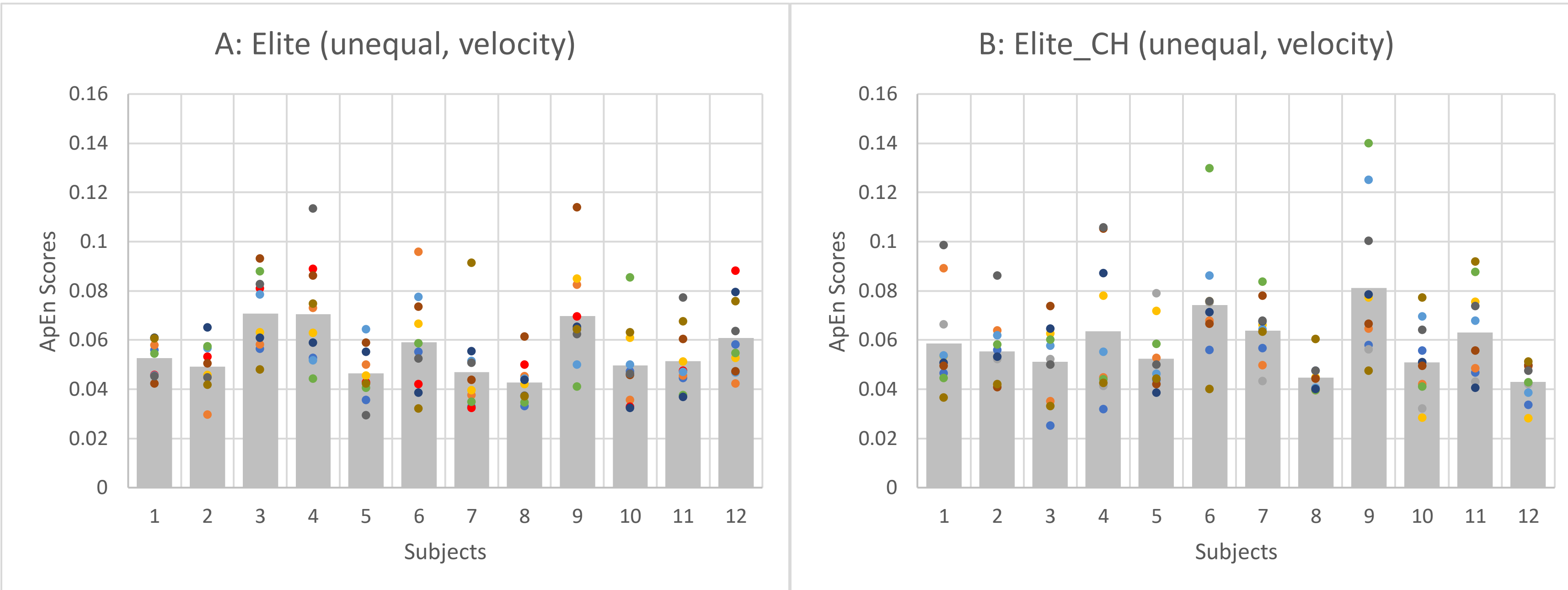

Figure 9. Side by side comparison of ApEn scores for the elite and elite group with concussion history for the variable velocity. Bars represent the average ApEn score for each subject over the course of ten trials. Dots represent ApEn scores on individual trials. Length of dataset for each ApEn score differed to match the full trial length. A) Data representing ApEn scores for the elite group. B) Data representing ApEn scores for the elite group with concussion history.

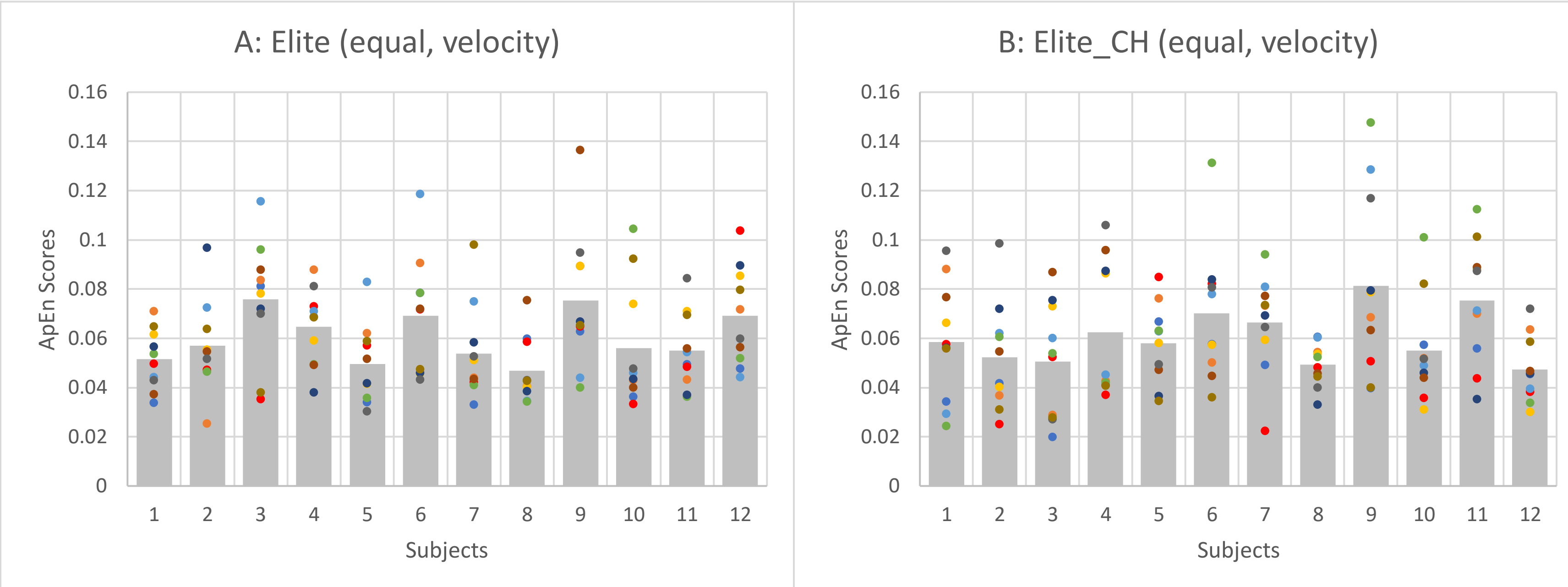

Figure 10. Side by side comparison of ApEn scores for the elite and elite group with concussion history for the variable velocity. Bars represent the average ApEn score for each subject over the course of ten trials. Dots represent ApEn scores on individual trials. Length of dataset for each ApEn score was of equal length. A) Data representing ApEn scores for the elite group. B) Data representing ApEn scores for the elite group with concussion history.

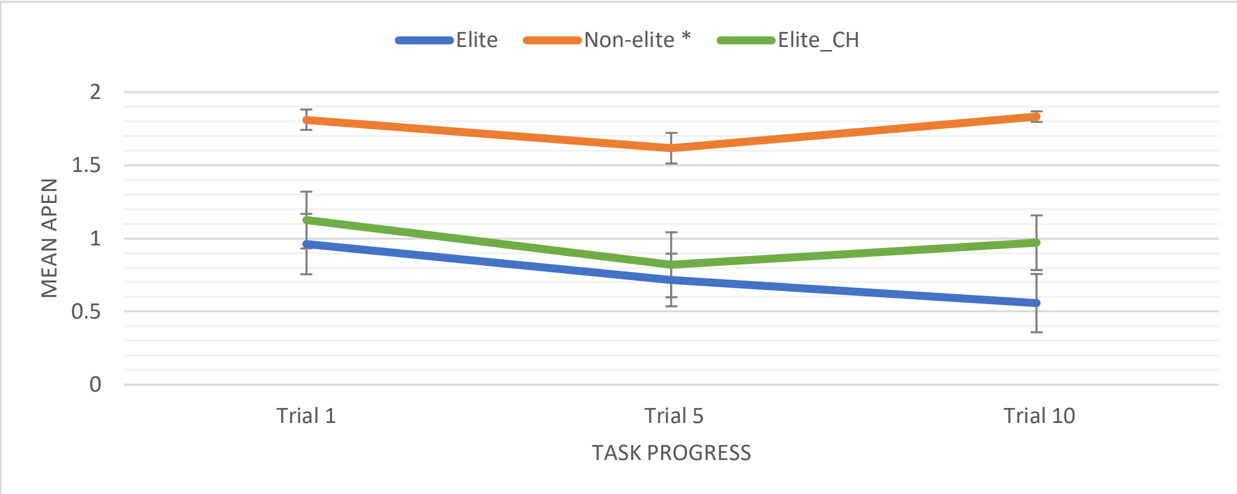

Figure 11. Evolution of approximate entropy (ApEn) values from beginning to end of the experiment in the equal length dataset condition for the variable jerk. Non-elite athletes show significantly higher ApEn values by the end of the experiment relative to the middle

