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SPEAKERS

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So what we're doing here is looking at the limits of some common sequences. And we've looked at limits of, of these sequences here, numbers going from 1, 2, 3, up to N. So that limit is going off to infinity. And if we have numbers going down, becoming more and more negative, that's going off to minus infinity. So let's look at one more. So this is a where we start with 1, then the second number in the sequence is 1 over 2, that's half, then the third number is sorry, the third number here should be 1/3. The fourth number is 1/4, the fifth number 1/5, and so on. And so this is a sequence, which is basically the Nth term of the sequence is 1 over N. So, and you can ask the question, where does the limit of this go to as N becomes really big? Then you can see what's happening to these numbers, it starting with 1, then it's becoming 0.5, then becoming 0.33, then it becoming 0.25, 0.2, and it's getting smaller and smaller and smaller and smaller. So essentially, what's happening is that if you look at 1 over N, the denominator is becoming bigger and bigger and bigger and bigger. Right? Another way of saying it is that if you have a cake, right, first you start with the whole cake, then you make, divide it into two, then you divided into three parts, then you divided into four parts, then into five parts, and so on. So as you become as N becomes larger and larger, you're slicing up that cake into smaller and smaller and smaller and smaller segments, right. So where does this limit go to as N becomes really large? So this goes off to zero, because the numbers become smaller and smaller. So this is another type of a sequence. And in this case, the limit of this sequence is zero, because basically, the denominator becomes bigger and bigger, while the numerator is, is one. So the number itself goes off to zero.

Now, once you know this, right, you can use this to derive limits of other sequences as well, right. So for example, if you look at this, this sequence, this is 10 plus N divided by N. So what sort of numbers are these, so if I start with the N equal to 1, that means this is 11 over 1, then the next number in the sequence is N equal to 2, that's 12 over 2, that 6, right, then the third one will be 13 over 3, so that will be like 4 point something, the fourth number in the sequence will be 14 over 4, and so on. And the idea is where does this go to as N becomes really big. Now if you look at the numerator in the denominator, this 10 over N divided by N. So the numerator itself is going off to, it's becoming bigger and bigger with N. The denominator is also becoming bigger and bigger with N. But what about the ratio of the two? Now what we can look at is if we break this up this fraction into two parts, so 10 over N and the other part is N over N. So this is 10 over N plus 1. And now if you want to look at the limit of this as N goes to infinity, so this becomes the limit of this expression as N goes to infinity. If you look

at this, this part always remains 1. But this thing here, the second part, which is 10 over N, as N becomes really big, big big, this number becomes small and small and small. So this part goes off to zero while this part remains 1, so the limit of this is going to be 1.

So the limit of this is equal to 1. So what it means is this set of numbers, you start with 11 and then it becomes 6, then it becomes 4 point something, then it becomes 3 point something, and so on. So where is this limit headed to? It's headed to 1. And so what we have made use of is knowing facts like this. And we have used it to derive limits of other sequences. Let me do one more like that. What about N over N plus 1? So this is a sequence where if you put N equal to 1, so the first number is 1 over 2, second number is 2 over 3, third number for N equal to 3, so the numerator is 3, denominator is 4, so that's three quarters, four fifth, and so on. So what, so what's the limit of this, limit of N over N plus 1 as N goes to infinity? Now if we just look at the numerator and denominator, so the numerator, this is N, so that means this goes off becomes bigger and bigger. What about the denominator? This is N plus 1 that also becomes bigger and bigger and bigger, right? But what about the ratio of the two? So how would we derive that.

So what I'm going to do is I'm going to divide both top and bottom by N. So if I take the top and divide it by N, and I'm also going to do the same for the bottom. So I've divided both top and bottom by N. So, what that becomes is 1 on the top, and this becomes 1 and this becomes 1 over N. And we are looking at limit of this as N goes to infinity. Have a look at this, this is 1, this is 1, these are not affected by N. But what's affected by N is this thing here, right? So this thing here goes off to zero as N becomes bigger and bigger and bigger. So, what happens to this whole thing as N becomes bigger and bigger, this part becomes zero. This part remains 1, this part remains 1, So the whole thing is 1. So the limit of this sequence as N becomes large is equal to 1. So this, that means the set of numbers, they head off to 1 as N becomes bigger as you go out further and further and further out. They come closer and closer to 1. Okay, so this way you can look at limits of other sequences just by knowing some particular facts like these.