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tangent plane, point, approximation, function, partial derivative, equation, partial, graph, approximate, evaluated, tangent line, x minus, error, linearization, 2f, picture, equal, give, input, swapped

SPEAKERS

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Welcome. In this lecture, we're going to go through how we would approximate a two variable function using a tangent plane, just in a kind of analogy with how we approximated a one variable function using a tangent line. Okay, so let's get started. So here's our pictures. So we have our X axis like this, it's going to go back like that, and our Y axis, and we have our Z axis maybe would give us, you know, a third axis. And then we're going to say we have the graph of a function is going to be up here. So we have some kind of graph, we have a function, and we have its graph up here. So this is Z equals F of, and this would be, you know, F of X and Y. Actually, I want to label it over there.

So this is going to be Z equals F, this is the graph of this. So what does that kind of mean? Putting all my caps back on, it's not necessary. So what I mean here, is that, right, so if I take a point, so we're going to take some kind of point, A, B. So I'm going to take some point, and we're going to look at it in like in the XY plane here. So maybe this point is, like here. So this is some kind of point A, B. I planned things out carefully in my notes to make sure everything's where I want it. And then I get excited drawing, and I draw things exactly where I purposely didn't put them. So that's, that's life when you get excited easily as I do. Okay, so then we have, so maybe this is some kind of point A, B. And then if I go up, so this is where actually like, again, we call this A, B, but it's really A, B, 0. And if I go up until I hit the graph, right, I get some kind of point here. And this point here, is going to be A, B, F of A, B, right? So this is going to be BF of A, B. So this is going to be A, B, F of A, B, right? And then at this particular point, I have this tangent plane, right? So this is this plane. And what is this planes equation, so the equation of this plane is going to be, so this is the tangent plane.

And it's going to be 2F, so our function F. So 2F, at this kind of this point, right, this point, A and then B. Right? And what is its equation? So its equation is going to be Z equals, and then I have, it starts with F of A, B. So that same point A, B. And that basically, like moves me up to the that particular location. And then I go over some amount in the partial, you know, based on the partial of X, you know, but I plug into the partial derivative of access point A, B. And then I multiply that by X minus A. So X minus, and I and I'm making sure to always put this like these A and B so that you can see, right because this is the, the tangent plane 2F at A, B. So this stuff all depends on your function and your points. So, and then we take the partial in the Y direction. And this is again A, B. And now instead, we're taking that X minus, sorry now this is the Y minus B. Okay, so this is the tangent plane. This is the equation for the tangent plane. And then what does that do for me? That tells me if I take a point that's very close by, right, so let's take a point, and now we're taking a point in here that's close by. Okay, so this point is going to look like.

So this point is going to be X, Y, F of X, Y. So this looks like X, Y, F of X, Y like this. So this is a point here. And what's going to happen is that, you know, these two points are on here, but then I can approximate, right, so this is my graph here. But I can approximate this by going up to the tangent plane, right? And getting a point here, which is going to look like, right, what is this point look like? This point's on the tangent plane. So I have X, Y, and then I have this whole big huge thing. So I have, oh, my A, B got kind of, the order there somehow got swapped in terms of color. I've been doing my, I don't know if you noticed this, that I swapped these in this particular one, I swapped the colors for some reason. Only in this particular one. So this was A and that was B.

Okay, and so now instead of this going into the function, this actually goes into our whole tangent line equation. So this goes into, so this is, we're going to call this the linearization, right? This is the linearization at, the linearization at A, B. Right, but then we're evaluating it at the point X, Y. Okay, so then this third coordinate here is going to be L of, well that linearization, so I can put down there, this is like A, B, but this is going to be evaluated at X, Y. So I'm just sticking this X and Y into that equation over there. But it's like kind of long, so I don't want to rewrite it all here. But it's this equation here. Okay, so just like with the tangent line, this approximates. These points should be pretty close, as long as the X and Y are pretty close, then those output should be pretty close. So this gives a pretty good approximation for the graph for the values of the function. So what are some kind of consequential approximations? So here are some consequential approximations.

Okay, so the first one is that if I take F of X, Y, this is going to be approximately equal to, why we get, like I was just saying before, if I actually kind of put this into so this is F of A, B plus FX A, evaluated at A, B. So the partial with respect to X, evaluated at A, B. And I'm actually going to take my X minus A, right, and then I want to add to that, so I have the partial with respect to Y evaluated A, B, and then this is Y minus B. Okay? And so this actually gives me an approximation for my output of my function evaluated at anything that's kind of close to A, B. The other thing that happens is if I want to look at my output error, so this is like my output error, that this is approximately equal to, so I have the partial X, A, B, and then I have this X input error. So this is my X input error. And then I'm going to add to that, right, if I do that, my partial with respect to Y, A, B, and my Y input error, then that would actually give my output error. Like my, my error in F.

Okay, so what's going on here? We have just like in the circumstance of having the, just like in the circumstance of the tangent line approximating like it gets, it's close to the graph of the function at that point, which means that the equation for it gives an approximation for the value of that function at that point. And we have a similar thing here. But now this is like a two dimensional picture where I have a plane, that's like, you know, in a, in a, on a graph on a two dimensional graph. And the Y, you know, the value that I would get out at the tangent plane now, like its height, would actually give me a good approximation for the function as long as my X and Y are close to my A and B. Notice that the tangent plane depends on my A and B, just kind of like our tangent line dependent on your input point. But it also gives you, so if I want to approximate my output error, then it's going to be approximately equal to what I get when I take the partial derivative with respect to X and then my input error, and then add that to the partial derivative with respect to Y evaluated A, B with my output error. Okay, so this is just an approximation technique with a really nice picture that goes with it. This is again a linear approximation, there are higher order approximations, okay? So, I hope that made some sense and I will see you in the next lecture.