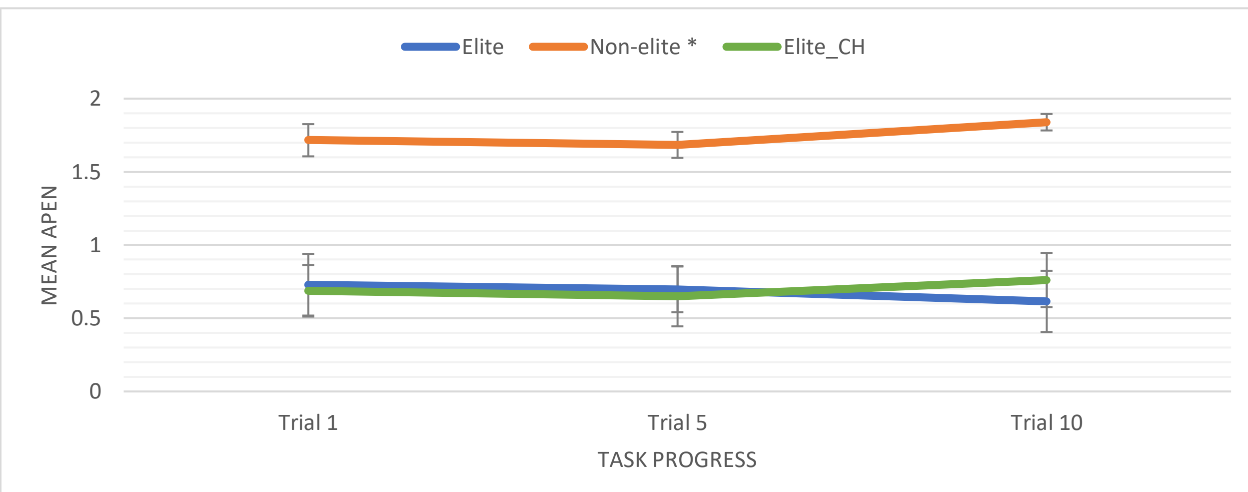

Figure 12. Evolution of approximate entropy (ApEn) values from beginning to end of the experiment in the unequal length dataset condition for the variable jerk. Non-elite group shows significantly higher ApEn values by the end of the experiment relative to the beginning *($p<0.01$)

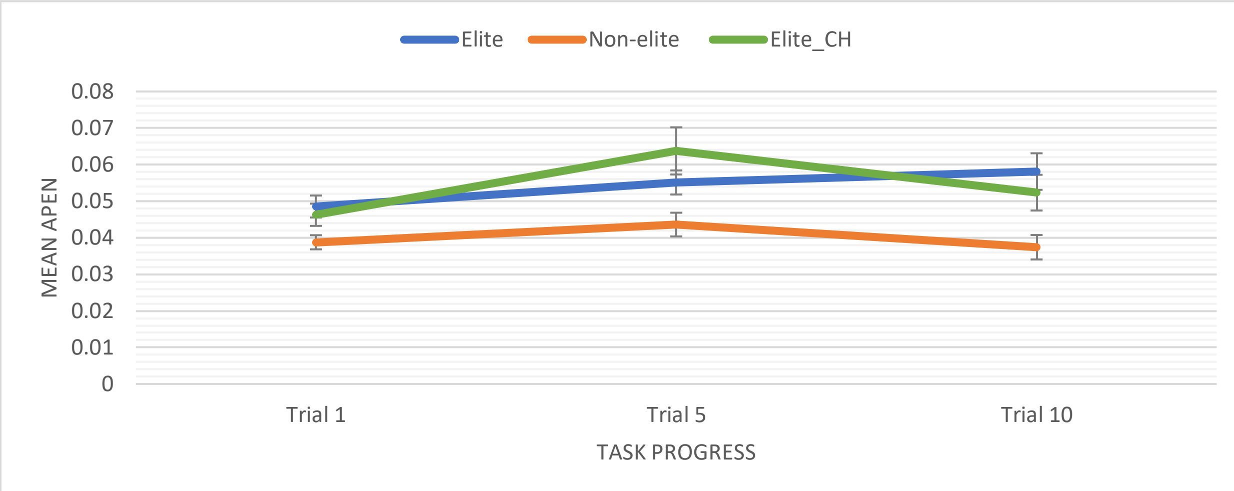

Figure 13. Evolution of approximate entropy (ApEn) values from beginning to end of the experiment in the unequal length dataset condition for the variable velocity. No significant difference was found between the trials in any of the three groups.

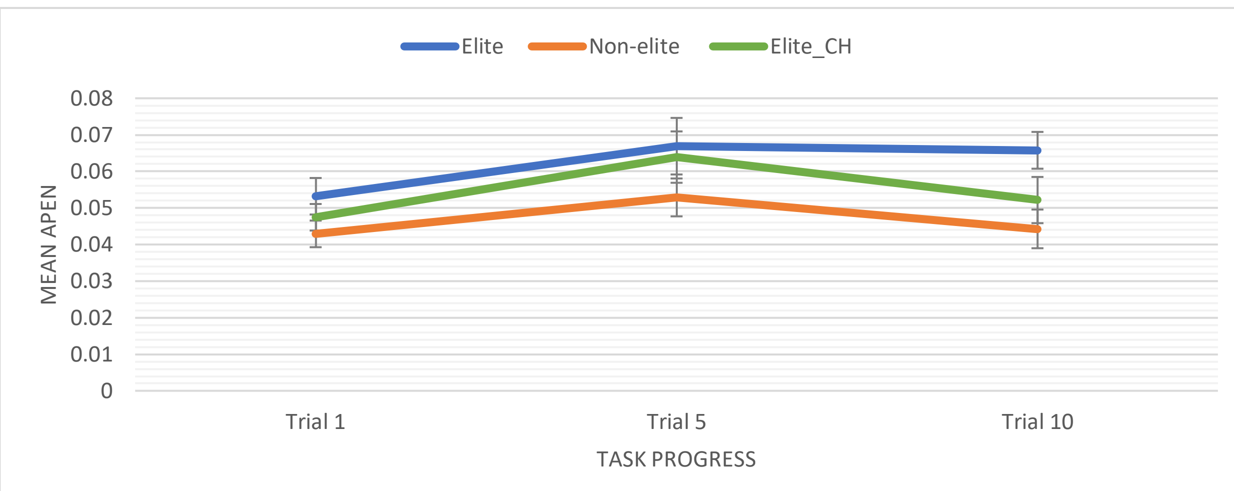

Figure 14. Evolution of approximate entropy (ApEn) values from beginning to end of the experiment in the equal length dataset condition for the variable velocity. No significant difference was found between the trials in any of the three groups.

Table 1. Mean approximate entropy values for the elite and non-elite groups using a combination of input parameters. m = embedding dimension, r = similarity criterion.

