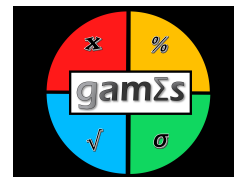


GAMES Practice Problems – Multivariable Calculus



- Find $\frac{dz}{dt}$ using the chain rule in the following cases. Assume the variables are restricted to domains on which the functions are defined.
 - $z = xy^2$, $x = e^{-t}$, $y = \sin t$
 - $z = xe^y$, $x = 2t$, $y = 1 - t^2$
- Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ in the following cases. Variables have restricted domains.
 - $z = xe^y$, $x = u^2 + v^2$, $y = u^2 - v^2$
 - $z = \sin \frac{x}{y}$, $x = \ln u$, $y = v$
- The voltage, V , (in volts) in a circuit follows Ohm's law: $V = IR$, where I is the current in amps flowing through the circuit and R is the resistance (in ohms). If we place two circuits, with resistance R_1 and R_2 , in parallel, then their combined resistance, R , is given by:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Suppose the current is 2 amps and increasing at 10^{-2} amp/sec and R_1 is 3 ohms and increasing at 0.5 ohm/sec, while R_2 is 5 ohms and decreasing at 0.1 ohm/sec. Calculate the rate at which the voltage is changing.

- In the following cases, calculate all four second-order partial derivatives and check that $f_{xy} = f_{yx}$. Assume the variables are restricted to a domain on which the function is defined.
 - $f(x, y) = 3x^2y + 5xy^3$
 - $f(x, y) = (x + y)^3$
 - $f(x, y) = e^{2xy}$
- Find the quadratic Taylor polynomial about $(0, 0)$ for the function $f(x, y) = (x - y + 1)^2$
- The quantity of salmon s purchased at a store, in kilograms per week, is a function of the price of salmon, p_S , and the price of trout, p_T , both in dollars per kilogram.
 - Money in a bank account earns interest at a continuous rate, i . The amount of money, $\$m$, in the account depends on the amount deposited (the principal) $\$P$, and the time, t , it has been in the bank and is given by the following formula:

$$m = Pe^{rt}$$

Find $\frac{\partial m}{\partial t}$ and $\frac{\partial m}{\partial P}$. What do these represent in financial terms?

- Do you expect $\frac{\partial S}{\partial p_S}$ to be positive or negative? Why?
 - Do you expect $\frac{\partial S}{\partial p_T}$ to be positive or negative? Why?
 - Interpret the statement $\frac{\partial S}{\partial p_S} = -50$ in terms of quantity of salmon purchased.
- Find the global maximum and minimum of the following functions on $-1 \leq x \leq 1$, $-1 \leq y \leq 1$, and say whether it occurs on the boundary of the square.

(a) $z = -x^2 - y^2$

(b) $z = x^2 - y^2$

8. Do the following functions have global maxima and minima on their natural domains?

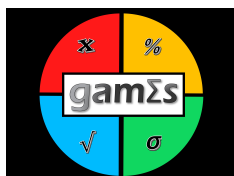
(a) $f(x, y) = x^2 - 2y^2$

(b) $f(x, y) = x^2 y^2$

(c) $h(x, y) = x^3 + y^3$

(d) $f(x, y) = -2x^2 - 7y^2$

(e) $f(x, y) = \frac{x^2}{2} + 3y^3 + 9y^2 - 3x$



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