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SUMMARY KEYWORDS

derivative, squared, partial derivative, equal, chain rule, respect, points, function, cubed, critical, orange, saddle, part, 8x, order, circumstance, partial, compute, step, greater

SPEAKERS

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Welcome. In this lecture we're going to go through how to kind of classify critical points into being local minima and maxima and saddle points. This is actually usually a very involved long process, because you have to take second order partial derivatives all over the place. And, right, because you have to get this function D that's going to tell you how to classify them. And so I decided to start with the example where we've already kind of done part of the work so that we can just kind of do that next steps or that like new parts of it. So we're going to assume that we know that the partial derivative with respect to X of F is 8Y minus X plus Y cubed, and the partial derivative with respect to Y of X, Y is equal to 8X minus X plus Y cubed, okay? And that these are the critical points, and we're going to classify them. So let's get started. So in this, the first step is going to be, because we know that we're going to need to compute this function, D. So what we want to have is that for D equals, and then we have F X, right, this second order partial derivative, and this one, then we're going to subtract off this, right, so we're going to need, like looking at this, we know we're going to need this, and this, and this. Okay, so let's get started. So let's, let's, you know, start with the first one. So this is going to be.

Actually, let's, let's, it'll be orange once it's all the way done. So this, so what does that equal? So we've got to take the derivative with respect to, so we're going to have to take the derivative with respect to X of this one here, so we have the derivative with respect to X of, and then we're just going to have that up there, right, so this is going to be just going to be of F X, right? So this is equal to the derivative with respect to X of, and then we already know what this is, that's up there. So we're going to go ahead and put that in. So this is going to be 8Y minus X plus Y cubed. And then we remember that how we do this is to treat Y as a constant and then take the derivative with respect to X. And when we do this, so we'll do kind of like a little bit careful in here, which is that we get, so we have to use a chain rule here, that part's just going to drop out, because it's a constant. But here, we actually need to use the chain rule. So we need to do that a little bit carefully. So we're going to get minus 3, and then we have X plus Y squared, and then we have to take the derivative with respect to X, we can't forget that we have to do that of, and then we're going to do of, right the inside here. So this is X plus Y. But this is going to actually just be 1, right, because the Y is a constant and the X, when you take the derivative of X with respect to X, you get 1. So this is in the end, just going to equal minus 3 times X plus Y squared. Okay, so now let's get started on the next one. It's going to be this is the, we want to take this second order partial. So in this circumstance, now I need to take the derivative with respect to Y of, and then I have, so here, this is this partial. So this is going to equal, so, right, this has to, I have to take the derivative with respect to Y. And then this part I can just take from right up here. So this is going to equal, so it's almost the same thing as this, but it's 8X minus X plus Y cubed. And then again, we're going to have to be very careful in using the chain rule. And again, so now we're treating X as a constant, right? So we're taking the derivative with respect to Y. So this drops out because it was treated as a constant. And then we have, you know, again, we get the same thing. But now we have the derivative with respect to Y of the inside as the second part of the chain rule, but still, this is 1. So I end up getting very coolly the same exact thing there. Okay? There's some baked in what we call symmetry, which is how we'd end up with the exact same thing there, okay? Now, we want.

We want this right, so remember that this one tells us the outside thing we have to do. So we have to do the derivative with respect to X of, and then of this one, the partial derivative of F with respect to Y. So then we get the derivative with respect to X of, and then we just plug it in like we did there. So this is the same partial derivative with respect to Y. So this is 8X minus X plus Y cubed. Okay, and you can kind of see, you would be applying the chain rule again. But by now I'm going to kind of trust that we could see that with a new part being that we're going to pick up an 8 there. So this is going to equal 8 minus 3X plus Y squared, right? And we know that this is the same thing as if you swap those. Okay, great. So my D is going to equal, I'm going to have, or my D X,Y, this is actually a function of X, Y, I could write it like that. Let's just write D, but it's actually D of X, Y. Okay? So I'm looking up here, this is the first thing I need. So it's right there. And then actually, you look and you see that and then you also see, right, these are the same thing. So I'm going to have the same thing twice. So I'm actually going to be squaring it there. So I've got minus 3 times X plus Y squared, squared, right? Because this was right, this was looking up here, I have that these two multiplied by each other, but those are these two which are exactly the same. Okay, and then we've got, and then we just need to do minus, and then we have this critter square, but that critter is this one here. So I've got 8 minus 3 times X plus Y squared, squared. Okay? So now let's actually use this to compute in the next step. So step two.