ApEn (m, r)	Elite	Non_Elite
ApEn (2, 0.2)	0.6046	1.7382
ApEn (2, 0.1)	0.9037	2.2318
ApEn (2, 0.15)	0.7265	1.9657
ApEn (3, 0.2)	0.5332	1.5631
ApEn (3, 0.1)	0.6741	1.4848
ApEn (3, 0.15)	0.6056	1.6267

Table 2. Approximate Entropy values representing each group for both the jerk and velocity variable. ApEn values in this chart were generated from datasets of equal length.

Group (Variable)	Mean ApEn	Std. Dev.
Elite (Jerk)	0.6915	0.6756
Non_elite (Jerk)	1.7672	0.2709
Elite_CH (Jerk)	0.7222	0.6710
Elite (Velocity)	0.0604	0.0214
Non_elite (Velocity)	0.0471	0.0164
Elite_CH (Velocity)	0.0606	0.0245

Table 3. Approximate Entropy values representing each group for both the jerk and velocity variable. ApEn values in this chart were generated from datasets of unequal length.

Group (Variable)	Mean ApEn	Std. Dev.
Elite (Jerk)	0.6046	0.6699
Non_elite (Jerk)	1.7382	0.3991
Elite_CH (Jerk)	0.5355	0.6011
Elite (Velocity)	0.0559	0.0175
Non_elite (Velocity)	0.0398	0.0091
Elite_CH (Velocity)	0.0585	0.0207

Table 4. Statistical values for the elite and non-elite groups. T-Tests based on approximate entropy values generated using datasets of both the equal length dataset condition and unequal length datasets condition (‡)

Groups (Variable)	DF	T-value	P-value
Elite vs Non_Elite (Jerk)	22	9.34	< 0.0001
Elite vs Non_Elite (Velocity)	22	3.37	0.0028
Elite vs Non_Elite (Jerk) ‡	22	7.95	<.0001
Elite vs Non_Elite (Velocity) ‡	22	4.73	0.0001

Table 5. Statistical values for the elite group and elite group with history of concussion. T-Tests based on approximate entropy values generated using datasets of both the equal length dataset condition and unequal length datasets condition (‡)

Groups (Variable)	DF	T-value	P-value
Elite vs Elite_CH (Jerk)	22	0.22	0.8315
Elite vs Elite_CH (Velocity)	22	0.05	0.9608
Elite vs Elite_CH (Jerk) ‡	22	0.44	0.6619
Elite vs Elite_CH (Velocity) ‡	22	0.62	0.5435

Table 6. ANOVA results comparing approximate entropy values from trials 1, 5, and 10 for each group. Results are based on entropy values generated from equal length datasets

Groups (Variable)	DF	F-value	P-value
Elite (Jerk)	2,21	1.66	0.1455
Non_Elite (Jerk)	2,22	3.42	0.0055
Elite_CH (Jerk)	2,22	1.97	0.0779
Elite (Velocity)	2,21	0.96	0.5134
Non_Elite (Velocity)	2,22	1.89	0.0914
Elite_CH (Velocity)	2,22	1.28	0.2967

Table 7. ANOVA results comparing approximate entropy values from trials 1, 5, and 10 for each group. Results are based on entropy values generated from unequal length datasets. † post-hoc Tukey test showed no significant difference between trials.

Groups (Variable)	DF	F-value	P-value
Elite (Jerk)	2,21	0.72	0.7232
Non_Elite (Jerk)	2,22	4.46	0.001
Elite_CH (Jerk)	2,22	1.44	0.2196
Elite (Velocity)	2,21	0.96	0.5203
Non_Elite (Velocity) †	2,22	3.04	0.0125
Elite_CH (Velocity)	2,22	2.12	0.0577

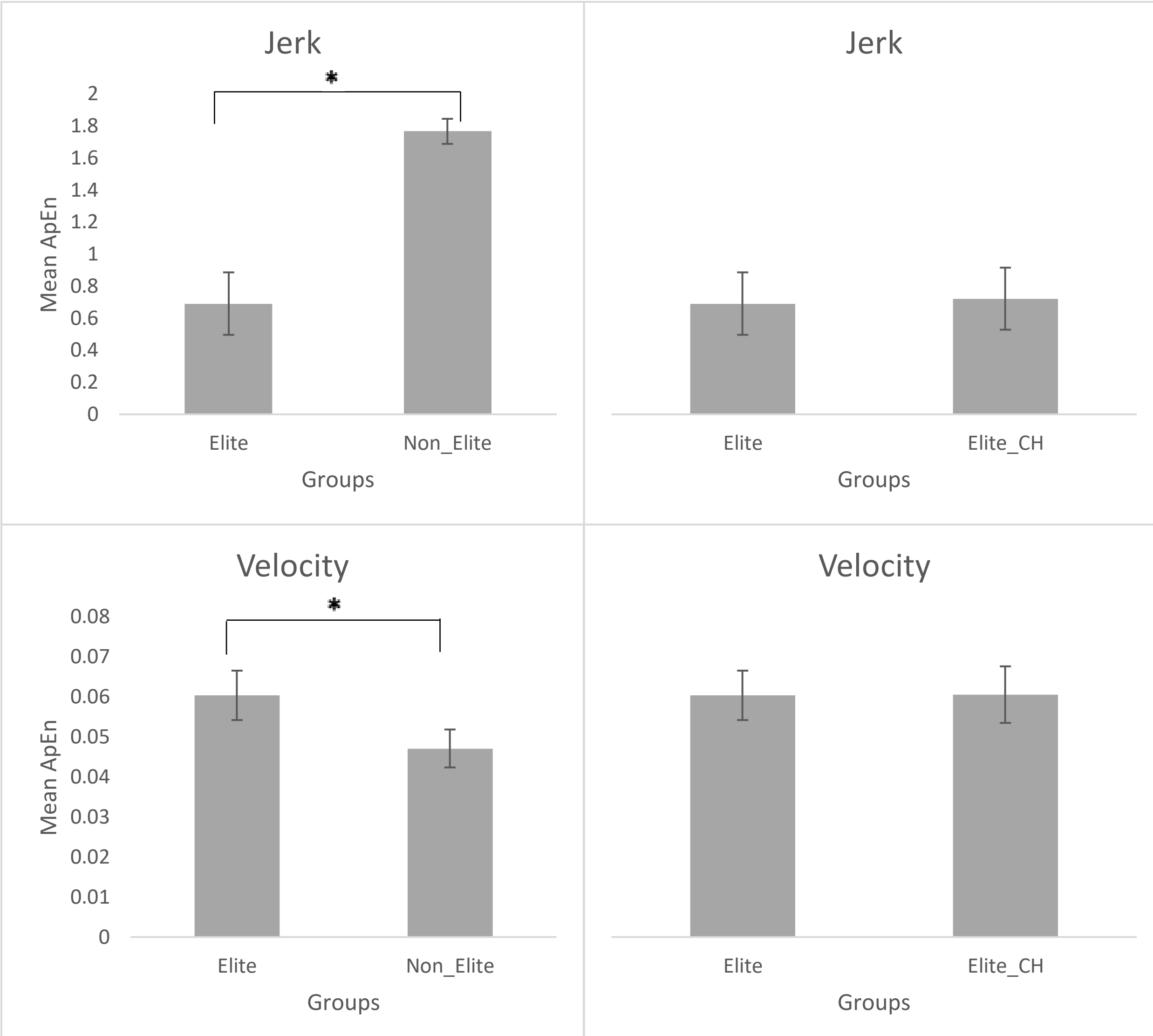

Figure 15. Group data for both kinematic variables for approximate entropy values. Bars represent mean ApEn values for each group, and error bars are standard error. Top panel: comparisons between the elite versus non-elite, and elite versus elite with concussion history for the variable jerk. Bottom panel: comparisons between the same groups, but for the movement velocity variable. * represents significant difference between groups.

