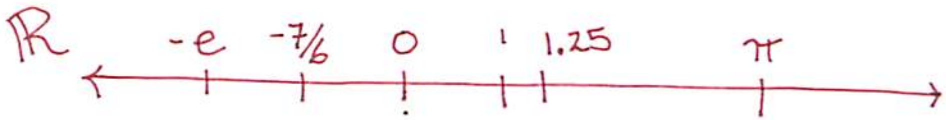
Real Numbers, Integers, and Their Notation

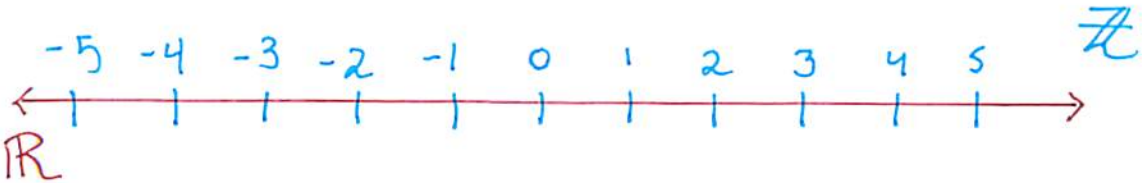
We use $\mathbb{R}$ for the collection (set) of all real numbers,
i.e. everything on the number line



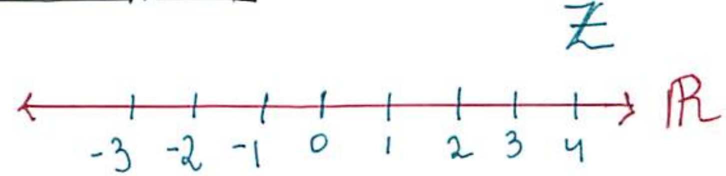
<u>SO</u> when we write $x \in \mathbb{R}$ we mean "x is some real number"

<u>Q</u>: What if we want to allow integers? (such as in the increase of goods produced)

<u>A</u>: We require $x \in \mathbb{Z}$ (x is an integer)



Integer Examples / Nonexamples



For $x \in \mathbb{Z}$ we mean x is an integer

SO

- x could be a number like:

  -532 or

  0 or

  $3^2 = 9$ or

  $4/2 = 2$

BUT

- x could not be a number like

  $3/2 = 1.5$ or

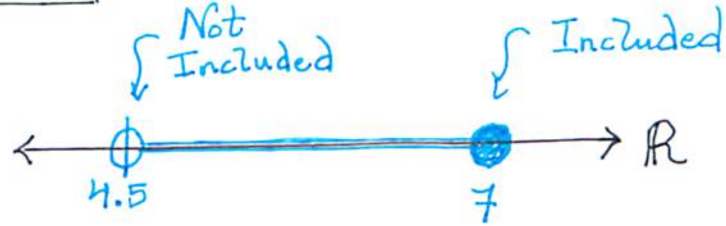
  $-\pi = -3.14\ldots$

Examples of Numbers that Must Be Integers

- Increase in the number of active cases of a disease (if decreases, negative integer)
- Car is located x spots to the right of the red car (if to the left, negative integer)

Some Ways to Write a Set

Q: How can we write $(4.5, 7]$, or other infite sets, in set notation?



$\{A \mid B\}$ means:

"everything of the form A satisfying B"

SO

Can write $(4.5, 7]$ as

$$\{x \in \mathbb{R} \mid 4.5 < x \leq 7\}$$

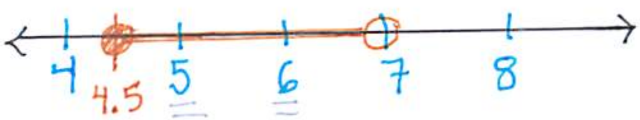
set of all real numbers x satisfying $4.5 < x \leq 7$

Inequalities and Finite Sets : Part 1

Q: Can we write $\{x \in \mathbb{Z} \mid 4.5 \leq x < 7\}$ more simply?

integers x satisfying $4.5 \leq x < 7$

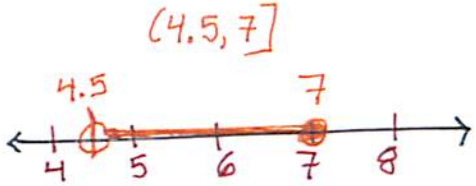
A: $\{5, 6\}$



Breaking It Down

- 5 and 6 are solidly between the endpoints, and **integers**, so are definitely included
- 4.5 is in the interval, but not an integer so out
- 7 is an integer, but is not in the interval, so is out

Inequalities and Finite Sets: Part 2



__Q__: Can we write $\{x \in \mathbb{Z} \mid 4.5 < x \leq 7\}$ more simply?

__Answer__: $\{5, 6, 7\}$ (7 is now included because it's in $(4.5, 7]$ __and__ also an integer!)

__Question for Going Forward__:

$\{x \in \mathbb{Z} \mid 4.5 \leq x < 7\}$ and $\{x \in \mathbb{Z} \mid 4.5 < x \leq 7\}$ were both finite sets

What about a set like the even integers? Next time!

Writing Infinite Sets with Patterns

Q: How can we write the set of all even integers?