I'll still do orange. It's okay, I can keep reusing colors. Okay, so step two, is actually going to be to sub in all of our critical points into here. So sub the critical points. So I want to sub my critical points into D. Okay? So what do I get there, so I have D of 0, 0. So this is going to end up equaling, so I have, right, so I have minus 3, and then I have zero plus zero, squared, right, so I just put zero in for X and Y there, and then this whole thing has to be squared. And then I have to subtract off this where I'm going to put zero in for X and Y again, so I'm going to get minus 8 minus 3 and then I'm going to have a zero plus a zero there again squared, squared, so then I can put my zeros in here. I have more of these in my hand than I need for sure. Okay, the next critical point we have is, so minus 1 minus 1.

So now it's exactly the same thing except instead of putting in zeros, I'm going to be putting in minus ones all over the place. So I have a minus 1 here and a minus 1 here. I'm going to add those, and then squared, squared minus 8 minus 3. And then these are now minus ones. I've been forgetting what the outputs of these are. So for the first one, it was 64, okay? Which is, we're going to want to keep track. So it's minus 64. So this is going to equal minus 64, which is going to be less than zero. And that doesn't quite fit on there. So I tried to make this a little bit more compact, so that I can fit that on there. Let's do this. You can fit it all for done, do it more carefully. Okay, so this is equal to minus 64, which is less than zero. There I did it, okay. And this one also let's just kind of get it a little bit more compact here, so this is squared, and this is squared and so this one here is going to equal so I get 128 which is greater than zero. Okay? And then in the last circumstance, right, so now let's put those ones in.

So now I'm just going to put in 1 everywhere, so minus 3. And I'll have ones everywhere, everywhere, everywhere. So now these are all ones for my X and my Y. And then in this one, I'm going to get 128 again, which is again greater than zero. Okay, so now I'm actually ready for my chart. So step three. We're using a lot of orange today. I don't usually use that much, but that's cool. Okay, so let's make our chart. Let's find a, this might be better for making a chart, okay. Okay, so we have, so we have 0, 0. And we have minus 1 minus 1. And we have 1, 1. Okay, and then we need our D right, and we can just get these from here, so we have minus 64, which is less than zero, right? For the next one, we have 128, which is greater than zero. And then for last one, we have 128, which is greater than zero. Okay? And then this, now you're, we're looking at this one here. So maybe I'll put that actually in, so we're looking at this here, right, coming from here. So that's going to go over here, so we have that second order partial, we have the, the derivative of F with respect to X with respect to X. And so from here, we're going to get, so we can just kind of plug in. So first, we need to plug in 0, 0. Oops, I'm supposed to have two of these in here. Okay, and maybe we can actually write out with this. So this is equal to minus 3 of X plus Y squared. So you can kind of see when we do these, that this is going to equal, and so I have minus 3 of zero plus zero squared. Right? Which is going to come out to zero. And then I, with the, so we're going to do for each of these.

So we're going to get, and then we're going to put in minus 1 minus 1, and then we'll put in 1, 1. Okay, so we're going to get for each of these, we just go minus 3 and that thing self squared okay? So in this one here, I'm putting in minus 1, right? And in this one here I'm putting 1, okay? And so I should be getting, okay, I should be getting minus 12 which is less than zero. Okay? So in this circumstance, so these should actually be, so the first one is going to give me a saddle point. Okay, so looking at our chart, this is going to be a saddle point, a local maximum, and a local maximum. So that if we're being careful, this is a local maximum, right? It's actually, you need to actually stick your critical point into the original function. This is where a lot of students make a lot of mistakes. They want to stick it into one of the derivatives. You want to stick it into the original function, and this is a local maximum. Okay, so I hope that made some sense and I will see you in the next lecture.