CHAPTER 4: DISCUSSION

The aim of this study was to test the efficacy of nonlinear analysis to effectively discriminate between elite and nonelite athletes' data comprising movement kinematic variables such as jerk and velocity. Additionally, we were interested in assessing the analytic method itself, by examining the effects of data length on entropy values. Furthermore, we were interested in using this analytical tool to find differences in athletes with a history of concussion and healthy elite athletes. The aim here was to assess if experiencing concussion in the past has an effect on the motor learning abilities of an elite athlete.

The use of entropy as an algorithm to describe predictability of a physiological time-series data is an emerging practice in human movement research. Previous studies have applied various nonlinear analyses to biological signals, but the application of entropy calculations in motor control data require further exploration. One of the things we looked at in our current analysis was the ability of entropy to discriminate between two experimental groups. ApEn was able to discriminate between the elite and non-elite athletes by discerning the level of regularity present in each group's time series data of kinematic variables.

Measures of kinematic variables provide important information about performance on a novel visuomotor task. However, such measures only provide us a snapshot of the greater picture, they do not tell us the patterns of activity that emerge throughout a trial in a task. Fluctuations in kinematic activity are best quantified using measures of variation. Linear measures of variation allow us to characterize distribution of values around a mean value and how far apart they are spread while disregarding the temporal order in which the values have been collected. In contrast, nonlinear measures of variation consider the level to which the serial order of values is complex. Such an analysis therefore implies that the data series itself may contain meaningful information.

Essentially, nonlinear analysis of datasets is a method of data compression, where we compute characteristic numbers representing a particular system. By interpreting the compressed information, we enhance our knowledge of the underlying dynamics of the system, and it becomes interpretable because the computed number carries a specific meaning.[44] The complexity of signals has been previously used to draw conclusions about underlying physiological systems. For example, disease or injury have been linked with diminished physiological complexity and adaptability to environmental or task related stress.[58,87]

Similar to most analyses, when computing entropy using variables produced by a robotic software, a high noise level could be a potential confounder and may hinder our attempts to discriminate between system complexities. The noise present could arise from hardware imperfections and biological fluctuations. This can be controlled by choosing an appropriate noise filter via the *r* value. To avoid a significant contribution from noise in the computation of the entropy analysis, it is best to choose *r* larger than most of the noise. A higher *r* value shows more robustness to reduced noise when discriminating the nonlinear system dynamics of the two groups. The problem with a smaller *r* value (noise filter) is that the algorithm will end up recognizing two sections being compared as dissimilar when the difference may be brought about by noise. However, a larger *r* value can result in certain signal details being lost. So, essentially it is a compromise between these two: large enough to allow the algorithm to tell signal and noise apart, but small enough to pick out details present in the system's signal. After testing few different *r* values, we saw an *r* value of 0.2 as being appropriate.

The purpose of our study was to explore the complexity of fluctuations in movement jerk and velocity values recorded over a trial spaced out in millisecond frames, and we used a measure of entropy to do so. The characterization of kinematic variables was an important feature of our analysis. We speculated that characterization of movement jerk and velocity values have the potential for determining the extent to which athletes adapt performance to changes in task demands and/or environmental conditions. Measures of complexity allow us to analyze the extent to which the ordering of data values in a long series of frequent measurements produces nonlinear patterns. Our results show that the two groups manifested differences in both the movement jerk and movement velocity pattern.

Most of the literature using nonlinear analysis to distinguish variability between groups has focused on distinguishing healthy from disease states. For example, in studies of gait, nonlinear analyses have been used to locate irregularities in gait patterns and to identify individuals at greater risk of falling.[87-89] When characterizing a dataset as either deterministic or stochastic, it also helps us with the purpose of classification, or for comparison with another time series. In this case distinguishing elite from non-elite performers.

We are not necessarily interested in whether one group is faster than the other, or which one maintains overall lower jerk values in this secondary analysis. Rather, we care to see how much regularity is present in the data series that represents those variables. So, instead of focusing primarily on the performance outcome, we are focused on the behaviour manifested during performance. We are interested in what patterns are preferred, or what does the pattern of behaviour tell us about the signal produced by the system responsible in the production of movement jerk and movement velocity values. This is conveyed to us by the time-series of the variable of interest. Furthermore, we are not entirely concerned with discovering the system responsible for the production of the signal detected. This is something beyond the scope of our current investigation. Rather, we are interested in detecting what is the signal, the level of complexity or regularity present in the signal, and more importantly if they are significantly different between one group and the other. Here we observed that our ApEn analysis does a great job of distinguishing the two datasets as being different from one another. It allows us to conclude that each group has a difference in their behavioural approach to the novel visuomotor task. In the case of both variables there is a significant difference between groups, suggesting a difference in the dynamic information produced by their behavioural performance in the novel visuomotor task.

Before selecting the variable of interest for our ApEn analysis, we predicted higher values for the elite group overall. This was inspired by literature reporting higher variability in the evolution of data values in studies exploring differences in healthy and unhealthy populations. The healthy population tend to report higher ApEn values, in particular with movement data. Higher ApEn values is regarded as promoting the exploration of a novel motor space. When looking at our velocity analysis of the elite athletes, this seems to be the case, but not for the jerk analysis. A significantly higher ApEn value for velocity implies a more variable system. The system is able to achieve a better performance in terms of time to complete a trial by producing a more complex pattern of velocity values. The elite athletes tend to be more exploratory with their velocity selection. The suggestion here is that the randomness in their velocity behaviour may serve as a signal that elite athletes like to perform a thorough exploration of the novel environment, in order to create the best map possible. Furthermore, not only do the elite group display a more random signal than the non-elite group, but they also show greater variability between each dataset. Meaning they display greater variability from trial to trial in their dynamic behaviour, fluctuating between regularity and randomness. This further emphasizes the capability and willingness to try and adapt to a new motor space. The system responsible for the control of movement velocity of the arm should be regarded as more stable in the case of a high ApEn value. Based on existing literature, this signals a flexible system that is adaptable to new environments rather than a rigid system.[58,87] In the case of adapting to a new motor space, rigidity is maladaptive. Being able to fluctuate in movement behaviour is beneficial in exploration of a new motor space. The ability to change and update the motor map is what separates the skilled from the less skilled population. This is translated well in our analysis of distinguishing the elite athletes from the non-elite athletes.

Motor learning in movement tasks can be characterized by the smoothening of movement. Movements tend to become smoother as learning takes place, so we can assume the underlying objective of the motor system is to achieve the smoothest movement. Mathematically, maximizing smoothness can be equated to a reduction of jerk values. Looking at our ApEn results analyzing evolution of jerk values, it shows that the elite athletes exhibit increased regularity relative to the non-elite athletes, signaled by their overall lower ApEn values. Given that our experiment design would only allow us to investigate the early stages of motor learning, we can allude this finding to the ability of the elite athlete to produce more sophisticated synergies between their independent joints, thus allowing jerk minimization. Jerk minimization provides a rigorous way of planning trajectories of motions between points while guaranteeing predictable and well-behaved trajectories. Maintaining less complexity in the pattern of jerk values allowed the elite athletes to produce lower overall jerk values. So, when assessing a system that analyzes jerk, a third derivation of position, it benefits the system to be more deterministic than chaotic.

Alternatively, it has been suggested that values of jerk may also capture the noise generated by the robot arm. Hence we could be looking at not only values generated by the athlete but also the noise within the measurement system in the robot arm. To address this directly, future experiments could assess a measure of ApEn using the data recorded from a static robot to serve as a comparison or baseline measure. It is possible that our analysis reflects both neuromotor noise and measurement noise. In conclusion, we can speculate that the elite athletes are doing a better job of dampening the noise produced by the robot arm itself than the non-elite athletes.

