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SUMMARY KEYWORDS

partial derivative, derivative, respect, xe, equal, yx, partial, notation, ey, leave, squared, correspond, pretend, inverse, constant, orange, branch, compute, match, function

SPEAKERS

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Welcome. In this lecture, I'm going to show you how to do kind of second order partial derivatives. And I'm just going to show you by example, because I think that that's actually the real way to see what's going on. So we're going to start out with this function. So we're, suppose we have the function, and the function we're going to start with is F of X, Y equals, and then I have XE to the Y, so XE to the Y, and then I'm going to do plus Y over X. Okay? And then, so let's start with, we're going to need to start with our first partial derivatives. So let's just kind of start with that. And then we'll kind of branch out from those, because if I have the partial derivative of, I'm going to want a lot of room here, so I'm going to start all the way on the side.

And I'm going to have, so I have the partial derivative of F. And this is just going to be with respect to X, right, we'll do it like this. So with respect to X, just to kind of remind you that what I'm doing there is I'm going to take that this is equal to, I meant to get a new orange and forgot. So this is equal to, so I have the derivative with respect to X of, and then I want to take it of this function here. So that's XE to the Y plus Y over X. And I'm not going to go through this computation, because we've done this kind of thing before. But remember, what we're going to do is we're going to pretend that Y is a constant and differentiate with respect to X. And what I'm going to get here is that this is going to equal, and then I'm going to go ahead and make this red so that you can see what I'm doing more easily in the next steps. So this is EY minus YX to the minus 2. Okay, so now I can branch off in two ways from here. So the first one that I can do using this is if I wanted to take the second derivative with respect to F of, ad now we're going to go, we're going to do so this is going to be partial, and I'm going to take X squared. And there's more than one notation for this actually. So I'm going to give you okay there's another notation here, which is going to be, I've used all my colors all of a sudden, so that's just going to be I'm just going to tell you what this other notation is, that this other notation for this just goes like F, and then two X's down next to it. Okay, just so that you have that for future reference. So how do I take this? So I'm going to take the derivative with respect to X of, right, the first, you know, the partial with respect to X. So this is of the partial derivative of F with respect to X.

Okay? But I have that as being this. So I'm going to take the derivative with respect to X of this partial that I already took. So of this critter over here, so this is EY minus YX to the minus 2. And then again, this is just, I pretend that Y is a constant, and I took the derivative with respect to X. And so what I'm going to get here is going to be, so this is going to be equal to 2YX to the 3. Okay? So that's the second part, like the partial derivative like the the second order partial derivative of F with respect to X. I don't know that there's actually a term for that one in particular. Okay, so then I want to take the, the other one that I want to look at is so, okay. So now what I want to do is I want to do YX. So this one has both, so Y is going to be the new one. So maybe I'll just make that one orange and leave the other because we already took that one. Okay, and then there's the other notation for this one is FYX. So whatever you have kind of on the bottom there, comes up there is how this works. Okay, and what I need to do with this one is to take the derivative with respect to Y. And so notice here, that the, it's going to look like the outside thing here really is the outside thing here. As in the partial derivative of F with respect to X is what's actually going on the inside here, and the partial derivative of F with respect to Y is the thing that's going to be leftmost here when I'm actually computing. So this is going to be in the partial derivative of F with respect to Y now, sorry, X. Again, because I'm just doing the, right this X is from there. So this is going to equal the derivative with respect to, maybe let's just leave that one like that. Okay. So this is going to be the derivative with respect to.

The derivative with respect to now Y of, and then again, this is the one that we've already computed, right? This is over here. So I can just put that in. So again, that's going to be E to the Y minus YX to the minus 2. So again, so now because I'm taking a derivative with respect to Y, I'm pretending the X is constant, and then taking the derivative with respect to Y. And what I ultimately get here is going to be E to the Y and then minus X to the minus 2. And that's going to be my answer there. So now I can go through, so the other ones, I need to start out with the partial derivative of F with respect to Y. So let's start out with that kind of down here. So the partial derivative of F with respect to Y.

Right, to compute this, this was equal to, so we have the derivative with respect to Y. And then I have the function goes here. So this is the same function. So this is just XE to the Y plus Y over X. And now I'm taking the partial or the partial derivative with respect to Y. So I'm leaving X as a constant, and I'm taking the derivative with respect to Y. And so then you can work out that this is going to equal, so I have XE to the Y plus X inverse. Okay, and this is the first step of two second order partial derivatives. So now we'll do those. Okay? Noticing, right, so maybe I can even kind of indicate on here like what are, what are these two arrows correspond to? They correspond to the derivative with respect to X, you know, as the next thing that happens. And the next thing that happens here is the derivative with respect to Y. So those were kind of the two choices that could happen there. And then I have the same two choices as to what can happen here is I can take the derivative with respect to X. Or I could take the derivative with respect to Y.

Okay? So let's start with taking the derivative with respect to X. And this is going to give me, so now I've got the second, like this partial where I have, so I've got, I've already done the y part, it's the X part that's new. So we'll put that there, like that. And this one down here is going to be the one that I'm going to get where I get the same thing going on twice. Right, I'm taking the derivative with respect to Y again, a partial derivative with respect to Y again. And notationally you could probably guess that this one is FXY. And this one is FYY. Okay, so let's get started on this. So this is going to equal, so I had to take the derivative, the outside one, right, of the inside one, so I have the derivative with respect to X of, and then I'm going to do, I already did, so we'll do this in parentheses. So but it's going to now it's of the inside one. So it's a partial derivative of F with respect to Y, which I've already done there. Right, over there. So this is the one that's going to show up as, so this is a derivative with respect to X of the one I did there. So, I want this which is actually the same thing as this. So this is going to equal, well and maybe I can actually, can I, it might be too much, it's going to get too cluttered. Okay, so this is going to equal, and then I have to take that. So I have XE to the Y plus X inverse. Okay? And again, so I'm taking the derivative with respect to X of this. And so what's happening is that I'm leaving Y as a constant, and then I'm differentiating with respect to X. And so what I should get here is going to equal E to the Y, right, because this is a constant, because Y is a constant. And then and then I go with respect to X, and I've got a minus X to the minus 2. And this is my final answer for this one. And then for this one, I need to take the derivative with respect to Y.

And then I have the partial derivative, so with respect to X of F. No, this is the partial derivative with respect to Y. I knew something was wrong there. Right, because this is the, right, this Y and Y correspond with the Y squared, like the X and Y correspond with the X and Y there. So then I'm getting the derivative with respect to Y, right, of this partial derivative that I've already taken, right. It's the same one is here, this is going to be XE to the Y plus X inverse. Okay? So this is going to equal here, this is going to equal XE to the Y. Okay, so if I wanted to get, right, if I want to get this derivative where I have, you know, the the X squared here, I have to take the derivative with respect to X of the the partial derivative of F with respect to X. If I want to get this one, then I have to take the derivative with respect to Y and the partial derivative of F with respect to X. And like notice that these match up. And then over here, right, I need to take the derivative with respect to X of the partial derivative of F with respect to Y. Again, these match up. And then here, I've got to take the derivative with respect to Y, or the partial derivative with respect to Y. Again, those match up. Something else that matches up here, which is not a coincidence that this matches up and it's really kind of cool, is that, I'll just write this with orange, right. So something like cool and important to notice, this is, this is, these are the same, this is not a coincidence. So not a coincidence that the same, not a coincidence that they're the same. We're always going to have that FXY equals FYX. So it's not going to matter which direction you do this in, and sometimes that might actually make things easier for you. Okay, so I hope that was helpful and made some sense and I will see you in the next lecture.