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partial derivative, notation, derivative, respect, naught, constant, function, cosine, evaluate, computation, equal, x naught, parentheses, varying, sine, y naught, pi, treat, put, point

SPEAKERS

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Welcome. In this lecture, we're going to talk about partial derivatives. And we're going to have an underlying example here. So let me just kind of put this here to start with. So this is the example when we're talking about the function F. So underline example. Which is going to be that, okay, we're going to have this function. So this is a two variable function with the variables being X and Y. And then this is going to equal, so in our case, we're going to have this equal X squared, plus, or sorry, X squared Y plus, and then we're going to have X cosine of Y. So X, cosine of Y. Great. And then we're going to kind of go through notation, and then how you compute because we're going to need to talk about the derivative with respect to X and the derivative with respect to Y. And what these are kind of, before we jump into like computation and all these things, just to kind of understand, and we'll draw pictures also. But the way that you should think about them is how the function changes with respect to that variable. So if I fix a particular Y value, and I'm allowing my X input to vary, how will the function F's output changes. So that would be the derivative with respect to X because X is varying. And then the taking the derivative with respect to Y, what we want to do is we want to fix a particular Y value, and then see what happens as we, sorry a particular X value and see what happens as we vary Y. And because Y is what's varying, this is the derivative with respect to Y. So this is what we're going to do. So let's kind of get started. So I'm going to break this into we're going to have notation over here. And then for each one that we give, after we give the notation, and then the, kind of computation method over here will give an example of what this kind of look like. So this is the computation method is going to go over here.

Okay, so the first one that we're going to go through is we're going to go the part, through the partial derivative of F with respect to. So this is going to be the partial derivative of, and then we have F. And then this is going to be with respect to, so our first coordinate X. So with respect to X, but it's going to be at, and then this is going to be our point is going to be, so just like when we do derivatives, you could evaluate at a particular point or you can leave it as a function. If you want to evaluate it at a particular point X naught Y naught, we're going to show you how to do that. Okay, so this is the point X naught Y naught. Okay? So we're going to look at this first. So this is the partial derivative of F with respect to this. Okay, so we'll do kind of the both sides. So the notation of how to do this, so we have F at X, so putting a little X there tells me where it's not at, it's differentiating with respect to X, okay? X is what's varying, and then evaluated at the point X naught, Y naught.

Oops, that suddenly is not working as well as I'd like. Okay, I didn't even notice it didn't show up super well, there. Okay, I'll be careful about that. That this is going to, okay. And so this is one kind of notation, there's another kind of notation for it. I'm writing both of these. They're both notation that are perfectly legitimate for this. And so then we have F, and then this is kind of more similar to be forward telling us with respect to X, but we kind of have these curly things to say that it's a partial derivative. That there actually is more than one variable to worry about. And then we have X naught and Y naught. So evaluated kind of at that point.

So computationally what we're going to do, after we switch out and get ourselves a marker that will hopefully work a little bit better. So computationally, what we want to do is we're going to be, this seems to be working better, and we want to treat Y as a constant. Right? And then we want to differentiate with respect to X. And so then, we want to differentiate with respect to X as if we were in a circumstance where you know, now that we've treated Y as a constant, this is, you know, a familiar question how to differentiate with respect to X. And then if we want to evaluate it right at this point, we have a function. So at this point have a function in two variables, at this point have a function of X and Y. Of X and Y, right, it's still a function, we haven't put anything in yet. But if we want, so, the next step would be and plug in X naught, Y naught. So X naught and Y naught into this function, right, we have a function after that and we can put those in. If evaluated at, so, if we actually wanted to evaluate that function at something. Evaluated at X naught and Y naught.

Okay, so let's do an example. So here's our example. So we're looking at that underlying example up there. So we want to take the partial derivative, the first thing we want to do is to take the partial derivative with respect to, and we said we're taking it with respect to X, we're going to use this notation. Okay? So this is going to equal, right, we're looking at that function, we're pretending Y is a constant, and so we're going to get 2 times XY, right? Because we treated Y as a constant, so you can basically pull it out and then we get 2X, since we're taking the derivative that. And now we're treating cosine of Y as a, as a constant, right, because Y is being treated as constants, so cosine of Y is a constant. So it's just like a constant times X. So you just get out the constant, right? So then we're going to add to it, cosine of Y. And now looking at this, this is a function of, this is a function of X and Y, like I said you would get. But then let's go ahead and evaluate it a point. So you know, there is an X and Y, we have some place to input. So let's go ahead and do that. So we want to take this at, and then we're going to look at the point, 1, 2. So this is like my, just to be clear on what's going on here. This is my, this is equal to my X naught Y naught. So this is my X naught, Y naught. So we're going to kind of put that in, okay. So then we're going to get, so we have to put that into here or put it into here. So we have the derivative with respect to X, evaluated at this point, 1, 2.