If we are to believe that jerk minimization is a hallmark of skill acquisition,[12] then it may be appropriate for us to link it to the events described by Bernstein[90] and later Vereijken and colleagues.[8] Just as jerk minimization is linked with learning, so is the initial freezing out of the degrees of freedom. Both of these phenomena describe the same outcome. If the athlete is doing a good job at one of them, then it could mean they are doing a good job at the other phenomenon as well. According to our results, these events are linked with a dataset of values that contain high regularity. So, in addition for the difference in skill level, we observe a difference in their movement dynamics in a novel motor space. There is a significant difference in the dynamical evolution of their jerk values. Although a rigid or spastically fixed system is considered undesirable in the analysis of movement, there appears to be a benefit to rigidity early on in the acquisition of a new motor skill when analyzing jerk. Regarding the evolution of jerk values, it appears to be beneficial for the system in control to be rigid. Although, the later stages of learning entail a gradual releasing of these degrees of freedoms, we do not expect to see this in our experiment. Given the length of our experiment, it is safe to conclude that the elite athlete only exhibits the early phase, that which is marked by the rigidity in the joints involved.

Lower entropy values in movement jerk exhibited by the elite athletes appears to be counterintuitive with previous findings that link lower complexity levels with unhealthy systems.[91] However, other studies do not necessarily follow this generalization. Some have observed greater complexity of a system being linked with worse performance on a task.[84] So it appears to be specific to the variable under analysis. It is possible that higher complexity in a system can be a sign that the system is becoming less sustainable.[92] This is one assumption that can be related to traditional idea of variability as a measure of disorder.

Despite what the pre-existing literature suggested, relative to the non-elite athletes, the elite athletes in our study displayed significantly higher entropy values for one variable (velocity) and significantly lower entropy values in another variable (jerk). Traditional interpretation of entropy values suggests that those with injury and/or less ability should exhibit lower entropy. The reason for this can be attributed to the presence of less physiological complexity, which can be translated to an inability to be adaptable in the face of a novel visuomotor task. However, a lower entropy value does not necessarily mean less complexity, it only exhibits higher regularity based on one particular timescale.[61,69] Higher entropy values found in a particular group may be an indication of mechanisms that are too random to properly command control of the variable of interest. So, it could be that the higher complexity observed in one group, such as in our non-elite athletes, may be associated with an unstructured system that becomes less sustainable.

We employed two approaches in this analysis. In one approach we used the entire length of the trial as the dataset, meaning the datasets' lengths were not equal to each other. In the second approach, we cut off a middle equal portion of each trial to represent our data length. We were interested to see if the two different methods would yield different results, something suggested by the previous literature on this technique [44]. Given that each dataset contained at least around 4000 data points, we reported no significant change in the level of significant difference between the data type groups. Suggesting that entropy analysis stabilizes after a certain dataset length. Our findings show that once data lengths become this big we do not see a significant effect on the entropy values calculated.

The methodological approach of having two different dataset lengths provided great insight into the characteristics of this nonlinear analysis. It shows that for future experiments similar to ours, where length of each trial cannot be uniform, experimenters have the freedom to run the entropy analysis despite knowing the length of each dataset is not the same. We can assume that the effect of data length to the ApEn value plateaus at a certain point. The greater the length of the dataset (N), the closer we can expect to get to stabilization. However, it is important that length be long enough to capture the dynamics of the system being analyzed.

Our findings add to a growing body of literature supporting the utility of nonlinear analytical tools to detect biological signals. The results of our analysis highlight the robustness of nonlinear tools for contributing to the understanding of fundamental motor control functions. Furthermore, our study has implications for other researchers or recruiters interested in measuring learning abilities in athletes.

A real-world application of our analytic method can be extended to the process of talent identification in a sport such as hockey. Nonlinear analytical methods can serve as an addition to the talent identification toolbox. Higher order cognitive functioning, such as executive function, might be particularly relevant for identifying talent in sports. The importance of executive function to successful athletic performance is that it facilitates adaptation to novel situations and recall of strategies.[93] Maintaining motor control in a novel sport environment demands complex integration of continually changing inputs whose processing is dependent on cognitive functions.[94] A particular study working with soccer players found that executive function can serve as a predictive measure of future success in relation with scoring and assisting.[95] Given that motor learning capacity is an important prerequisite of learning a particular skillset, it is beneficial to search and select talent based on the ability to develop rather than performance level during testing.[96] Talent identification models encompass multiple factors.[97-99] So, adding the application of nonlinear analysis on motor learning tasks may prove to be a step in the right direction with regards to distinguishing elite athletes.

We have performed our entropy analysis on two different variables. Interestingly, while the elite athletes display a significantly more predictable system for the jerk variable, they show a significantly more random signal for the system controlling movement velocity relative to the non-elite group. The highly variable behaviour in the dynamics of movement velocity seem to negatively correlate with the highly predictable jerk variable. We believe that the signal for the jerk variable contains more regularity because it is something the elite athletes have learned to control in order to minimize their energy expenditure. This falls in line with the minimum jerk hypothesis of motor control. So, in the context of this novel visuomotor task perhaps the elite athletes allow more 'boundary testing' and variability to their velocity profiles while trying to simultaneously increase smoothness. This is something important for hockey players in particular, given the speed-focused nature of the sport. The increased variability in velocity profiles allows their motor system to exploit task relevant areas, thus better serving their motor learning.

4.1 Effects of concussion on motor learning abilities in elite athletes

Regardless of concussion history, our nonlinear analysis found no significant difference between the two elite group of athletes during the novel motor learning task. One possible reason that ApEn was not able to discriminate between elite athletes and elite athletes with a history of concussion may be due to the limited discriminatory ability of the ApEn analysis. Perhaps a multiscale ApEn could be better suited given its supposed superior discriminatory ability. A potential solution would be to augment the sensitivity of our ApEn analysis by running it in conjunction with other nonlinear measures such as fractal dimensions or Lyapunov exponent. Fractal dimension provides information about the change of the control strategies being used for control of the variable being tested.[100] The Lyapunov exponent affords us the ability to measure the local stability of a dynamical system. It is a useful analytical tool for characterizing the chaotic behaviour present in a signal. It serves as an indication of the ability of the system to adapt to a changing environment. The algorithm investigates how the system states change over time in terms of the exponential divergence of initially nearby trajectories, and the rate of separation between nearby trajectories serves as a reflection of the sensitivity of the system to initial conditions.[101]

From a physiological standpoint, failure to see a significant difference between the elite athletes and the elite athletes with a history of concussion might also be linked to a neuro-motor skill ‘reserve’. Young athletes that have greater athletic experience may have more skill-related motor ‘reserve’ generated from lifelong participation in eye-hand coordination sports, such as hockey, that allow performance compensation.[28,102] This is one of the factors that have been suggested in aiding behavioural recovery post-concussion. Relative to non-elite athletes, there seems to be reduced effect of concussion on motor tasks in elite athletes. The years of skilled training may provide a protective effect in the elite brain leading to a superior frontoparietal network. According to Dalecki and colleagues,[102] the suggestion is that the elite athletes have a superior frontoparietal network that allows for greater compensation following concussion. Other studies have reported more efficient fronto-parietal networks that stem from high levels of eye-hand coordination related sport experience,[103,104] something that is also seen in elite level musicians.[105] Additionally, compared to non-elite athletes, the elite athletes require less neural activation in order to correctly execute a visuomotor task.

The concept of ‘reserve’ is more often linked with studies assessing cognition primarily, claimed to provide a protective effect against cognitive decline associated with aging or disease (i.e. Alzheimer’s).[106,107] In this manner, we can link elite skill performance stemming from years of training as offering this same level of protection against performance decline following concussion through more efficient and resilient brain networks needed in the control of complex skills. Evidence from EEG and fMRI studies show that elite athletes demonstrate 'neural efficiency' due to more specific neural circuitry and minimal energy consumption. However, while elite athletes with prior concussion may not exhibit noticeable behavioural deficits due to their superior brain networks, the underlying neural effects may still be present. So, an extension of our current analytical approach could allow us to explore these underlying effects that could still be present.

