module2_lecture5

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So in the last clip, I talked about how you can denote the total score in a course, as the, as the weight times the score on, on the number of assignments. So we got a term like this, which is W1 S1 plus W2 S2, up to W5 S5. And we can represent this as in the summation notation, as Wi times Si, i running from 1 to 5. So this is sometimes called the weighted average of scores, because what you're doing is you're putting different weights on different scores and then adding them up. So in this clip, I'll be talking about two particular properties of summation, which sometimes come in quite useful. And both of these properties are relatively simple. But I think I should still mention it, so that in certain particular circumstances, you will see that it makes things pretty easy. So for example, if you're adding up a set of numbers like this, which is 10 plus 20 up to 100. So all of these are multiples of 10. So this is, the first one is 10 times one, the second one is 10 times two, and so on. And the last one is 10 times 10.

Now see, there's a common multiple 10 for each of these numbers, right, a common factor here. So I can take that out. So if that, if I take that outside the bracket, then this, what remains is 1 plus 2 plus so on to 10. So this is the same as the sum of these numbers 1 through 10 multiplied by 10. So this holds, even if the numbers are not just 1, 2, and 3, and so on, if you take any set of numbers, which are all multiples of 10. So this is 10 times X1, this is 10 times X2 so on up to 10 times XN, you can take the common factor 10 outside, and then add up the X1, X2 up to XN. So if I portray this in summation notation, what this means is that if I were to add up these numbers, which are all multiples of X, of 10, sorry, so if I write 10 times Xi, as each of these numbers, and i running from 1 through N, this is the same as 10 times the summation of the Xi's, i running from 1 through N.

So this here is the left hand side. So this is the summation of the 10 times X1 plus 10 times X2, so on up to 10 times XN. And on the right hand side, this is just 10 times the summation of the Xi's. So what this means is that if the numbers are all come multiples of some common number, you can take that common number outside and, and then add up whatever remains, right? Or another way of saying this is that the 10 times this, the sum of the Xi's, is the sum of the 10 Xi's. And this holds more generally. It need not be 10, right. If there is a common multiple like C, this is C times X1, C times X2

so on up to C times XN, you can do the same trick. Pull that common factor C outside then add up the X's. Okay, so what that means is that if you add up C times Xi's with i running from 1 through N, this is the same as if you added up just the Xi's from 1 through N and multiply that by C.

So this is property number one is a really straightforward property but it sometimes comes in quite handy. And you'll see that in the next clicker question where, where Aya is measuring the heights of trees in a forest. And she measures it in, in meters, while her cousin Maya measures the same thing in feet. So let me ask you now to do this particular clicker question, which is based on the property that we just did. So please stop the video at this point, and attempt this clicker question. Okay, so I hope you got the chance to do the clicker question. And what it is, is that Aya measured the heights of trees in a forest in meters, and found that the total height of all the trees was 9000 meters. Maya measured the same but in feet. And we know that 1 meter is equal to 3.3 feet. So if we look at Aya, what she did is, she added up the height of all of the trees, I mean from 1 to N. On the other hand, if we look at Maya, she also added up the height, but she added them up in feet, right, instead of meters. And we know that 1 meter is 3.3 feet. So the relationship between her and Aya's calculations are each of these, Aya's calculation, these are in meters, if Maya converted them to feet, that means she multiplied each of these by 3.3 and then added them.

Now, recall the property that we just did. So if each of these numbers are multiplied by 3.3, we can pull that outside. And then it becomes just the summation of the Hi's from 1 to N. So if we look at Maya's calculation, it must be 3.3 times whatever Aya calculated, because this is what Aya had calculated, right? So this is the part that Aya had calculated. So Maya's calculation is 3.3 times what Aya had calculated, which was 9000. So the right answer to this particular clicker question is B, where it's Maya's calculation of total height is 3.3 times what Aya had calculated, which was 9000.

