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So let me do one other type of sequence, which is also very interesting and comes up a lot in the social sciences. This is the geometric sequence. And this is what we have seen before. It's the sequence, which is 1, R, R squared, R cubed, R to the power N and so on. And so the question is, what is the limit as N goes to infinity? Now, what it turns out is that this depends a lot on the number R. So for example, if it's the case R is equal to 1, then the sequence is 1, 1, 1, 1, and so on. Right? So in this case, this limit in the case R is equal to 1, the limit of the sequence is just going to be 1. What if R is a number, which is is less than 1, let's say R is equal to half.

So in this case, we start with one, then next number is half, the third number is half squared. So that's 1 over 2 squared, which is 1/4. Then the next number is 1 over 2 cubed, which is equal to 1/8. And next number is 1 over 16. And it goes this way. And here, you can notice what's happening is that it at each stage, we start with 1, but then the number becomes 1/4, then it becomes 1/8, then it becomes 1/16. And the Nth term is 2 to the power N, or this 2 to the power N minus 1, right? So as N becomes big, what's happening here is that this denominator, 2 to the power N minus 1, that becomes bigger and bigger and bigger. The denominator, right? So what does that mean? That means this whole number, it becomes smaller and smaller and smaller, right? So in this case, this sequence is heading off to zero. And this will be true for any number, which is less than 1. Because for example, I could do exactly the same thing. If this was 1, if instead of, instead of half it was 1/5, then the next number would be 1 over 5 squared, that's 1 over 25, the third number would be 1 over 5 cubed, that's 1 over 125, the fourth number would be 1 over 5 to the power 4, which is 1 over 625. So it becomes smaller and smaller and smaller, as N becomes large. So for R less than 1, right? So this limit will be, will be zero. What about the other case, right? What if R is greater than one? So for example, if you take for example, R is equal to 5. So how would the sequence look? It would start with 1, the next term would be 5, the third term would be 5 squared, that's 25. The fourth term would be 5 cubed, that's 125. The fifth term would be 625, and so on, right? And so this would be like, the Nth term would be would be 5 to the power N minus 1, and so on.

And as you can see, in this case, the numbers are getting bigger and bigger and bigger as you draw the sequence more. So what happens is that in this case, the limit of the sequence it goes off to infinity. And this is true anytime R is bigger than 1, because anytime you're raising it to bigger and bigger powers, the number becomes bigger and bigger and more and more humongous. This is like the exponential growth that we saw before with the grains of rice. So what, so the result is that in the case of the geometric sequence, right, where does the limit go to? That depends on the value of R. If R is less than 1, then the limit is zero. If R is greater than one, then the limit is infinity. And if R is equal to 1, then the limit is 1. So, so that's why, for example, in the, in the case of infections, remember, this was the sequence of infections, this is on day 1, day 2, day 3, day 4, day N plus 1, right. So if the, and remember R was the reproduction number. So what this tells you is that if that R, or if the reproduction number is less than 1, the number of sequences as the days go on, that becomes lower and lower and heads towards zero. On the other hand, if your reproduction number R is bigger than 1, that means the number of infections just blew up and becomes bigger and bigger, and it heads towards infinity.

So that's why this reproduction number is such an important number in figuring out the dynamics of epidemics. But also in lots of other contexts of in the social sciences, this geometric sequence occurs, and, and where it goes to in the long run depends on whether that number R is less than 1 or greater than 1. So let me end this limits with a one last thing that so far, we've seen all the sequences which have limits, some sequences don't. And here is an example. So this is a limit, this is a sequence which has 2, minus 2, 3, minus 3, 4, minus 4. So this is not going to any particular value, right? So this is just alternating in sign and becomes more and more wildly oscillating. So this sequence doesn't have a limit.

So this is the end of the module. And what I would like to end the module with is a clicker question on limits, just to make sure that you're on top of this material. So here is the clicker question. So you may want to stop the video here and try this out. So I hope you got a chance to do the clicker question. So what it asks you is the limit of this sequence, which is given by 2N plus 5 divided by N. As N goes to infinity, so see, again, if you look at the numerator, this also shoots off to infinity, the denominator also shoots off to infinity. But what about the ratio? That's the question. So like before, if we break it up, if we divide the first term by N and divide the second term by N, so we divide 2N, N, plus 5 over N, and we want to look at the limit of this as N goes to infinity. So we're looking at the limit of, of 2 plus 5 over N. So this 5 over N, as N becomes bigger and bigger, this will go off to zero. So the, so the only thing that will remain is 2. So the correct answer here is B. So that ends this module. And just to quickly recap, the elements that we did in this module include looking at the summation notation, and how the summation notation can naturally occur in a lot of different circumstances. We looked then at sums of two particular type of series, the arithmetic series and geometric series. Then I showed you how to calculate sums in Excel. And, and last of all, we looked at sequences and limits of sequences.