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SPEAKERS

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Okay, so so far, we have done the summation notation. And in this part of the module, we'll be covering some common sums that you see in different, arise in different areas of the social sciences. So the first one that we'll be dealing with is a sum like this, which is just adding up all the numbers from 1 to 100. 1 plus 2 plus 3 plus 4, and so on up 100. So this is called an arithmetic series, because see, each number can be derived from the previous one by adding one. So this type of a series where each number can be derived from the previous one by adding a particular value, in this particular case it's one, it's called an arithmetic series. So first of all, right, how would you denote this in our traditional summation notation, a series like this? Again, we do summation, and see the numbers are 1, 2, 3, 4, and so on, right, starting from 1 to 100. So what we can do is, we can add up i, where i runs from 1 to 100. This denotes this particular series, because see, when i is equal to 1 that means it's 1, when i is 2 that's 2, i is 3, it's three, and so on up 100. Right, so we can denote this particular series by this notation, summation over i from i equal to 1 to 100. But now comes the main idea of how to actually find this sum. And the idea is first ascribed to a famous mathematician called Karl Friedrich Gauss. He was German mathematician who did very phenomenal work, both in mathematics and in physics. And the story goes that in elementary school Gauss' teacher gave them this problem, just to keep the students occupied, add up all the numbers from one to 100. And while his classmates, most of his classmates, were actually trying 1 plus 2 is 3, 3 plus 3 is 5, and so on, Gauss hit upon the following ingenious idea. So what he said is that, okay, if you're adding up the numbers, 1, 2, 3, and so on, up to 99 and 100. So if you add up the, if you add up numbers 1 and 100, that's 101. If you add a 2 and 99, that's also 101. If you add up 3, and the next number here, which will be 98, 98 plus 3, that's also 101. So that means anytime we add up the first and the last number, the second number and the second last number, the third number and the third last number, right, they all add up to the same one, which is 101.

So if you do it this way, right, so how many such pairs are there? There's a total of 100 numbers, so the number of such pairs is half of that, which is 50. So the total sum will be 50 times 101. So that was Gauss' ingenious idea. So the summation from 1 through 100 is just going to be 50 times 101, that's 5050. And imagine he's doing this in elementary school. Anyways, I don't know, that that's a story which is famously ascribed to Gauss. But this has this idea, right, and the formula that comes out of it has been referred to as the Gauss' formula, right. So, if you want to, and the, what is the

Gauss formula? That means If you're adding up a set of numbers 1, 2, 3, 4, up to N, and again, the same idea if you're adding up 1 plus 2, and so on, the last number is N, the second last number is N minus one. Again, if you add the first and the last number, that N plus one, if you add up the second and the second last number, so that's 2 plus N minus 1, that's also N plus 1, add the third and the third last, that's also going to be N plus 1. Okay, and how many such pairs are there? So if there are N numbers, the number of pairs will be half of that, right? So this will be N over two pairs, each of them sum up to N plus one. So the summation of these numbers from 1 through N must be equal to N over 2 times N plus 1.

So this is Gauss' formula for adding up numbers 1 through N. So this is the Gauss' form. Now the good thing about this is that see, you can apply it to any N, right? So for example, if your teacher gave you instead of Gauss, where his teacher gave him to add up 100, what if your teacher said okay, now that you are in university, why don't you add up numbers from 1 through 1000. You can do it again in one step, just use Gauss' formula. So there are 1000 numbers here. So N is 1000 here, so this will be 1000 times 1001 divided by 2. Right? So this is again, you cancel this, this is 500. So this will be 00 and 5005.

So that will be the sum of numbers from one through 1000. Now, if you want to do a really, a little bit of rigorous proof of this, right, and I don't usually do proofs, proofs in this course, but this is a pretty simple one. So I'll show it to you just to give you a flavor of how mathematical proofs actually work. Right? What's the problem with our, with our previous thing here? So see here, I, in this argument, I agreed that I argued that each of these two things add up to N plus 1, right? And because if there are N numbers, number of such pairs will be N over 2. Now that works fine if the number of numbers is even. What if they were in odd set of numbers? If they were 100, if you're adding up from 1 to 101. Right? In that case, you're not going to get pairs, like a whole, whole number of pairs, because you because the number of numbers we have is 101, which is not divisible by 2. Okay? Do we have to change the formula? We don't, okay. And that's why I'm going to show you this quick proof, right? So so the idea is this, right, you want to find the sum of these numbers, 1 plus 2 plus 3, plus so on up to N. Now, if we reverse the order of the numbers, so if I instead add up from the end, I do N plus N minus 1 plus N minus 2 and do it backwards. But that sum is still going to be the same. It's still going to be S. Okay, and remember, S is what we're trying to find out. What is that sum, right. Now, if we add up,if I add these two things up. So this is S plus S, that's 2S. And if I add up on this side, so if I add these two up, right, that's N plus 1, if I add these two up, N minus 1 plus 2, that's another N plus 1. If I add these two up, that's another N plus 1, and I go on. The last one is also N plus one. So that means 2S must be equal to how many of them are here, right, there are N of them here. So there are N times N plus 1. So what that means is that if I'm trying to isolate out for S, remember, this is what I'm trying to find out. So that S must be equal to N times N plus 1 divided by 2. So that's the Gauss' formula. And this holds for whether N is odd, whether N is even, it doesn't matter, it's the same formula.

Now, you may be thinking in your mind that okay, he has added up from 1 through N. What if it didn't start at 1, it started at somewhere in between. Let's say you've started at 51, 52, 53 and you want to add up to 80. How do we do that? So that's what I'll be doing in the next clip. So let's stop this clip here. And in the next clip, I'll be doing a more general version of Gauss' formula and also applying it into, into an example.