4.2 Conclusion

The application of nonlinear analytical tools to motor control studies is shaping to be a promising approach. Measuring the complexity of a time-series of kinematic variables to explore motor learning differences allows us to discriminate between elite and non-elite athletes. The elite athletes display greater complexity in the control of their velocity dynamics when compared to the non-elite athletes. However, when analyzing jerk values, they display less complexity in their data relative to the non-elite athletes. The group with elite skill exhibits a variable system in the production of their velocity profiles, and a more deterministic system in the production of jerk behaviour. An extension of our entropy analysis in conjunction with other nonlinear analytical tools affords us the possibility to better explore underlying neuromotor effects that may still be present in elite athletes with prior concussion.

REFERENCES

1. Shadmehr R, Wise S. *The Computational Neurobiology Of Reaching And Pointing*. Cambridge, MA ; London: MIT Press; 2005.

2. Krakauer JW. Motor learning: its relevance to stroke recovery and neurorehabilitation. *Curr Opin Neurol*. 2006;19(1):84-90. doi:10.1097/01.wco.0000200544.29915.cc

3. Wolpert DM, Diedrichsen J, Flanagan JR. Principles of sensorimotor learning. *Nat Rev Neurosci*. 2011;12(12):739-751. Published 2011 Oct 27. doi:10.1038/nrn3112

4. Hallett M, Grafman J. Executive function and motor skill learning. *Int Rev Neurobiol*. 1997;41:297-323. doi:10.1016/s0074-7742(08)60357-8

5. Seidler RD. Neural correlates of motor learning, transfer of learning, and learning to learn. *Exerc Sport Sci Rev*. 2010;38(1):3-9. doi:10.1097/JES.0b013e3181c5cce7

6. Henriques DY, Cressman EK. Visuomotor adaptation and proprioceptive recalibration. *J Mot Behav*. 2012;44(6):435-444. doi:10.1080/00222895.2012.659232

7. Fitts PM, Posner MI. Human Performance. Brooks/Cole Pub. Co; Belmont, CA: 1967.

8. Vereijken B, van Emmerik R, Whiting H, Newell K. Free(z)ing Degrees of Freedom in Skill Acquisition, *Journal of Motor Behavior*. 1992;24(1):133-142. DOI: 10.1080/00222895.1992.9941608

9. Latash ML. The bliss (not the problem) of motor abundance (not redundancy). *Exp Brain Res*. 2012;217(1):1-5. doi:10.1007/s00221-012-3000-4

10. Gandolfo F, Mussa-Ivaldi F, Bizzi E. Motor learning by field approximation. *Proceedings of the National Academy of Sciences*. 1996;93(9):3843-3846. doi:10.1073/pnas.93.9.3843

11. Shadmehr R, Mussa-Ivaldi F. Adaptive representation of dynamics during learning of a motor task. *The Journal of Neuroscience*. 1994;14(5):3208-3224. doi:10.1523/jneurosci.14-05-03208.1994

12. Flash T, Hogan N. The coordination of arm movements: an experimentally confirmed mathematical model. *The Journal of Neuroscience.* 1985;5(7):1688-1703. doi: 10.1523/JNEUROSCI.05-07-01688.1985

13. Seidler RD, Purushotham A, Kim SG, Uğurbil K, Willingham D, Ashe J. Cerebellum activation associated with performance change but not motor learning. *Science*. 2002;296(5575):2043-2046. doi:10.1126/science.1068524

14. Sweatt J. *Mechanisms Of Memory*. 2nd ed. Academic Press; 2010:24-47. doi:10.1016/B978-0-12-374951-2.00002-0

15. Ungerleider LG, Doyon J, Karni A. Imaging brain plasticity during motor skill learning. *Neurobiol Learn Mem*. 2002;78(3):553-564. doi:10.1006/nlme.2002.4091

16. Marinelli L, Quartarone A, Hallett M, Frazzitta G, Ghilardi MF. The many facets of motor learning and their relevance for Parkinson's disease. *Clin Neurophysiol*. 2017;128(7):1127-1141. doi:10.1016/j.clinph.2017.03.042

17. Shadmehr R, Holcomb HH. Neural correlates of motor memory consolidation. *Science*. 1997;277(5327):821-825. doi:10.1126/science.277.5327.821

18. Cacciola A, Calamuneri A, Milardi D, et al. A Connectomic Analysis of the Human Basal Ganglia Network. *Front Neuroanat*. 2017;11:85. Published 2017 Sep 26. doi:10.3389/fnana.2017.00085

19. Nakahara H, Doya K, Hikosaka O. Parallel cortico-basal ganglia mechanisms for acquisition and execution of visuomotor sequences - a computational approach. *J Cogn Neurosci*. 2001;13(5):626-647. doi:10.1162/089892901750363208

20. Wolpert DM, Miall RC. Forward Models for Physiological Motor Control. *Neural Netw*. 1996;9(8):1265-1279. doi:10.1016/s0893-6080(96)00035-4

21. Smith MA, Shadmehr R. Intact ability to learn internal models of arm dynamics in Huntington's disease but not cerebellar degeneration. *J Neurophysiol*. 2005;93(5):2809-2821. doi:10.1152/jn.00943.2004

22. Rabe K, Livne O, Gizewski ER, et al. Adaptation to visuomotor rotation and force field perturbation is correlated to different brain areas in patients with cerebellar degeneration. *J Neurophysiol*. 2009;101(4):1961-1971. doi:10.1152/jn.91069.2008

23. Seidler RD, Noll DC, Chintalapati P. Bilateral basal ganglia activation associated with sensorimotor adaptation. *Exp Brain Res*. 2006;175(3):544-555. doi:10.1007/s00221-006-0571-y

24. Hamid AA, Pettibone JR, Mabrouk OS, et al. Mesolimbic dopamine signals the value of work. *Nat Neurosci*. 2016;19(1):117-126. doi:10.1038/nn.4173

25. Musumeci G, Ravalli S, Amorini AM, Lazzarino G. Concussion in Sports. *J Funct Morphol Kinesiol*. 2019;4(2):37. Published 2019 Jun 19. doi:10.3390/jfmk4020037

26. Banks RE, Domínguez DC. Sports-Related Concussion: Neurometabolic Aspects. *Semin Speech Lang*. 2019;40(5):333-343. doi:10.1055/s-0039-1679887

27. Brown JA, Dalecki M, Hughes C, Macpherson AK, Sergio LE. Cognitive-motor integration deficits in young adult athletes following concussion. *BMC Sports Sci Med Rehabil*. 2015;7:25. Published 2015 Oct 19. doi:10.1186/s13102-015-0019-4

28. Hurtubise J, Gorbet D, Hamandi Y, Macpherson A, Sergio L. The effect of concussion history on cognitive-motor integration in elite hockey players. *Concussion*. 2016;1(3):CNC17. doi:10.2217/cnc-2016-0006

29. Choe MC. The Pathophysiology of Concussion. *Curr Pain Headache Rep*. 2016;20(6):42. doi:10.1007/s11916-016-0573-9

30. Giza CC, Hovda DA. The new neurometabolic cascade of concussion. *Neurosurgery*. 2014;75 Suppl 4(0 4):S24-S33. doi:10.1227/NEU.0000000000000505

31. De Beaumont L, Tremblay S, Poirier J, Lassonde M, Théoret H. Altered bidirectional plasticity and reduced implicit motor learning in concussed athletes. *Cereb Cortex*. 2012;22(1):112-121. doi:10.1093/cercor/bhr096

32. Cantarero G, Choynowski J, St Pierre M, et al. Repeated Concussions Impair Behavioral and Neurophysiological Changes in the Motor Learning System. *Neurorehabil Neural Repair*. 2020;34(9):804-813. doi:10.1177/1545968320943578

33. van Donkelaar P, Osternig L, Chou LS. Attentional and biomechanical deficits interact after mild traumatic brain injury. *Exerc Sport Sci Rev*. 2006;34(2):77-82. doi:10.1249/00003677-200604000-00007

