module2_lecture8

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So in the last clip we looked at Gauss' formula for adding up numbers from 1 through N. And this was given by this particular formula, which is N times N plus 1 divided by 2. And then I asked the question, what if the number didn't start from 1, but it started from something else, like let's say 51 through 80? If we wanted to add up the numbers from 51 through 80, do we need to change the formula? So luckily, Gauss' ingenious method works in this case, too. So if we were to add up the numbers from 51 through 80, so again, we can represent it by the summation notation as we had before, this is adding up i from i starting from 51, and then ending up at 80. So now, if we actually wanted to find the sum, right, how would we do it? Let's use again Gauss' method, so 51, plus 52, and so on, up to 79 plus 80. We'll see if we add up 51 and 81, sorry, 51 and 80. So that'll give you 131.

So if I add up the next two set of numbers, which is 52 and 79. So 9 plus 2 is 11. And then carry over 1, 5 and 712. So that's again, 131. If we added up the third set of numbers, which would be 53 and 78, that would also be 131. So again, it's just very much reminiscent of how we added up numbers from one through 100. So again, you see, if I add up the first and the last number, that's 131. If I add up the second and the second last number, that's the same sum 131. If I add up the third, and the third last number, that's also 131. So if we want to find out the summation of all of these, these are all 131. And how many pairs are there? So there are a total of, going from 51 through 80, there are a total of 30 numbers. So the number of pairs will be 30 divided by 2. So, so the answer here will be 15 times 131. And whatever that is, that's the summation of the numbers from 51 through 80.

And you can use this idea very generally. So instead of instead of this particular set of numbers 51 through 80, suppose it's a general set of numbers starting from 1, going up to A2, A3, so on up to AN, but with this particular property that the next number, so the number Ai plus 1 is 1 more than the previous term. So this is my arithmetic series that my next number is, can be, is just 1 plus the previous number. So here, the same idea works that if we want to find the sum of this, you take the first number, which is A1, add it to the last number, which is AN, like we did for 51 through 80. Remember, the formula was 51 plus 80, that's 131, times, see the number of terms there were were 30 divided by 2, that gives the number of pairs, right? The same idea here. So if for the arithmetic series with N terms, where you're starting with A1, ending up at AN, you do A1 plus AN, the first number times plus the last number, times the number of terms which is N divided by 2.

So this is your formula for arithmetic series, where the numbers just all add up by, so all increase by one. In fact, the the formula is even more general than that. The numbers don't even have to increase by one. They can increase by two or they can increase by three. It's, so let me again show you how that goes. So let's consider the case where we want to add up the following set of numbers. 55 plus 60 plus 65 and so on up to 75 plus 80. So this is a case where the numbers, they increase not by one, but by five. But we can still use the same idea. Of course, even in this case, see if I add up the first and the last number 55 and 80, that's 135. If I add up the second, and the second last, that's 60 and 175, and 75, that's also going to be 135, and so on. So if I want to get what's the, the sum of this set of numbers, each of these additions are going to be the same as that of the first number and the last number. And now the only thing that we need to figure out is how many such pairs of numbers are there. So see, we go from 55 to 60, that's two, 65 to 70, that's four, right, and then 75 to 80, that's six, so there are six numbers divided by two. So this is 3 times 135. So if you calculate that, that'll be 405.

But the main point is that this Gauss' formula, where you add up the first number, plus the last number, multiplied by the number of terms and divided by two, this holds even when the numbers don't go up by one, so long as they go by a constant. So if Ai plus 1 is Ai plus a constant C, Gauss' formula works. So now, having got a hang of this general formula for any arithmetic series, it doesn't have to start at one doesn't have to go up by one. But it can start anywhere, so long as it goes up by a constant term, this formula applies.

So now let's apply it to actually some examples. So let's do this following example, that suppose Kim joins a company with a starting salary of $501. This is his yearly salary. And every year, his salary goes up by $1. And now, suppose Kim works at the same company for 40 years. So the idea is, so the question is, what's his total lifetime earnings. So he starts off with 501. Then next year, his income is 502, the following year, it's 503. And since he has worked for 40 years, this is his salary in year one, the salary in two, year three, his ending salaries 540. So this is probably quite a few years in the past, where $500 would be your yearly salary, and, and people worked at the same company for the lifetime. But the question here that I'm asking is that what is his lifetime earnings? So his lifetime earnings is given by this sum here.

So how would we find that sum? So we're going to make use of Gauss' formula. Remember, first term last term, add those two things up, so that's 501 plus 540. How many terms are there? We start at 501, end up at 40. So that's 40 terms here, divided by 2. So that's 20 times 1041. So that's 28, 2082. So that's Kim's lifetime. So what we've just done is made use of Gauss' formula to figure out the sum and thereby figure out Kim's lifetime earnings. So, so we can apply this sort of logic or this formula to lots of other applications as well. So let me stop here and in the next clip I'll show you one other application and then I'll have a clicker question just to make sure that you're on top of, of this Gauss' formula.