Whoops, 2, is going to equal, I've done at least one of these, is going to equal. Well, now let's look at our function here. So we have 2, and then we've got to put in for, you know, this is where we get to put in our 1, and then we've got parentheses here that we're going to have to feel and then cosine of, and then parentheses maybe I'll, you know, it's a little bit overboard in parentheses, but gives the idea that these are both places where you put in your Y. So now we can put our Y into those like that. Okay, so what did I do so far? Right, I differentiated with respect to X by treating Y as a constant. And so that's what we did, and we got this. This is still a function, I wanted to evaluate it at X naught Y naught equals 1, 2. And so I did that by just sticking 1 in for X and 2 in for Y. So I stuck those into the partial derivative, because I was evaluating the partial derivative at that particular point. So now let's look at the derivative with respect to Y. Okay, so now we're going to look at the partial derivative of F with respect to Y.

So partial derivative of F, and now we're going to be with respect to Y. And then this is at the point X naught Y naught. I feel like this board is getting a little bit cluttered, so I'm going to box a few things so that it's easier for them to stand out from everything else. So this is what we're talking about here. I'll box this one too. Okay, so this is what we're going to look at now. So terms of notation, we're going to get this is, we have the, you know, we have both of the same notations except that now we have Y. So this is X naught Y naught here. And in this, we again have something that looks like this, except that we have, right, there's a Y there. So this is with respect to Y evaluated at X naught Y naught.

So that's a notation and then how do we actually go ahead and compute this? Well, now we want to treat X as a constant. Okay, so to compute here, we want to treat X as a constant, so treat X as a constant, right? And then we want to differentiate with respect to Y. So differentiate with respect to Y. And then, right, at this point we have a function. And then we're going to plug in if we want to actually evaluate it at something, we're going to want to plug in just like we did in the other circumstance, we want to plug in X naught Y naught if we actually want to look at its value at a particular point. If evaluated at X naught Y naught.

Okay, and just to emphasize, so this here is a function of X and Y. So, this is a function of X and Y. Function of X and Y, right? And then this is actually a value. Like a real number, okay? So, let's go ahead and look at an example. So we're going to look at the example up there. So let's go ahead and find this. So we want to find the derivative of this function F with respect to Y, right? So to do this, right, so we follow our instructions we're doing with respect to Y. So Y is going to vary, so X needs to be constant. So what happens if we treat X constant? This is like having a constant in front of Y, so we just get left with a constant, right? So we get left with X squared. And then looking at this, if X is a constant, like having a constant in front of cosine of Y, so it's like constant times negative sine of Y. Okay, so this ends up giving us minus, the constant just kind of sits there, sine of Y. Right, and where minus sine of Y was a derivative of cosine of Y with respect to Y. Okay? So now let's go ahead and actually substitute in point. So now we're going to look at, at where we have, maybe I'll just kind of write it like this that it's actually, so this is X naught and this is Y naught equals 3, pi.

Okay, so then we have the derivative with respect to Y, and then we have 3, pi. Right? Okay, so that's what I'm going to be plugging in to this derivative with respect to Y over here. So this is going to give me, and then we can just go ahead and kind of write that in such a way that we kind of know everything is going to go. So all of our X's will be placed with kind of orange parentheses. And then wherever my Y's go I'll replace it with red parentheses, so I can keep track of what I need. And then I put my X value into the orange ones, so that's 3. Put my Y value into the red one, so that's pi. Okay, and so that would be my answer. And if you wanted to, you could simplify this one a little bit more. This is actually zero. I guess I never actually simplified this one up here. It's okay, this could be my answer. Or you could simplify it down. But this one's easier to simplify because sine of pi is actually known to be zero. So that one's actually much easier that this is just going to equal 9.

Okay, so let's kind of go through everything we did, right? If I want to take the partial derivative with respect to X, I'm looking at how the function varies when X varies. So I keep Y constant, I look at how the function varies by taking the derivative with respect to X. Which is what I did here, right, and kind of looking at that, if Y is a constant, then it's just like, well what's the derivative of X squared with respect to X, that's 2X. So that's how I got the 2X. And then Y was a constant, so it just was kind of left there. Now we have like a constant, right, because if Y is a constant, then cosine of Y is a constant. So we've got a constant times X, we just get left with that constant. Right now, this is a function of X and Y. But I wanted to evaluate it at a particular point to give me a real number, which is like actually the slope in that direction. So to do that, I end up getting, you know, I just go ahead and like kind of the easiest way maybe to do is every time you see an X, put orange parentheses, every time you see a Y put red parentheses. Put my X equals 1 value in for the orange, and then my Y equals 2 value in for the red. Okay? This is very similar, I'm going to take the partial derivative of F with respect to Y, but now Y is varying, and X is treated like a constant. So in our particular example, up here, if X is a constant, then X squared is a constant, right? So we have X squared times like a constant times Y, so we get left with that constant. And then again, if X is a constant, right, this is a constant, so we've got a constant times, sorry we've got a constant times cosine of Y. Well, the derivative of cosine of Y with respect to Y is minus sine of Y. So we get constant times minus sine of Y, which is exactly what we have here. And again, if we have a point we want, with just a function of X and Y, if we have a point we want evaluated at, we go ahead and put that in. So you can see kind of the orange. If colors mean something to you, I have for X, that's where the 3 is going to go. And so I put parentheses for that, and then stuck that in. And then the Y, right, it's going to be this pi. And so I put parentheses there and stuck it in. I happen to know that sine of pi is zero. So I did that quick computation and I got out 9. Okay, so we'll do a little bit more on this, but I hope this made some sense and I will see you in the next lecture.