34. Murdaugh DL, King TZ, Sun B, et al. Longitudinal Changes in Resting State Connectivity and White Matter Integrity in Adolescents With Sports-Related Concussion [published correction appears in J Int Neuropsychol Soc. 2018 Sep;24(8):890]. *J Int Neuropsychol Soc*. 2018;24(8):781-792. doi:10.1017/S1355617718000413

35. Wright DK, Gardner AJ, Wojtowicz M, et al. White Matter Abnormalities in Retired Professional Rugby League Players with a History of Concussion. *J Neurotrauma*. 2021;38(8):983-988. doi:10.1089/neu.2019.6886

36. Hristopulos DT, Babul A, Babul S, Brucar LR, Virji-Babul N. Disrupted Information Flow in Resting-State in Adolescents With Sports Related Concussion. *Front Hum Neurosci*. 2019;13:419. Published 2019 Dec 12. doi:10.3389/fnhum.2019.00419

37. Morasso P. Spatial control of arm movements. *Exp Brain Res*. 1981;42(2):223-227. doi:10.1007/BF00236911

38. Bizzi E, Mussa-Ivaldi FA, Giszter S. Computations underlying the execution of movement: a biological perspective. *Science*. 1991;253(5017):287-291. doi:10.1126/science.1857964

39. Stergiou N (Ed.). Nonlinear Analysis for Human Movement Variability. *Boca Raton: CRC Press.* 2016:173-259. doi:10.1201/9781315370651

40. BAR-YAM Y. *DYNAMICS OF COMPLEX SYSTEMS*. Boulder, Colorado: Westview Press; 2003.

41. Gell-Mann M, Mermin D. The Quark and the Jaguar: Adventures in the Simple and the Complex. *Phys Today*. 1994;47(9):89-89. doi:10.1063/1.2808634

42. Takens F. *Detecting Strange Attractors In Turbulence*. Groningen: Rijksuniversiteit Groningen. Mathematisch Instituut; 1981:366-381

43. Mayer-Kress G, Liu Y, Newell K. Complex systems and human movement. *Complexity*. 2006;12(2):40-51. doi:10.1002/cplx.20151

44. Bradley E, Kantz H. Nonlinear time-series analysis revisited. *Chaos*. 2015;25(9):097610. doi:10.1063/1.4917289

45. Stergiou N, Harbourne R, Cavanaugh J. Optimal movement variability: a new theoretical perspective for neurologic physical therapy. *J Neurol Phys Ther*. 2006;30(3):120-129. doi:10.1097/01.npt.0000281949.48193.d9

46. Faisal AA, Selen LP, Wolpert DM. Noise in the nervous system. *Nat Rev Neurosci*. 2008;9(4):292-303. doi:10.1038/nrn2258

47. Renart A, Machens CK. Variability in neural activity and behavior. *Curr Opin Neurobiol*. 2014;25:211-220. doi:10.1016/j.conb.2014.02.013

48. Mainen ZF, Sejnowski TJ. Reliability of spike timing in neocortical neurons. *Science*. 1995;268(5216):1503-1506. doi:10.1126/science.7770778

49. van Vreeswijk C, Sompolinsky H. Chaos in neuronal networks with balanced excitatory and inhibitory activity. *Science*. 1996;274(5293):1724-1726. doi:10.1126/science.274.5293.1724

50. Herzfeld DJ, Shadmehr R. Motor variability is not noise, but grist for the learning mill. *Nat Neurosci*. 2014;17(2):149-150. doi:10.1038/nn.3633

51. Shadmehr R, Huang HJ, Ahmed AA. A Representation of Effort in Decision-Making and Motor Control. *Curr Biol*. 2016;26(14):1929-1934. doi:10.1016/j.cub.2016.05.065

52. Sutton R, Barto A. *Reinforcement Learning*. The MIT Press; 1998.

53. Dhawale AK, Smith MA, Ölveczky BP. The Role of Variability in Motor Learning. *Annu Rev Neurosci*. 2017;40:479-498. doi:10.1146/annurev-neuro-072116-031548

54. van Beers RJ, Brenner E, Smeets JB. Random walk of motor planning in task-irrelevant dimensions. *J Neurophysiol*. 2013;109(4):969-977. doi:10.1152/jn.00706.2012

55. Latash ML, Anson JG. Synergies in health and disease: relations to adaptive changes in motor coordination. *Phys Ther*. 2006;86(8):1151-1160.

56. Scholz JP, Schöner G. The uncontrolled manifold concept: identifying control variables for a functional task. *Exp Brain Res*. 1999;126(3):289-306. doi:10.1007/s002210050738

57. Wu HG, Miyamoto YR, Gonzalez Castro LN, Ölveczky BP, Smith MA. Temporal structure of motor variability is dynamically regulated and predicts motor learning ability. *Nat Neurosci*. 2014;17(2):312-321. doi:10.1038/nn.3616

58. Stergiou N, Decker LM. Human movement variability, nonlinear dynamics, and pathology: is there a connection?. *Hum Mov Sci*. 2011;30(5):869-888. doi:10.1016/j.humov.2011.06.002

59. Lipsitz LA. Dynamics of stability: the physiologic basis of functional health and frailty. *J Gerontol A Biol Sci Med Sci*. 2002;57(3):B115-B125. doi:10.1093/gerona/57.3.b115

60. Pincus SM. Approximate entropy as a measure of system complexity. *Proc Natl Acad Sci U S A*. 1991;88(6):2297-2301. doi:10.1073/pnas.88.6.2297

61. Richman JS, Moorman JR. Physiological time-series analysis using approximate entropy and sample entropy. *Am J Physiol Heart Circ Physiol*. 2000;278(6):H2039-H2049. doi:10.1152/ajpheart.2000.278.6.H2039

62. Shannon C.E. A mathematical theory of communication. *Bell Syst Tech*. 1948;27:379-423

63. Ladyman J, Lambert J, Wiesner K, What is a complex system?. *Euro Jnl Phil Sci.* 2013;3:33-67. doi:10.1007/s13194-012-0056-8

64. Kolmogorov AN. A new metric invariant of transitive dynamical systems and of automorphisms of Lebesgue spaces (Russian). *Dokl Akad Nauk SSSR (N.S*). 1958;119:861-864

65. Kolmogorov AN. Logical basis for information theory and probability theory. *IEEE Trans Inf Theory*. 1968;14(5):662-664

66. Sinai Y. On the notion of entropy of a dynamical system. Dokl Russ Acad Sci. 1959;124:768-771

67. Delgado-Bonal A, Marshak A. Approximate Entropy and Sample Entropy: A comprehensive tutorial. *Entropy (Basel)*. 2019;21(6):541

68. Cover TM, Thomas JA. *Elements of Information Theory*. 99th ed. John Wiley & Sons; 1991

69. Pincus S. Approximate entropy (ApEn) as a complexity measure. *Chaos*. 1995;5(1):110-117. doi:10.1063/1.166092

70. Pincus SM, Gladstone IM, Ehrenkranz RA. A regularity statistic for medical data analysis. *J Clin Monit*. 1991;7(4):335-345. doi:10.1007/BF01619355

71. Pincus SM, Goldberger AL. Physiological time-series analysis: what does regularity quantify?. *Am J Physiol*. 1994;266(4 Pt 2):H1643-H1656. doi:10.1152/ajpheart.1994.266.4.H1643

72. Lake DE, Richman JS, Griffin MP, Moorman JR. Sample entropy analysis of neonatal heart rate variability. *Am J Physiol Regul Integr Comp Physiol*. 2002;283(3):R789-R797. doi:10.1152/ajpregu.00069.2002

73. Chen X, Solomon I, Chon K. Comparison of the use of approximate entropy and sample entropy: applications to neural respiratory signal. *Conf Proc IEEE Eng Med Biol Soc*. 2005;2005:4212-4215. doi:10.1109/IEMBS.2005.1615393

