PfaffModule11L10

Thu, 2/17 2:12PM 16:42

SUMMARY KEYWORDS

directional derivative, partial derivative, direction, u1, slope, y direction, graph, u2, evaluate, equal, tangent line, inputs, gradient, dot, outputs, starting, change, vector, point, tells

SPEAKERS

Catherine Pfaff

Welcome. In this lecture I'm going to introduce directional derivatives. And so let's get started. So here's kind of an underlying question which is going to be, how do we find the slope of, so how do we find the slope of, and then we have the graph of F. So the graph of F. Okay? And like the rate of change, or you know, the rate of change of F, right, the slope of the graph of F is the rate of change of F.

These are only slightly different colors, but we're going to, we're going to live with that, that you should be able to tell when I'm using blue okay, versus white. So in some direction, so in some direction, and then this is my direction here. So this is direction U equals, and then the, then U equals U1, U2, okay? And we are going to require that the length of this, so we are assuming that the length, so we're assuming that the length of U is equal to 1, okay? So we want to know in that direction basically starting at the point A, B, so from A, B, okay? So these are going to be, this is the point A, B.

Okay? And the way to do this is the directional derivative. So this is the directional derivative. Answer, so the answer is the directional, okay, I want to make sure you can see it. So the directional derivative. Okay? So what is this directional derivative? So this is going to be, so I'm getting, so I have, so I have D, and then this U, and then we have F of, and then we have, well actually, I have to do this a little bit more carefully. Because this is the directional derivative of F, and then this is like evaluated at A, B. Okay, so it's purposely not the same as at F. I'm wondering if this, this is actually so the graph of F, I think these two are distinct, and I think that I want, I'm going to change the color of this. So let's make this green, the reason I'm doing that is to kind of it's a little too close to the white. So A, B.

Okay? and then we have this, this is going to equal. So I'm going to take this F, and I'm going to put, so then this is going to go in the X direction. And then we have A, B. So this is the partial derivative with respect to X, and then we're going to evaluate this at A, B. And then I'm going to take the U1, so this is the first coordinate from my U. And then I'm going to add that to, so I have this F and then I have, so this is the partial derivative with respect to Y, and then I'm going to evaluate that at A, B.

Because remember, it's a function and so we can evaluate it at a point like this. And then, so what is this? This actually, if you look at this, you might notice that this is equal to, this is actually equal to taking this vector. Right, which is the, I'm going to take the partial derivatives with respect to X and Y, and then evaluate them at the point A, B. So at the point A, and then B, right? And then dotting that with this vector U. Right? That's exactly what I did here, right? Because if i dotted, this is U1, U2, oops there should be a U2 here. So I would get this times U1 plus this times U2. That's how the dot product works. So these two things are the same. And then I'm going to get that these are equal to.

So these are going to be equal to, and then I'm going to have, well, this is going to be, so this is going to be the gradient. So I have the gradient of F, right, evaluated at this point A, B. Right, because that's precisely what this is. This right here is actually the gradient of F evaluated at A, B, right? And this is a vector, and then we dot that with U. Okay, so that gives me that this is actually, right, so each of these, these are each equal and then this so this is actually this. So in each of these, they're actually equal to this directional derivative

So the directional derivative of F in the direction of U at the point A, B. Okay, so that's what the directional derivative is. So we have a picture first so here's the picture, so here's my picture. Okay? We're going to have, so I have the X direction, and I have my Y direction, oops. Okay, so, I have my X direction and my Y direction and my Z direction, right? And then I have the graph of, so I have here this is Z equals F of X, Y, okay? And that's actually this F, okay? So F of X, Y. And then I have some point A, B, so the point is down here, so I have some kind of point like here, so this is my point A, B. And then if I go up above it and actually U goes in some direction, so, let's make U go maybe in this direction. And then if I lift up, I get a point. So, then I lift up, I get a point here, right which is, so this point is going to be, the point so, this is above here so it has A and then it has B, and then this is F of A, B.

So F of A,B. Right, because that's the point that lives above A, B. And if I can get, right, and then if I kind of take this direction and I lift I get a, right, I get this tangent line, right? Because I can lift this direction I get some kind of curve. Okay? And then the tangent line here has slope, so this tangent line has slope. And the slope is going to be this partial derivative. So its slope is, let's just do it like this, this is slope is D U F, like the directional derivative in that direction evaluated at A, B, okay? So right, so I lifted kind of this line up to like, imagine cutting through this surface here. I'm like cutting through that graph, or this surface here, where like right above this, this is like a line here, I cut through like that, I get some kind of curve, I look at the tangent line, and that's actually that derivative. Okay? Well, now there's another question. So when you're looking at a new object, you need to figure out, well what are the inputs and the outputs? So let's let's go through that. So what are the inputs? Okay, well the first input is that function F. Okay? And then the second input, so we have the function F, and then we're going to have the point right, there's a point here. So, we have the point A, B.

And then the third input is going to be that direction okay? So the third input is going to be that length one. So I have that length one, vector U giving the direction we want the slope in. Or the rate of change in oops, that's a little too far down, so giving, right, the direction we want the slope in. So giving the direction that we want the slope in. So those are my inputs, and then what are my outputs? So I take these inputs and then my outputs are, what is my output? So my output, like what kind of object is it? So the kind of object is it's a number. And what kind of number specifically? It's a number giving the slope of the graph of X, Y. So, the graph of F of X, Y okay? And in that direction, so in that direction U, and then starting at the point A, B. So starting at A, B.

Right? And I can kind of go and put, right, so this is like my inputs and then my directional derivative outputs this. Okay? So we've kind of talked about it in a variety of ways. But the directional derivative gives us how the slope, like the slope of the graph in a particular direction, right? We did the partial derivatives, which gave us specifically in the X direction, the Y direction. But what if you want to know in another direction, like how fast your function changes, or like the slope of the graph? And you do this by the directional derivative, and there's like a bunch of different ways to write it. Really, it's a gradient dotted with U, like that vector U that you want to go in that direction. Breaking apart where the gradient is, it's those partial derivatives, just like the partial derivative of this, the partial derivative in the X direction times U1, plus the partial derivative in the Y direction, times U2. Okay? So it's kind of like picking up how much like, the U1 tells me how much of the, of the, you know, change in the X direction am I picking up, and the U2 tells me how much in the change in the Y direction am I picking up? And it's actually, if you look at this U here, like starting at the point A, B, and then you get this line here, and you kind of lift that up, and you get a curve that goes through this, this point A, B, and then you look at the tangent line at that. So this point is actually A, B, F of A, B, because it's on the graph of the function. And the slope of that tangent line is precisely also this directional derivative. So it's a very powerful notion. It tells you a lot about how things change, and not just the standard directions. Okay, so I hope that makes sense, and I will see you in the next lecture.