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SPEAKERS

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Okay, let me finish this module with a couple of clips on an idea related to something that we, to things that we have been already been doing in this module. And this is the idea of sequences. So mathematical definition of a sequence is numbers which occur in a particular pattern. So, so far we have seen things like, like Kim's salary. Remember Kim, who started out working in a company at a at a yearly salary of $501. And then every year his salary increased by 1. So, his, so his salary in year one was 501, year 2 was 502, 503, 504, and so on. So these are numbers, which are increasing by 1 every year. So that's the particular pattern. Now, we also saw an example of infections where we started with 1, then in year 2, it became R zero, year 3, or sorry, day 2 it became R zero, day 3 it became R zero squared, day 4 it's R zero cubed, and so on. So in this case, every day, the number got multiplied by a new R zero, right, so that was the pattern. So these are all examples of, of sequences. So more generally, so these were two particular patterns, but more generally, any sequence of numbers which follow a particular pattern, that's called a sequence. So it could be A1, A2, A3, A4, and so on AN and continuing, right, with a particular distinction, a particular distinct pattern that, that will be will be a sequence. For example, the primes, right, so 1, 2, 3, 5, 7, 11, 13. So that's a sequence.

Now, in the social sciences some more, some of, I'll now talk about sequences in general. In mathematics, you can do a full week, or like at least half a course on sequences. But what I'll be talking about is a couple of common sequences. And one of them is the arithmetic sequence pattern, where each term in the sequence is found by adding a constant to the previous term. So, so it so the pattern is that if you look at the N plus 1, at the number in that sequence, it's AN plus C. So for example, if I start from 4 and go up by 5 every time, right, so that that's, that's an example of an arithmetic sequence. In this case, the N plus 1 member of that sequence is equal to 4. Sorry, that's 5 added to the previous term, which is A. So this is an example of an arithmetic sequence. Now, the other type of sequence which occurs commonly is the geometric sequence. And we've already encountered this as what I call the geometric series, right? So here, the idea is that the next number can be found by multiplying the previous number by a constant. So it again, if it was something like this, it's 4, then 4 times R, 4 times R squared, 4 times R cubed. So every time I'm multiplying the previous number by R. So in this case, if you look at the N plus 1 at the term, is R times the previous term. So that's an example of a geometric sequence. And we already saw that. Remember the examples we did about infections or a crime rate growing, right? So those were examples of things which occurred in a geometric sequence.

Now, of course, these are not the only sequences. Like, there's another very interesting sequence which is called the Fibonacci sequence. And in this sequence, what happens is that the next number is found by adding up the previous two. Okay? So for example, not for example, this is the Fibonacci sequence. You start with one and one, okay, then the third term is found by adding up the previous two, so that's two, then the next term is found by adding up the previous two, so this is three, then the next term will be five, then the next term will be found by adding up the previous two, that's eight, then the next add up these two, that's 13, 21, 34, and so on. And this is a, so this sequence, it's named after an Italian mathematician. But this apparently is a sequence which has been known from a long time ago. But what's interesting about the sequence is that this, you can see a lot of, a lot of things in nature following the Fibonacci sequence. So I'm not going to go too deep into it. But, but for example, the number of petals in flowers, right, there either are 3, or there are 5, or there are 8, or 13, or 21, and so on, right. So the number of petals in flowers that you find in nature usually follow, it's the numbers of the Fibonacci sequence. There's also the things like the spirals in a sunflower, or the spirals in a pine cone, in and in a pineapple, they follow the Fibonacci sequence. So there are lots of cool things about this Fibonacci sequence, which occur in nature. So if you want to know more about this, you can watch this YouTube video, which goes, goes into that. And there you can also find lots of other videos on the web about the Fibonacci sequence, because it's a pretty cool sequence in the sense of occurring in, in nature.

Now, so what we will be doing here is talking about what's called limits of sequences. So the limit of a sequence is the idea that because it follows a particular pattern, what do you want to see is where does the sequence go towards, or head towards as, as you as you draw it out further and further? Right. So the idea is that so remember, in all of the sequences, I told you, okay, the N plus 1 term is found by some relation with the previous terms, right? The question is that, as that N becomes larger and larger, right, where does the sequence head towards? And this is important from the perspective of social sciences, because we will often want to, would like to know, where is this headed towards? Right, so for example, under the pandemic, right, so a big question is, where is this number of infections headed towards if we don't do anything? Okay? Or if we vaccinate people, after that, where is this level of infections headed to in the long run? Right? Or with lockdowns, where is this going to be headed to as N becomes really large, right? So the the way, we denote this, the notation for this is the lim. So this denotes the limit as N goes to infinity of AN. So that's typically the, that's how the, the idea of a limit is denoted mathematically. It's denoted by lim of AN as N goes to infinity.

So let's do a couple of, couple of limits. Right, so one limit is, is for example, if you look at Kim, right, so Kim's income started with 501, then next year it became 502, the following year 503, and in year N it became 500 plus N, right? And now suppose you asked the question that suppose Kim turned into Yoda and lived forever. Okay, so suppose okay, we talk about Yoda, and if this was the pattern of his salary, where would his yearly salary head towards as N becomes very large. Now you can see, what's happening here is that as this numbers are increasing, right, they're continually increasing. So as you go bigger and bigger and bigger and bigger, right, this number becomes bigger and bigger and bigger, there's nothing stopping it. So that means the limit of this, where does this head towards is infinity. So the limit of, as N goes to infinity, of of salary N, that's infinite. So that's pretty easy. Now, the same thing occurs if you have any sequence of numbers like this, right? 1, 2, 3, 4, 5, 6, N and N plus one and N plus 2. Right, so this set of numbers just heads up to infinity. Same thing, if you take squares of numbers, 1, 2 squared, 3 squared, and so on up to N squared, and so on, right, and as N becomes bigger and bigger and bigger, this becomes bigger and bigger and bigger. And that heads up to infinity. You can also go the other way, right? What if you start from 1 and start decreasing numbers, right, from 1 you go to zero, then you go to minus 1 minus 2 minus 3, so on minus N and you become, and you keep going, right? So the numbers become smaller in the sense that they become negative numbers and they become larger and larger negative numbers, right? So the limit of this sequence will head off to minus infinity. So, so these were easy, right? So next what I'll be doing is a couple of other limits, which are, which we see a lot in, in the social sciences. And one of them will be the geometric sequence. The other will be will be a sequence of 1 over N.