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SPEAKERS

Sumon Majumdar

So in the last clip I introduced you to the idea of a geometric series, where each term is derived by multiplying the previous term by a particular number. And in this clip, I'll do a couple of applications. So start off with this application, which is a mythical story coming from India and involves the chessboard. So apparently, the inventor of the chessboard came to a king and showed him this new invention of a game. And then the king asked, oh this is very neat, what can I offer you in reward? So what the inventor said is that I don't want much, all I want is that, put one grain of rice on the first square. On the second square, put the double, double of that, that's two, on the third square, put double of what you put on the second, right, so that would be four, and so on, and fill up the whole chessboard with grains of rice. That would be my reward. Seems simple enough. So the king asked his minister to do that. And soon they realized that they had a huge problem. So what's the problem? So, so if we look at the number of grains, right, so you put one on the first, you put two on the second, then to get the third or amount on the third square, it's two times what you had put on the previous one. So that's four. And then on the next square, it's double what you put on the on the previous one, and so on.

So the number of grains of rice would be 1 plus 2 plus 2 squared plus 2 cubed, and this is how it would go. And remember in a, in the chessboard, there are 64 squares, right? So you started with this is 2 to the power zero, this is 2 to the power 1, and so on. So on the 64th square, it would be 2 to the power 63. So this is the number of grains that the inventor wanted. So the question is, what is the sum? So, so so now we know how to get the sum of a geometric series, right, because if you look at this, this is nothing but the geometric series. So, so what we need to do is add this up, 1 plus 2 plus 2 squared plus 2 cubed and so on up to 2 to the power 63, right? So if I'm writing this in terms of summation notation, so what I'm doing is 2 to the power i and i running from zero, that's the first term right, remember, this is 2 to the power of zero, and running up to 63.

And we know the formula for that, which we did in the previous clip, this is 1 minus 2 to the power N. Remember, the last term here is 64 minus 1. So this N that means is 64 divided by 1 minus 2. So to get the number of grains of rice, you just need to compute this. Now, if we actually used a calculator to compute this, and this is where they realize that they had a huge problem, if you compute this, this is a humongous number. This is like 18, I calculated this out 446,744,073,709, comma 551,615. Okay, so this is 18 followed by, followed by 18 zeros. So this is what's called 18 quintillion rice grains. And someone figured this out that if they had to actually give him that amount of rice grains, they would be, it would basically be equivalent to covering the entire surface area of India with about a meter high of rice. So obviously, that was not possible. But what this shows you is that it's the power of what's called exponential growth. That if you go this way, where each, the previous number gets multiplied, if you keep doubling the previous number, or like, or tripling the previous number, and if you go that route, the total number soon just becomes humongous, humongous. But again, this is an application of geometric series, because in this case, the number of rice grains, right, you get the new number of rice grain we're multiplying what was on the previous square by 2.

So let me do one other application. And this is an application, which is you see a lot in the social sciences in lots of different areas. So I've picked as an example, this is from something you may see in sociology, right? So let's say, the crime rate in a city or in a district or in a province, right? Let's say the crime rate grows by 5% every year. So what that means is that suppose you start off year one with 100 crimes, then in year 2, the number of crimes grows by 5%. So 5% is 2 over 100 times 100. Or another way of writing this is 0.05 times 100 is the number of extra crimes in the second year. So what do you have in the year 2, the number of crimes will be 100 plus this. So if I were to take 100 common, the common factor between these two, that would be 100 times 1, 0.05, 1 plus 0.05. This is the number of crimes in year 2. Now, if you go to, so this is the number of crimes in year 2, right? So remember how it is, it's 100, the number you started with, and then multiplied by 1 plus 0.05. Now, if you want to go to year 2, right, this is the number of crimes in year 2. And how many more crimes will there be in, in in that year, it will be an extra 0.05 times the number of crimes in in the previous year, right? So this will be the total number of crimes in year 3. So if I take, so see this is a common factor between both of these terms. So if I take that outside, it'll be 100 times 1 plus 0.05. What's left? And it's 1 plus this term here, so that's 0.05. So in year 3, the total number of crimes will be 100 times 1.05 times 1.05, right? So I can write this out, it will be 100 times 1 point 0.05, 1 plus 0.05 squared. And if you continue like this, you see the pattern, right? In year 4, it will be 100 times 1 point, 1 plus 0.05 cubed. If you continue in year 5, it will be 100 times 1 plus 0.05 to the power 4, and so on. This is how your crime, number of crimes will evolve year upon year.

So now suppose they ask the question, how many crimes will there be in the city over 10 years? So I start with 100. So that's my year 1. Year 2 will be 100 times a 1.05. Year 3 will be 100 times 1.05 squared, and so on over 10 years. So in year 10, this will be 100 times 1.05 to the power 9. Now to calculate something like this, we already have a formula, which was the formula for the geometric series that we did in the last clip. And the formula, if you recall, was this, it's A plus A times R, plus A times R squared, and so on, up to A times R to the power N minus 1 was equal to A times 1 minus R to the power N, divided by 1 minus R. So here is a geometric series with N terms. So this is the first term, this is the second term, this is the third term, this is the Nth term. And the sum of this geometric series is given by A times 1 minus R to the power N divided by 1 minus R. So in our particular case, here, that A is 100, and the R is 1.05. So this is R squared, and so on up to R to the power 9. So there are 10 terms here, this is the term first term, second term, so on up to the 10th term. So what we're going to do in this case, when we want to know how many crimes are there, over the 10 years, we'll just going to make use of this formula. So A is 100 times 1 minus 1.05 to the power 10 divided by 1 minus 1.05. Now you can use your calculator to figure, figure out this term here. And if you do that, you'll find that it's equal to 12.578 multiplied by 100. And so this is 1257.8. So this gives you the total number of crimes committed in the city or in this particular province over 10 years.

So what we're just doing is realizing that the progress, the progression of crimes over the years is a geometric series, and then using the formula to figure out the sum of that. Now, this idea of a geometric series, it, this occurs a lot in the social sciences, okay. So for example, in this particular example, the crime rate grew by 5% every year. It could have happened that the crime rate decreases by 5% every year. So in that case, the number of crimes in year 2 would be less than the number of crimes in year 1 by 5%. So the only thing that would change in this is that instead of 1 plus 0.05, this would become 1 minus 0.05. Same thing, in year 3, this is the amount in year 2, and then in year 3, it would decrease by another 5%. And now if you're in this scenario where crime decreases, if you ask the same question, how many crimes over 10 years, you'd still be using the same formula, the only thing that would change is that instead of 1.05, this would become now 1 minus 0.05 which, so that would be 0.95. Otherwise, you can still use the same formula.

So, as I was saying, in the social sciences, this sort of situation occurs all the time. For example, as I said, in the case of infections, here in the case of crime, a completely different example would be, suppose in kinesiology, you've you've recommended to someone to jog or, or walk, okay, for physical exercise. And you have told this person to say that, told this person that why don't you start out by jogging for 2 kilometers, but every day you increase the amount to jog by 5% What is the distance that this person would have jogged over 10 days, or over 20 days, or over a month? Right, we can again cast it exactly the same way that we did with the crime rate. And again use the geometric series idea formula to figure out the total distance this person would have walked over 10 days or 20 days or a month. So this idea of a geometric series this is a very powerful idea. And it shows up all the time in economics, in sociology, demography, kinesiology, and in a variety of ways in the social sciences. So I hope you will remember this idea of the geometric series, and even better, the formula for adding up this geometric series.