PfaffModule11L16

Mon, 2/21 1:55PM 10:08

SUMMARY KEYWORDS

partial derivative, equals, squared, constant, critical, 3x, respect, minima, evaluated, gradient, 3y, function, coordinate, derivative, cubed, treat, point, function f, pick, multiplying

SPEAKERS

Catherine Pfaff

Welcome. In this lecture we're going to go through an example of how to find the critical points for a two variable function. This is remember, this is going to be useful because it's going to be necessary when you're trying to find the local maxima and minima. Okay, so here's our task. So our task is going to be that we want to find the critical points, and I'll put them in red, because we often have been putting critical points in red. So we want the critical points, and this is going to be for the function, so the function that we're going to be looking at is going to be F of X, Y. So F, or F of Y, X is going to equal, so we have X cubed, and then we have minus 3X times Y square. Okay, so what are we going to need? Right, in order to find critical points, we know that we need that the gradient is going to equal zero. So we need that the gradient, so this is the gradient of this function F, and then evaluated at our critical point A, B is going to actually equal zero. So that would be a critical point, if this was true, i.e., i.e. what we need would be that, actually maybe I'll do this over here that we would have, so there are two partial derivatives we need to worry about. So the first, right, so maybe I'll put it in, do both of them like that. And then I'm just going to put these just to kind of, so that you can see we're taking the partial derivative with respect to X and with respect to Y.

So these are both, right remembering that the gradient equals, so the first coordinate comes from the partial derivative with respect to X. And since it's evaluated at A, B, I actually am looking for this evaluated at A, B. So that'd be the first coordinate to make that zero, right? That has to be zero. And then to make the second coordinate zero, again, we're going to take the partial with respect to Y evaluated at this point A, B, and we have to get zero. Okay, so this is what we actually need. Now, then, let's, let's take these two partial derivatives, and you're actually going to have to combine information from both of them. So let's go ahead and start with the first one of these, okay? So in order to find our critical points, we're going to have to look for where each of these is zero. So let's, let's start where, where this one is zero. So if I want to take the partial derivative with respect to X.

Actually, I'm going to put a little, so that it's clear that I'm starting something new here. And I'm evaluating this at X, Y. I don't know I'm at the critical point yet. So for now, let's just kind of write this as X, Y, because this is just kind of general. So how do I, so what does it mean for this to be zero? Well, let's take what does it mean to take the partial derivative with respect to X of this function? So I want to take this partial derivative, but with respect to X of, and then it's going to be this function up here, right? Because that's my F is this function here. So X cubed minus 3XY squared. Okay? And we're going to look for when that equals zero. Now, how do we take a partial derivative with respect to X? So let's just kind of remind ourselves over here what I need to do. And what I'm going to need to do here is right, we treat Y like a constant and differentiate with respect to X.

So let's go ahead and do that. Right, this part only has an X thing. So that part is not so bad. So I'm just going to get 3X squared. This is the one where I have to like, Y is a constant, Y is it constant? So Y squared is a constant. So everything here is a constant except for the X, right? Which means that we just pick out everything but the X, right. If I had a constant times X and I took the derivative with respect to X, I would just get that constant. So that's minus 3Y squared. And I'm saying that equal to zero. Okay? And then then I can just keep on rearranging. So I know that 3X squared equals 3Y squared, I'm just adding 3Y squared to both sides, I'm divided, then I'm going to divide both sides by 3. And then at this point, I can see that I would need that X equals plus or minus Y. Okay, so this is going to be one piece that I'm going to need. So now let's do the other one. So I'm going to take the partial derivative with respect to Y and set that equal to zero.

Okay? So the partial derivative with respect to Y of, and then we have this function, so I have X cubed minus 3XY squared. Okay? Now to do this one, so there is a treat Y like a constant and differentiate with respect to X. Now for this one, right, I need to treat X like a constant and differentiate with respect to Y. Okay, because this is, I'm differentiating with respect to Y here. So this is a constant, right? So that just completely drops out. And now we can look at, we have this constant in front of Y squared. So the constant part just kind of stays there. So this is minus 3X times, and then I have 2Y. Right? And I still want this to equal zero. So this equals zero, i.e. I have minus 6XY equals zero. Okay? Well, in order for minus 6XY to equal zero, I either need that X equals zero or Y equals zero. So either X equals zero or Y equals zero. Okay, well, now I know that my critical points have to satisfy this. And they have to satisfy this, right? Well, what satisfies that one of these is zero, and they're plus or minus of each other? Well, if one of them's zero, and the other one's plus or minus that, then the other ones zero. So they both have to be zero. Okay, so these are going to tell me that I'm going to need, so we need X, Y equals 0, 0. Right? So my critical point, so combining those two, I get this, so I only have one critical point, and it's going to be 0, 0.

And that's my final answer. Okay, so what did we do here? We remembered that we get critical points only where we have that the gradient is zero. But being the, for the gradient, the first coordinate of the gradient is a partial derivative with respect to X, the second coordinate the gradient is the partial derivative with respect to Y. So we need that both of those are zero. So I just took the partial derivative of each of them, or with respect to each of them, so with respect to X and with respect to Y, and set that equal to zero. Remembering when we take the partial derivative with respect to X, that's like you fix like Y, treat it like a constant, and then you differentiate with respect to X. So with Y being like a constant here, it's like I can just do this one like normal, this is 3X squared, but then it's like I have a constant times Xz so I just pick out the constant. Now I just said, you know, I added 3Y squared to both sides, you know, divided by 3, I got X squared equals Y squared, which would tell me that X had to equal plus or minus Y. Okay, which on its own isn't enough to tell you, however, I also needed that this was satisfied. So here, I'm going to take the, I'm treating X like a constant and differentiating with respect to Y. So if X is a constant, X cubed is a constant, so that just drops out when I take the derivative with respect to Y. And now you can look at this as minus 3X being a constant in front of Y squared. So I just pull out that minus 3X, and I take the derivative of this with respect to Y, so I just get a 2Y. And so multiplying that all together, I get minus 6XY equals zero. And the only way to get like a, you know constant times a product like that being equal to zero is that one of these two things equals zero. So kind of bringing all that together, I need that, right? Because if X equals zero, and Y equals plus or minus X, then Y equals zero. If Y equals zero and X equals plus or minus Y, then X equals zero, so they both have to be zero. So our critical point is the point 0, 0. And that's where we could look for local maxima and minima. Okay, so I hope that was helpful and I will see you in the next lecture