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SPEAKERS

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Welcome. In this lecture, we're going to talk about the gradient, which is going to tell us something about, kind of, in what direction is the graph of the fun, like in which direction is the function like increasing the most? Like, how has it changed, where is it changing the fastest, okay? So the gradient of, so if I, if I'm looking for the gradient of, and then I have this function F at, and then I have this point A, B. So what is this? Box this. So this is the new definition, the new object we're excited to learn about. So this is notated like this. This is the gradient. That's how it's notated. And it's going to equal in, in particular, so it might be notated like that, but actually, probably, we're going to want to have it this is the gradient at A, B.

Because it depends on the point. But sometimes if that's clear, you leave that out. So either way, okay, so this is going to equal, so we have our function, but we're going to take these partial derivatives. So it's like FX of A, B, the partial derivative is evaluated at, so it's a partial derivative with respect to X, evaluated at A, B. And then I'm also going to take, so then I'm also going to take the partial derivative with respect to Y, evaluated again at A, B, right, which is why it means like, why this changes, why it's important to kind of indicate in some form or another that you're taking, evaluating the gradient, or you're looking for the gradient at A, B. Okay, so this is the definition of the gradient. This is the vector that first coordinates the partial derivative of F with respect to X evaluated at A, B. This is the partial derivative of the second coordinate the first derivative of F with respect to Y evaluated at A, B. Okay? It's important to know what kind of object we're talking about. So the gradient is a vector, right, it's a vector with two components depending on the point A, B. So depending on the point A, B.

And this gradient, right so, again, this tells me if this is the gradient at A, B, tells me the direction at A, B, right, so at A, B, with largest slope indicates this is the gradient at A, B with largest slope right. So this is going to be the direction in which, so the direction in which this F of X, Y increases the fastest. So F of X, Y is going to be increasing the fastest.

So starting at B, in which direction do I go like along the graph if I wanted to increase it faster? Do I want it, in which direction do I go if I want F to increase the fastest, like from A, B? Right? So I'm like want it, in which direction do I go if I want F to increase the fastest, like from A, B? Right? So I'm like in R2, where you could think of it in that X, Y plane in R3, and I'm at a point A, B, and it's, the question is right, and then above that is a graph of the function, I'm going to draw this in a moment. And the question is, right, if I'm moving from A, B here, I'm going to move around and that gives me different values of F. And I want to know like, you know, which direction do I move in to change that value of F the fastest, okay? And then we have if I look at it length, so of the gradient evaluated at A, B, then this length is the rate, that F is, the rate F, X, Y is increasing, okay? So this tells me the direction that I want to go in for it to increase the fastest, right? So in the direction, so in the direction, right the gradient of F, okay? And so I go in the direction of the gradient in order to, for it to change the fastest, and then the length of that actually gives you how fast it changes. Okay, so just to remind you, this is the length the gradient. Okay, so let's draw a picture, we have our X. So if this is X, and so this is like I is kind of, right, this goes to length one on X, and then I have J direction. So this is Y and then J goes that, that length here to one, and then I have my Z direction.

And then somewhere I have the graph of the function. Right? So this is the graph of the function. So this is Z equals F of X, Y. Right? And then if down below, I have, you know, somewhere down in this X,Y plane, right, there's this plane here. And here, I can have this point, right? So I can, I can have a point here, which is like A, B. And what happens above that point? So if I go straight up, right, so this is a point here, I go straight up, I hit the graph of the function, it's the graph of the function. So this is going to look like, right, I have my A is my first coordinate, and then I'm taking at, my second coordinate is B. And then my third coordinate is F of A, B, right, because that's what it means to be on the graph of the function. And then, so I can kind of decompose this. So if I go in the direction here, right? So if I go in this kind of direction, like which is kind of, it's like lifting up this direction, then this is that FX of A, B, right? This partial derivative with respect to X, evaluated at that point, tells me the slope in this direction. So tells me the slope in this direction.

Right, and then to go on the other way. Okay, so now I'm like lifting up this direction. And FY A,B tells me the slope in this direction. So the slope, you know, as I vary Y, because that's what I mean by the partial derivative with respect to Y is telling me what happens as I vary Y, a partial derivative with respect to X is telling me what happens as I vary X. So this is going in the direction that varies X, it's just not parallel to that, and this is going in the direction that varies Y which is parallel right to the Y axis, the slope in this direction of varying Y, right. And then the gradient gives me the direction that I changed the most rapidly in, okay? So by the way, this is actually A, B, 0, but I kind of think of it in this plane, this is like A, B, okay. So, I hope that made some sense, and I will see you in the next lecture.