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SUMMARY KEYWORDS

partial derivative, circumstance, partial, variable, points, evaluated, reread, equals, min, magic formula, critical, local minima, formula, max, test, very common mistake, function, stick, vector, local

SPEAKERS

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Welcome. In this example, we're going to talk about how you would find the local minima or maxima of a two variable function. So let's just kind of get started. So the first thing that we need, and a lot of times will be given to you, but a lot of times it won't actually, is you need to actually find what your two variable function is that you're going to maximize or minimize. And that might sound like silly, but you know, in the question, you may have to kind of combine more than one equation, or kind of reduce things down to two variables by solving for one and then plugging it in or something like that. So this sometimes is a real part of what you need to do. So, the first thing would be to, you need to, so you need a two variable function two variable function F X, Y, that you'll be kind of trying to maximize or minimize.

Okay, and as I mentioned here, for this, you may need to use more than one equation to reduce down to two variables. So you may need to use more than one equation to reduce down to two variables. As I said, like solve for, you know, X in and you know, Z in one of them and then plug that in to get rid of Z or something to reduce down to two variables. I seem to have an extra parenthese. I don't know what you call a single parenthese. Parenthese, maybe that's it. Okay, so and then the next thing is that our critical points are. So our critical points are A, B, right? Where are we have that if I'm going to take the, so I'm taking the gradient of F.

And then I have to actually evaluate it at A, B and I need to get zero, like I need to get this zero vector. Right? But what is this here, this is actually so we know that this is actually equal to, so this is going to be I'm going to take two different partial derivatives, I'm going to take the partial derivative with respect to X of A, B. And then I have to take the partial derivative with respect to Y. And then sorry, evaluated at A, B. Okay? So using that, what do we actually want? So we want, so we're going to want this A, B with, so we want the points A, B, where we have, so we need both of these things. We need both of these are equal to zero because that has to be the zero vector. Right? So this is going to be an and statement. Then we're going to need both of these. So the first thing, right, so we need both of these partials. So I need the partial with respect to X and the partial with respect to Y, and then I need to evaluate both of those at A, B and I have to get zero in both cases. So both of these have to be zero. Okay? And then at this stage, so I recommend to students that they reread the question for whether you want because this is a very common mistake for students. The other very common mistake to make in this step is to stick in your like, you know, if you stick in your A, B before you differentiate, you're always going to get zero. So that's not going to give you anything interesting. So wait to until the end, okay? so we want to reread the question for whether want, so for whether we want, I mean that would be an example where you're trying to test if some things are critical points. In this case, you don't know what they are yet, you're just looking for them. But if you're trying to test, make sure you don't put them in until after you've taken the partial derivatives. Okay? So reread the question for whether we want, so in the first case would be where the max min is at. Right? So in this case, A, B is the answer.

Or whether, right, we're looking for the output, so the max min value, so the max min value. And then to capitalize that, like I capitalized at. Okay? So in that circumstance, I want to actually have, you know, I'm going to stick my, my critical points A, B into that, and actually get the output of the local max min. So this is the answer. Okay, and then there's a final step, which allows you to actually check that these are indeed local maxima minima. So, so four, is you're going to want to check whether you actually have, so check whether you actually have.

A local max or min okay? And there's, there's a, there's a very beautiful, nice trick for this. So this is going to be, is going to come from where you have, so there's this very special formula, I get to write this formula. Even though I used orange up there for something else, I'm going to write this formula in orange. So this formula is D equals F, we take, you know, the partial derivative with respect to X twice of A, B. And then I'm going to multiply that by the partial derivative with respect to Y twice evaluated at A, B. And then I'm going to subtract off what I would get if I would square. so I want to do, you know the mixed partial here and then square that. So this is like this magic formula. And this is why this is a magic formula is that we have this chart, which will look like okay, so if I have D, and I can divide this into circumstances where it's, so I have two circumstances where it's greater than, one circumstance where it's less than zero, and one where equals zero. And then, so we'll draw a row for each of these because then I'm going to have additional information that's going to come into play.

Okay? so the next thing that I would be interested in is this partial derivative. I'm going to make this green even though it's got, I'm making the whole thing green, that's just going to be what it's going to be. So I got this, this double partial, and then I'm checking whether it is greater than zero, or less than zero. And then the last thing that happens like with this information is to know what you have. So then I guess I would just write this as local min or max. So, okay, and in this circumstance, I have a local min. Right? And this is this kind of picture where it looks kind of like this, maybe something like that. And so we've got a local min. In this circumstance, I'm going to have a local max.

So this is more a circumstance like this. Looks like close by, it looks a little bit like that. In this circumstance, I actually have a, what would be called a saddle point. Okay? So that's going to look more like I have in one direction it's concave up and in one direction it's concave down. And I'm kind of sitting there at where that changes like that. And then in the last circumstance the test fails. So this is actually kind of really cool. Because, right, I'm able to kind of use this this special magic formula to figure out, and maybe I should actually, this is evaluated at A, B, okay? And it's going to be able to tell me whether I'm actually in a local max or a local min circumstance, unless the test fails, whenever I figured out my critical point. So the first step is to figure out the critical points where you're gonna have to take the partials and then solve for where they equals zero. But then you can actually use this with this nice formula to figure out what kind of critical point you actually have, and whether you have a local max or a local min. Okay? So, I hope that was helpful and I will see you in the next lecture.