

Solutions to Module 5 Practice Problems: Probability Basics

1. During the 2000 US election, Edison Research conducted exit polls for the National Election Pool in which they interviewed 15,590 voters outside polling places or early-voting sites or by phone. Of these voters, 8069 said they had voted for Joe Biden, 9198 did not have a college degree, and 4145 voters without a college degree said they had voted for Joe Biden. The remaining voted for Donald Trump.

(a) Use a Venn diagram to depict this data.

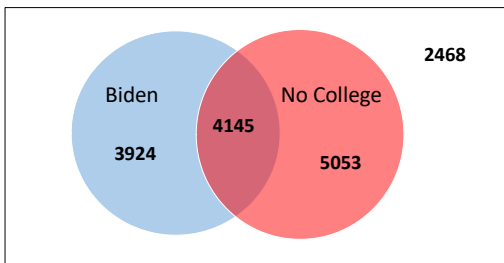


Figure 1: Venn diagram 1(a)

(b) How many of the surveyed voters had a college degree and had voted for Joe Biden?

3924

(c) If a random voter was chosen, what is the probability that he/she had voted for Joe Biden?

$$\frac{8069}{15590} = 0.52$$

(d) If a random voter was chosen, what is the probability that he/she had a college degree?

$$\frac{6392}{15590} = 0.41$$

2. 2000 households were surveyed about their subscriptions to Netflix and Amazon Prime. Of them, 1060 had a Netflix subscription, 1200 had an Amazon Prime subscription, and 630 had both.

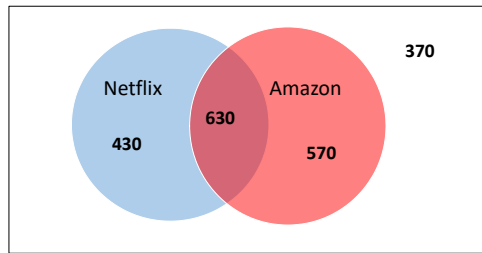


Figure 2: Venn diagram 2(a)

- (a) Use a Venn diagram to depict this data.
- (b) How many of the surveyed households did not have a subscription to either Netflix or Amazon Prime?

370

- (c) If a random household were chosen, what is the probability that it had a subscription to both Amazon and Netflix?

$$\frac{630}{2000} = 0.315$$

- (d) If a random household were chosen, what is the probability that it had a subscription to only Amazon, but not Netflix?

$$\frac{570}{2000} = 0.285$$

- (e) Are having a subscription to Netflix and having a subscription to Amazon Prime mutually exclusive? Are they independent?

Neither mutually exclusive, nor independent

3. Now suppose another 1000 households were surveyed about their subscriptions to Netflix, Amazon Prime and Disney Plus. Of them, 540 had a Netflix subscription, 610 had Amazon Prime, and 380 had Disney Plus. 324 subscribed to both Netflix and Amazon, 220 to both Netflix and Disney, 210 to both Disney and Amazon Prime, and 60 to all three.

- (a) Use a Venn diagram to depict this data.
- (b) How many of the surveyed households did not have a subscription to any of the three subscription services?

164

- (c) How many of the households had a subscription to Netflix or Amazon Prime?

826

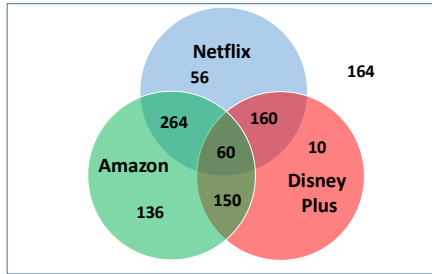


Figure 3: Venn diagram 3(a)

(d) If a random household were chosen, what is the probability that it had a subscription to both Amazon Prime and Netflix, but not Disney?

$$\frac{264}{1000} = 0.264$$

(e) If a random household were chosen, what is the probability that it had a subscription to only Amazon Prime, but not to either Netflix or Disney Plus?

$$\frac{136}{1000} = 0.136$$

4.(a) If $P(A) = 0.2$, $P(B) = 0.4$, $P(A \cap B) = 0.1$, determine $P(A \cup B)$.

$$0.5$$

(b) If $P(C) = 0.7$, $P(D) = 0.5$, $P(C \cup D) = 0.9$, determine $P(C \cap D)$.

$$0.3$$

(c) If $P(E) = 0.3$, $P(F) = 0.4$, $P(G) = 0.3$, $P(E \cap F) = 0.2$, $P(F \cap G) = 0.1$, $P(E \cap G) = 0.2$, $P(E \cap F \cap G) = 0.1$, determine $P(E \cup F \cup G)$.

$$0.6$$

(d) If $P(X) = 0.4$, $P(Y) = 0.7$, $P(Z) = 0.3$, $P(X \cap Y) = 0.3$, $P(Y \cap Z) = 0.3$, $P(X \cap Z) = 0.2$, $P(X \cap Y \cap Z) = 0.15$, determine $P(X \cup Y \cup Z)$.

$$0.75$$

5. A family has 4 children.

(a) What is the probability that none of them are girls?