Slick Answer: $\{2k \mid k \in \mathbb{Z}\}$

"The set of all $2k$ satisfying that k is an integer"

Much Easier to Understand

$\{\ldots, -4, -2, 0, 2, 4, 6, \ldots\}$

Breaking It Down

- Need dots to indicate pattern continues on forever
- Needed to include enough elements to make the pattern clear

  e.g. $\{2, 4, \ldots\}$ could be positive even integers OR positive powers of 2

Set Overlaps: Intersections

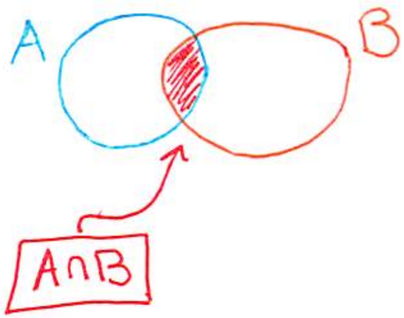
Set-Up: A and B are sets

$\boxed{A \cap B}$ is the $\boxed{\text{intersection}}$ of A & B

Technical Definition:

$$\boxed{A \cap B} := \{ x \mid x \in A \text{ and } x \in B \}$$

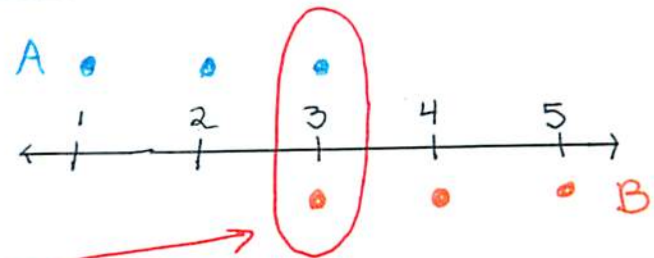
SO it's the "overlap" of A and B



Favorite Example:

$$\begin{cases} A = \{1, 2, 3\} \\ B = \{3, 4, 5\} \end{cases}$$

Image:



3 is the only number in the overlap, i.e. in both A and B

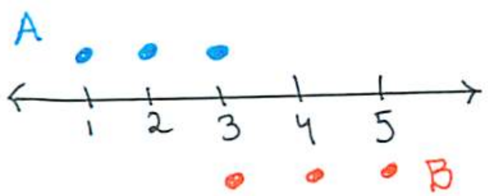
SO $\boxed{A \cap B = \{3\}}$

Combining Sets: Unions

Set-Up: A and B are sets

Favorite Example $\begin{cases} A = \{1, 2, 3\} \\ B = \{3, 4, 5\} \end{cases}$

In Example:



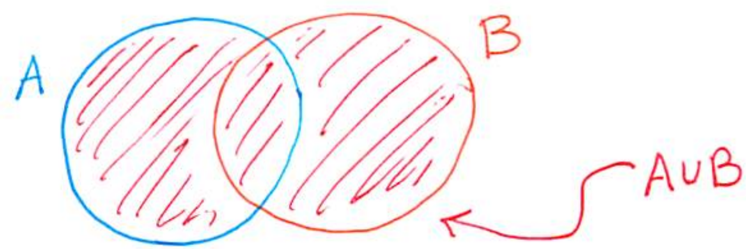
$\boxed{A \cup B} = \{1, 2, 3, 4, 5\}$

$\boxed{A \cup B}$ is the $\boxed{\text{union}}$ of A and B

Technical Definition:

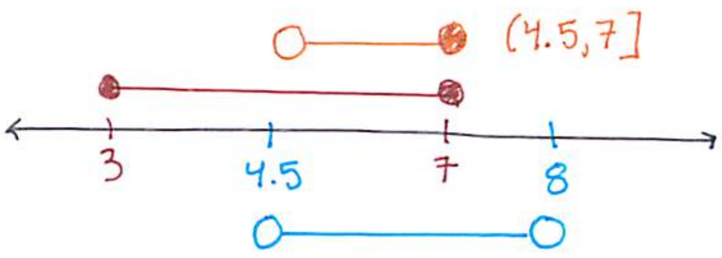
$\boxed{A \cup B} := \{x \in A \text{ OR } x \in B\}$

SO: It's the combination of A and B



Intersections of Intervals

Q: $[3,7] \cap (4.5,8)$?



Interval Venn Diagram

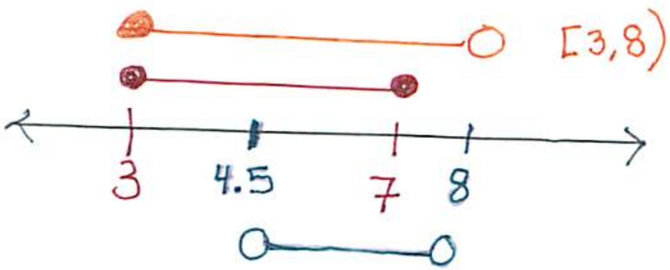
1. Start by writing all endpoints
2. Helps to draw 1 interval above & 1 interval below
3. Find where the intervals overlap

Breaking It Down (Looking for numbers in both [3,7] AND (4.5,8))

- Everything between 4.5 and 7 is in both intervals, so is in the intersection
- So check endpoints carefully
- 4.5 is in [3,7] / is not in (4.5,8) } one failed, so 4.5 is not in the intersection
- 7 is in [3,7] / is in (4.5,8) } both satisfied, so 7 is in

Unions of Intervals

Q: $[3,7] \cup (4.5,8)$



Looking for what is in <u>at least</u> one of them

"<u>Interval Venn Diagram</u>"

1. Start by writing all endpoints
2. Helps to draw 1 interval above & 1 below
3. Find where intervals overlap find combination for unions

Breakdown of Example

- Everything between 3 and 8 is either covered by [3,7] (above) or (4.5,8) (below), so is <u>in</u>
- 3 is in [3,7], so is <u>in</u>
- 8 is in <u>neither</u> interval, so is <u>out</u>