

# PfaffModule11L14

📅 Thu, 2/17 3:54PM ⌚ 5:58

## SUMMARY KEYWORDS

boundary, partial derivative, domain, perimeter, extrema, global, absolute minimum, minima, occur, remembering, closed interval, function, variables, inputs, circle, evaluated, defined, maxima, absolute maximum, minimum

## SPEAKERS

Catherine Pfaff

---

Welcome. In this lecture, we're going to talk about how to find the global extrema of a function of two variables. Remember that global extrema are absolute maxima and absolute minima. So like the very highest peaks and very lowest values, valleys. So here we go. So we want to suppose our setup is, so suppose that we have, so we're going to have some kind of, two, right, this is going to be a function of two variables so, got the wrong thing. So we want to have  $F$  of  $X, Y$ , and this is going to be defined on, so this function of two variables is going to be defined on its domain is going to be, so it's going to be closed. So what that's going to mean is that it contains the boundary or kind of like the outside perimeter points. Okay, so this means including the boundary. It's a little bit more technical than that, but you can think of it that way. Okay, kind of the perimeter of it, it's going to contain that it's not missing that. And then, right, and this would be like a closed interval versus an open interval kind of thing. And then the other part is that it's going to be bounded.

Which means basically means you can stick it inside a circle, or that the inputs can only get, you know, they can't get infinitely big. So the inputs can only get a certain amount big. The inputs can only get a certain amount big. Okay, and then this is our domain  $D$ . Okay? So the picture kind of being that I have, so down below, I have the domain on which it's defined on, but I said it has to be closed and bounded. So it has to look something like, you know it can be, nobody said it can't be a little funny. But it has to contain this boundary. Right? And then kind of inside and I need to be able to include it within a circle kind of there. And then I'm going to have the graph of the, right, this is, right, and then I'm going to have the graph of my function  $Y$  equals  $Z$  of  $X, Y$ , but this is kind of, it's on, it's just on that domain. So this is  $Z$  equals  $F$  of  $X, Y$ . Okay, and so then what how do I look for the global extrema? And this should look very familiar from when we were talking about the global, like the absolute values, the sorry, the absolute maxima and absolute minima on a closed interval. So then the global extrema, so the global extrema occur, either, okay, so you're either going to have, so this is where they occur. So they're either going to be on the boundary, let's make them red, so they occur. So this is where they're at. So this is going to happen either, so one, the boundary of your domain, so on the boundary of your domain.

Right? So here, you could think of the boundaries just like the perimeter kind of the outside part like

that. Or, the other place they can occur is at a point where, so at a point  $A, B$  where if I'm going to take this gradient, so I'm going to take the gradient of  $F$ , and I'm going to evaluate it at  $A, B$ . Okay? Then I have to get out, so my, I have to get out  $0, 0$ , right. But remembering, so this is kind of the important part is remembering that when we're talking about the gradient we're actually talking about, so this is going to be, so what vector is this? So this is the partial derivative of  $F$  with respect to  $X$ , evaluated at  $A, B$  and then the partial derivative of  $Y$ , sorry, of  $F$  with respect to  $Y$  evaluated at  $A, B$ . Okay? So what this is really telling me that this has to be zero is that each of these partial derivatives, both like both of them have to be zero in order for us to end up with absolute maximum or an absolute minimum, unless it's possible that actually the minimum occurs on the boundary. For example, like if you had something that kind of bowled down, then, or bowled up, you would get an absolute minimum, right? So if it bowled down, you'd get an absolute minimum on the boundary, and if it bowled up, you'd actually get a maximum on the boundary, but your minimum would be kind of down there. Okay? So, I hope that made some sense and I will see you in the next lecture.