$$\left(\frac{1}{2}\right)^4$$

(b) What is the probability that at least one of them is a girl?

$$1 - \left(\frac{1}{2}\right)^4$$

(c) What is the probability that exactly one of them is a girl?

$$4 \left(\frac{1}{2}\right)^4$$

(d) What is the probability that exactly two of them are girls?

$$6 \left(\frac{1}{2}\right)^4$$

(e) What is the probability that exactly three of them are girls?

$$4 \left(\frac{1}{2}\right)^4$$

(f) Suppose this family had 4 girls and are now expecting a fifth child. What is the probability that this fifth child will be a girl?

$$\frac{1}{2}$$

6. The Pew Research Center in the US is well-known for conducting surveys on a wide range of topics. Typically these surveys are conducted over the telephone. In 2017-18, the typical response rate for a telephone survey was about 6% i.e. out of 100 households contacted via telephone, about 6 agree to take the survey. We can assume that each household's response is independent of the others.

(a) What is the probability that the first household that an interviewer contacts agrees to take the survey?

$$0.06$$

(b) What is the probability that the first two households that an interviewer contacts both agree?

$$(0.06)^2$$

(c) What is the probability that the first household that agrees is the third one on the interviewer's list?

$$0.06 (0.94)^2$$

(d) Suppose the interviewer phoned 10 households and none of them agreed. What is the probability that the eleventh household will agree?

$$0.06$$

(e) What is the probability that if the interviewer phones 6 households, at least one will agree?

$$1 - (0.94)^6$$

(f) What is the probability that if the interviewer phones n households, at least one will agree? How large must n be to ensure that this probability is greater than 0.9?

$$1 - (0.94)^n; n \geq 38$$

7. The probability of scoring a goal from a penalty kick in soccer has been estimated (see instat-sport.com) to be about 75% i.e. $\frac{3}{4}$. If a game ends in a tie, each team gets to take 5 penalty kicks. Assume that the outcomes of the penalty kicks are independent of each other.

(a) What is the probability that a team scores on all 5 of its penalty kicks?

$$\left(\frac{3}{4}\right)^5$$

(b) What is the probability that a team scores on none of its penalty kicks?

$$\left(\frac{1}{4}\right)^5$$

(c) What is the probability that a team scores on at least one of its penalty kicks?

$$1 - \left(\frac{1}{4}\right)^5$$

(d) What is the probability that a team scores on exactly one of its penalty kicks?

$$\frac{15}{4} \left(\frac{1}{4}\right)^4$$

(e) What is the probability that a team scores on exactly two of its penalty kicks?

$$10 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^3$$

(f) Suppose the team has a top player like Lionel Messi, whose probability of success in a penalty kick is about 80% i.e. $\frac{4}{5}$. This player takes one of the 5 kicks. Now, what is the probability that a team scores on (i) none, and (ii) all, of its 5 penalty kicks.

$$(i) \frac{1}{5} \left(\frac{1}{4}\right)^4, (ii) \frac{4}{5} \left(\frac{3}{4}\right)^4$$

8. Mary owns stocks in three companies, Ford, Dell and RBC. An analyst tells her that the probability of the stock value going up in the next week is 20% for Ford, 60% for Dell and 30% for RBC. The analyst also tells her that these are independent events.

(a) What is the probability that all three of Mary's stocks will go up in value in the next week?

$$0.2 * 0.6 * 0.3 = 0.036$$

(b) What is the probability that none of the three will go up in value?

$$0.8 * 0.4 * 0.7 = 0.224$$

(c) What is the probability that exactly one of them will go up in value?

$$0.2 * 0.4 * 0.7 + 0.8 * 0.6 * 0.7 + 0.8 * 0.4 * 0.3 = 0.488$$

(d) What is the probability that exactly two of them will go up in value?

$$0.2 * 0.6 * 0.7 + 0.2 * 0.4 * 0.3 + 0.8 * 0.6 * 0.3 = 0.252$$

9. Determine in the following cases whether the events are independent, mutually exclusive or neither.

(a) $P(A) = 0.2$, $P(B) = 0.4$, $P(A \cap B) = 0$

A and B are mutually exclusive.

(b) $P(C) = 0.2$, $P(D) = 0.5$, $P(C \cap D) = 0.01$

C and D are independent.

(c) $P(E) = 0.7$, $P(F) = 0.8$, $P(E \cap F) = 0.6$

E and F are neither independent, nor mutually exclusive.

(d) $P(G) = 0.4$, $P(H) = 0.4$, $P(G \cup H) = 0.8$

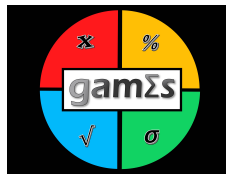
G and H are mutually exclusive.

(e) $P(I) = 0.5$, $P(J) = 0.3$, $P(I \cup J) = 0.65$

I and J independent.

(f) $P(S) = 0.8$, $P(T) = 0.1$, $P(S \cup T) = 0.85$

S and T are neither independent, nor mutually exclusive.



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