

GAMES Practice Problems – Applied Differential Calculus

1. Find $\frac{dw}{dv}$ using implicit differentiation. Make sure you have the correct answer by rearranging the expression to isolate w and taking the derivative w.r.t. v

$$3w + 6v^2 = 12$$

2. Find the first and second derivative ($\frac{dw}{dv}$, $\frac{d^2w}{dv^2}$) using implicit differentiation. Make sure you have the correct answer by rearranging the expression to isolate w and taking the derivative w.r.t. v

$$w^2v = 1$$

3. Find y' using implicit differentiation where b is a constant for the following

(a) $x^2 + y^2 = b^2$

(b) $\sqrt{x} - \sqrt{y} = \sqrt{b}$

(c) $x^4 - y^4 = x^2y^3$

(d) $2xy - 3y^2 = 16$

4. The differentiable function h is given. y is an implicit function of x . Find y' for each of the following:

(a) $g(x) - y^3 = -xy$

(b) $g(x + y) = -x^2 - y^2$

(c) $(xy - 1)^2 = g(xy^2)$

5. Excel: find the linear approximation of $\sqrt{1+x}$ when x is close to (about) 0. Use Excel to create a graph with both $y = 1 + 0.5x$ and $y = \sqrt{1+x}$ on the domain $x \in [-1, 3]$ and compare the two.

6. Find linear approximations for the following about $x = 0$

(a) $(1+x)^{-1}$

(b) $(1+x)^5$

(c) $(1-x)^{1/6}$

(d) $(5x+3)^{-2}$

7. Compute the actual change in y : $\Delta y = f(x + dx) - f(x)$ and the approximate change in y : $dy = f'(x)dx$ for each of the following:

(a) $f(x) = x^2 + 2x - 3$ when $x = 2$ and $dx = 0.1$

(b) $f(x) = \frac{1}{x}$ when $x = 3$ and $dx = -0.01$

(c) $f(x) = f(x) = \sqrt{4}$ when $x = 4$ and $dx = 0.05$

8. Find the linear approximation of $h(k) = B(1+k)^{\frac{c}{c+d}} - 2$ where c, d, B are positive constants.

9. An Economy with Labour and Capital – Consider an economy that produces output, Q , using both capital and labour K, L . All other influential factors are represented by the constant, A .

$$AK^{2/3}L^{1/3} = Q$$

When taking Q as a constant value, recognize that the value of K depends on the value of L .

- (a) Use implicit differentiation to find the marginal rate of substitution, $\frac{dK}{dL}$.
 - (b) When $K=100$ and $L=400$ and the economy gives up one unit of capital, approximately how much labour would the economy need to add to maintain the same amount of output?
10. Simple Macroeconomy – Consider a general version of the macroeconomy where output (Y) is equal to consumption (C) plus investment ($\bar{I}$). Note that consumption depends on output:

$$(i) Y = C + \bar{I}$$

$$(ii) C = f(Y)$$

- a) Combine these equations and find $\frac{dY}{d\bar{I}}$ assuming Y is an implicit function of $\bar{I}$.
 - b) If $f'(Y) = 2$, what is $\frac{dY}{d\bar{I}}$?
11. Supply-Demand Model with a Tax – Consider a linear supply and demand model where consumers pay a tax of τ per unit.

$$D = a - 500(P + \tau) \quad S = \alpha + 100P$$

a and α are positive constants. The equilibrium price is found when supply equals demand:

$$a - 500(P + \tau) = \alpha + 100P$$

- (a) Solve the above for Price, P , then find $\frac{dP}{d\tau}$ using the combined equation.
 - (b) When the tax rate increases, does the price, P , increase or decrease? Use implicit differentiation to find $\frac{dP}{d\tau}$ and interpret your results.
12. Consider the general forms of the supply and demand functions:

$$D = f(P + \tau) \quad S = g(P)$$

where $f' < 0$ and $g' > 0$

Determine if the Price paid by the consumer, $P + \tau$, increases or decreases when the tax rate, τ , is increased?

