module2_lecture10

Mon, 12/27 3:50PM 9:52

SUMMARY KEYWORDS

geometric series, r naught, infections, power, squared, formula, term, called, day, divided, sum, multiplied, cubed, summation, naught, r9, arithmetic, add, summation notation, series

SPEAKERS

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So, so far we have looked at the arithmetic series. The other type of series that we encounter quite commonly in the social sciences is what's called the geometric series. We've already seen an example of that in our first module when we were doing the model of infections. I never presented it as as a geometric series, but you can see the idea here. So if you recall that example that we did, so this was a model of infections and how infections can spread. So, we started off on day T equal to 1 with 1 infection. On day 2, the number of infection became NP. And I called that as the reproduction number or R naught. And if you go the same way, on day 3, the number of infections became NP squared, which was the R naught squared, right. So, what do you see is that every in every particular day, the number of infections is R naught to the power of N minus 1. So, for example, if we go up to the 10, the number of infections going to be R naught to the power 9.

And we had also done an Excel exercise using various values of N and P and therefore R naught, and showing how infection spread under different lockdowns, vaccinations, etc. So here, what I want to ask is the following question that so see, here we have one infection in day one, R naught in day 2, R naught squared in day 3, R naught cubed in day 4, and so on up to R naught 9 in day 10. So if I asked the question, that what's the total number of infections that happened until day 10? So that'll be 1, that's in the first day, plus R so I'm getting rid of the R naught, just I'm going to call it R. Give, I don't want to add this I'm being a bit lazy and not adding R naught, okay. But that's what I mean. So on day 1 it's 1, day 2 it's R, day 3 it's R squared, so on up to day 10 is R9, right. So it's this sum, which gives the total number of infections from day 1 through day 10. So if we were to write this in our summation notation, again, I use the sigma. And what I'm doing is R to the power i, because see, the first one is R to the power zero. That's why that's 1. This second one is R to the power 1, so that's R, this is R to the power 2, and so on. This is R to the power 9. So what we are doing is adding up R to the power i, and where is i running from? i is running from zero, that's the first term and going up to 9.

So this is a geometric series. And the difference between this and the arithmetic series is the following. See, in geometric series, if I'm trying to so the first term is 1, the second term I get R by multiplying the first term by R. If I want to get the third term, right R squared, that I get by multiplying R by another R. If I want to get the fourth term, which will be R cubed, that I'll get by multiplying the previous term by R. So unlike the arithmetic series, where I was getting the next term by adding a constant, here, I'm going to get the next term by multiplying by a constant. So that's why this is called the term geometric series. But my main objective here is to figure out the sum, how do I add up, add up the set of numbers, which is 1 plus R squared plus so on up to R9? And here, the formula is a little different from what we got for the arithmetic series. So if you want to add up the numbers 1, R squared so on up to R to the power N minus 1, that's given by 1 minus RN divided by 1 minus R. So this is my formula for adding up numbers in a geometric series. So it's 1 minus R to the power N divided by 1 minus R.

So let's put this into use. So, so in our model of infections, remember, we want the sorry, this should be Ri equal to zero to 9, right? We wanted to do R to the power i, right? So that's what I wanted to add up in my model of infections, right? So now, if I take a particular value of R, say R is equal to 2, right, what's, what is the sum? So this is the sum 1 plus R plus so on up to R to the power 9. So I'm going to make use of this formula here. So this will be 1 minus 2 to the power, so this is the last term is remember, it's N minus 1, right? So this, so this will be 1 minus 2 to the power 10 divided by 1 minus 2. And we use the calculator to figure out what is 2 to the power 10? That's going to be 1024 divided by 1 minus 2. So this is going to be 1023. It's going to be negative 1023 divided by negative 1, so that's going to be 1023. So in the case, where R is equal to 2, the sum of the set of numbers 1 plus R plus so on up to R to the power 9, that's given by 1023. So again, let me reiterate. The geometric series is that, if you want to get the next term, you multiply the previous term by a constant. In this particular case, you're multiplying by R, right? You started with 1 multiplied by R, you get R, multiplied by another R you get R squared, multiplied by another R then you will get R cubed, and so on. So that's your geometric series.

Now, so far, in the geometric series that I've done, I've started with 1. But I don't have to start with 1, I could have started with a different number. So for example, if I wanted to start with A, and then I multiplied by R, then I get A times R, I multiply it by another R, I get the next term, which is AR squared. And then I multiplied by another R, then I'll get AR cubed, and, and so on. And so if I want to add up this series, which starts with A, right, and goes up to A to the power R to the power N minus 1, how do we do that? So what I'm going to do is make use of one of the properties of, of a series summation that I talked about before. So see, the common factor here is A, so you can pull that out. So if I pull that out, so what I'm left with is 1 plus R plus R squared so on up to R to the power N minus 1. So this summation is then just going to be A times the summation of this. And for that, we already have the formula which is 1 minus R to the power N divided by 1 minus R. So this is going to be then A times that previous sum that I talked about, which is 1 minus A to the, A times 1 minus R to the power N divided by 1 minus R. So this is the formula for a geometric series. Again, the geometric series, to get the next term, you multiply the previous one by that constant. So what I want to do next is do a couple of applications of this formula. We have already done it for infections, but I'll do a couple of other applications to use this formula, which make use of this particular formula.