So now let's move on to a second property of of summation, and this is when you add up two sets of numbers. So suppose you have, so I've given an example here, suppose Vi denotes the total number of violent crimes in province i, and Zi is the number of non-violent crimes in province i. And you sometimes see this sort of statistics in sociology, right. So, and suppose you wanted to know what's the total number of crimes? So if you now wanted to know the total number of crimes in province i, this would be pretty simple. You just add up these two numbers Vi plus Zi.

And now suppose you wanted to do the same thing, but you now want to know the total number of crimes all across Canada. So the total number of crimes in province i is Vi plus Zi, but what I am interested in is the total number of crimes across all provinces. So if I want to do the total number of crimes across all provinces, right. So remember, in each province, it's Vi plus Zi. Now if I want to add this up, across all provinces, then what I would be doing is I would be adding up the Vi plus a Zi in province 1, plus province 2, plus province 3, so on up to province N. Right? So what I would be doing in summation notation would be adding up this Vi plus Zi terms, starting from i equals 1, to i equal to N.

Okay, so this is the total number of crimes across all provinces. Now, the property here is the following, and this is property two. That if you add, if you want to add a Vi plus Zi from i equal to 1 to i equal to N, you can also do it the following way, right, you add up the Vi's across all the provinces,

equal to N, you can also do it the following way, right, you add up the V's across all the provinces, and then you add up all the Zi's across the provinces. These two are equivalent. So, what it means, again is the following, right. So, suppose here is, here is our provinces, right, and these are the violent crimes and these are the non violent crimes, right? And let's say this is province 1, where the number of violent crimes is 10, this is 5, province 2 let's say this is 100, this is 500. Province three, this is 40, and this is 80 and so on, right? Now, if you want to add up what's the total number of crimes? You can do it in two ways. One is that you can add up the V's, the violent crimes across all the provinces, right? So you can do 10 plus 100 plus 40, that's 150. Okay, then you can add up all the Z's, right, so this is 5 plus 500 plus 80, that's 585. And then you can add these two up, that will give you your bottom line. And so first, you add all the V's, then you add all the z's. Right, so this is like here, you first add up all the violent crimes across all the provinces, then you add up all the non violent crimes across all the provinces, right, to get the total number of crimes across Canada.

You can do this addition, also another way, right. So the other way is that you first counted by provinces, right, so this you add up 10 plus 5, this is 15, right? Then you look at province 2, add up these two, which is 100 plus 500, that's 600, right? And then you add up these one, which is 40 plus 80 which is 120. And, and then you add up all these three, which is 15 plus 600 plus 120, to get the overall total number of crimes. And what this tells you, what this property tells you, is that these two numbers must be the same. So whether you add up first by province, right, and then you add up everything or vice versa, that you first add up here, and then here and then add these two up, these two are the same. So this is property two, that means if you're adding up two sets of numbers, you can either first add up, for each i, you can add up the two numbers, and then you add it up across all the i's. Okay, or you can first do, add up the first set of numbers across all the i's, right, then add up the second set of numbers across all the i's and then add those two things together, those are the same.

And I'm going to do this sort of thing also in, in an Excel exercise that we're going to do in the next clip, right. But what I wanted to point out to you is this general property of summation that if you're say, adding up a whole set of numbers, right, so if you're adding up Ai, Bi, Ci, Di, you can do it two ways. One is first add up for each particular i, you add up the Ai to Bi to Ci to Di, and then you add it up across all the i's. Or you can do that first add up Ai's for all the i's, then add up the Bi's for all the i's, then add up the Ci's for all the the i's, and then you add up the Di's for all the i's, right? And then you add all of these together. These two are going to give you exactly the same answer. So this is another property of summations, which sometimes comes in handy, but it's basically, what it tells you, it doesn't matter how you do the summation, like here, you could either do add up across, like first the columns and then add up at the bottom. Or you could first add up the rows and then add up the subtotals for each row, you'll get the same answer at the end. So this is property two in summations. So let me stop the clip here. And as I mentioned, the next clip will be on doing summations using Excel, using Excel and Google Sheets.