74. Kaipust JP, Huisinga JM, Filipi M, Stergiou N. Gait variability measures reveal differences between multiple sclerosis patients and healthy controls. *Motor Control*. 2012;16(2):229-244. doi:10.1123/mcj.16.2.229

75. Karmakar CK, Khandoker AH, Begg RK, Palaniswami M, Taylor S. Understanding ageing effects by approximate entropy analysis of gait variability. *Annu Int Conf IEEE Eng Med Biol Soc*. 2007;2007:1965-1968. doi:10.1109/IEMBS.2007.4352703

76. Khandoker AH, Palaniswami M, Begg RK. A comparative study on approximate entropy measure and poincaré plot indexes of minimum foot clearance variability in the elderly

during walking. *J Neuroeng Rehabil*. 2008;5:4. Published 2008 Feb 2. doi:10.1186/1743-0003-5-4

77. Hong SL, Newell KM. Motor entropy in response to task demands and environmental information. *Chaos*. 2008;18(3):033131

78. Smith BA, Teulier C, Sansom J, Stergiou N, Ulrich BD. Approximate entropy values demonstrate impaired neuromotor control of spontaneous leg activity in infants with myelomeningocele. *Pediatr Phys Ther*. 2011;23(3):241-247. doi:10.1097/PEP.0b013e3182289ae4

79. Rhea CK, Silver TA, Hong SL, et al. Noise and complexity in human postural control: interpreting the different estimations of entropy. *PLoS One*. 2011;6(3):e17696. Published 2011 Mar 17. doi:10.1371/journal.pone.0017696

80. Rose MH, Bandholm T, Jensen BR. Approximate entropy based on attempted steady isometric contractions with the ankle dorsal- and plantarflexors: reliability and optimal sampling frequency. *J Neurosci Methods*. 2009;177(1):212-216. doi:10.1016/j.jneumeth.2008.09.025

81. Weiss A, Brozgol M, Dorfman M, et al. Does the evaluation of gait quality during daily life provide insight into fall risk? A novel approach using 3-day accelerometer recordings. *Neurorehabil Neural Repair*. 2013;27(8):742-752. doi:10.1177/1545968313491004

82. Ihlen EAF, Weiss A, Bourke A, Helbostad JL, Hausdorff JM. The complexity of daily life walking in older adult community-dwelling fallers and non-fallers. *J Biomech*. 2016;49(9):1420-1428. doi:10.1016/j.jbiomech.2016.02.055

83. Cavanaugh JT, Guskiewicz KM, Giuliani C, Marshall S, Mercer VS, Stergiou N. Recovery of postural control after cerebral concussion: new insights using approximate entropy. *J Athl Train*. 2006;41(3):305-313.

84. Montesinos L, Castaldo R, Pecchia L. On the use of approximate entropy and sample entropy with centre of pressure time-series. *J Neuroeng Rehabil*. 2018;15(1):116. Published 2018 Dec 12. doi:10.1186/s12984-018-0465-9

85. Faure P, Korn H. Is there chaos in the brain? I. Concepts of nonlinear dynamics and methods of investigation. *C R Acad Sci III*. 2001;324(9):773-793. doi:10.1016/s0764-4469(01)01377-4

86. Guruparan N, Hurtubise J, Gorbet DJ, Sergio LE. The effect of concussion on the learning of a novel visuomotor task in elite athletes. Washington DC, USA; *Society for Neuroscience annual meeting*. 2017

87. Cavanaugh JT, Kochi N, Stergiou N. Nonlinear analysis of ambulatory activity patterns in community-dwelling older adults. *J Gerontol A Biol Sci Med Sci*. 2010;65(2):197-203. doi:10.1093/gerona/glp144

88. Vaz JR, Knarr BA, Stergiou N. Gait complexity is acutely restored in older adults when walking to a fractal-like visual stimulus. *Hum Mov Sci*. 2020;74:102677. doi:10.1016/j.humov.2020.102677

89. Buzzi UH, Stergiou N, Kurz MJ, Hageman PA, Heidel J. Nonlinear dynamics indicates aging affects variability during gait. *Clin Biomech (Bristol, Avon)*. 2003;18(5):435-443. doi:10.1016/s0268-0033(03)00029-9

90. Bernstein, N. The coordination and regulation of movements. New York: Pergamon Press. 1967.

91. Manor B, Lipsitz LA. Physiologic complexity and aging: implications for physical function and rehabilitation. *Prog Neuropsychopharmacol Biol Psychiatry*. 2013;45:287-293. doi:10.1016/j.pnpbp.2012.08.020

92. Borg FG, Laxåback G. Entropy of balance--some recent results. *J Neuroeng Rehabil*. 2010;7:38. Published 2010 Jul 30. doi:10.1186/1743-0003-7-38

93. Williams AM, Ericsson KA. Perceptual-cognitive expertise in sport: some considerations when applying the expert performance approach. *Hum Mov Sci*. 2005;24(3):283-307. doi:10.1016/j.humov.2005.06.002

94. Grooms DR, Onate JA. Neuroscience Application to Noncontact Anterior Cruciate Ligament Injury Prevention. *Sports Health*. 2016;8(2):149-152. doi:10.1177/1941738115619164

95. Vestberg T, Gustafson R, Maurex L, Ingvar M, Petrovic P. Executive functions predict the success of top-soccer players. *PLoS One*. 2012;7(4):e34731. doi:10.1371/journal.pone.0034731

96. Abbott A, Collins D. A Theoretical and Empirical Analysis of a 'State of the Art' Talent Identification Model. *High Ability Studies.* 2002;13:157 - 178.

97. Baker J, Cobley S, Schorer J. Talent Identification and Development in Sport. London: Routledge. 2012. doi.org/10.4324/9780203850312

98. Čoh, Milan, and Milan Žvan. " Identification of Young Talents in Sport", *Physical Education and Sport Through the Centuries* 6, 1 (2019): 70-79, doi: https://doi.org/10.2478/spes-2019-0007

99. Scharfen, H-E, Memmert, D. Measurement of cognitive functions in experts and elite athletes: A meta-analytic review. *Appl Cognit Psychol*. 2019; 33: 843– 860. https://doi.org/10.1002/acp.3526

100. Maragos P, Fan-Kon Sun. Measuring the Fractal Dimension of Signals: Morphological Covers and Iterative Optimization. *IEEE Transactions on Signal Processing*. 1993;41(1):108. doi:10.1109/tsp.1993.193131

101. Kędziorek J, Błażkiewicz M. Nonlinear Measures to Evaluate Upright Postural Stability: A Systematic Review. *Entropy*. 2020;22(12):1357. doi:10.3390/e22121357

102. Dalecki M, Gorbet D, Macpherson A, Sergio L. Sport experience is correlated with complex motor skill recovery in youth following concussion. *Eur J Sport Sci*. 2019;19(9):1257-1266. doi:10.1080/17461391.2019.1584249

103. Di X, Zhu S, Jin H, et al. Altered resting brain function and structure in professional badminton players. *Brain Connect*. 2012;2(4):225-233. doi:10.1089/brain.2011.0050

104. Park IS, Lee KJ, Han JW, et al. Experience-dependent plasticity of cerebellar vermis in basketball players. *Cerebellum*. 2009;8(3):334-339. doi:10.1007/s12311-009-0100-1

105. Medina D, Barraza P. Efficiency of attentional networks in musicians and non-musicians. *Heliyon*. 2019;5(3):e01315. Published 2019 Mar 26. doi:10.1016/j.heliyon.2019.e01315

106. Craik FI, Bialystok E, Freedman M. Delaying the onset of Alzheimer disease: bilingualism as a form of cognitive reserve. *Neurology*. 2010;75(19):1726-1729. doi:10.1212/WNL.0b013e3181fc2a1c

107. Soldan A, Pettigrew C, Albert M. Cognitive Reserve from the Perspective of Preclinical Alzheimer Disease: 2020 Update. *Clin Geriatr Med*. 2020;36(2):247-263. doi:10.1016/j.cger.2019.11.006