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SPEAKERS

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Welcome. In this lecture, we're going to talk about a dot product of factors. One of the reasons why you might be interested in the dot product is because when we start talking about gradients, and trying to understand how a, the graph of a function is changing, when you're talking about a multivariable function, then the dot products come into play. But they come into play all over the place. So let's just kind of get started. So here we go. So the dot product of vectors.

Okay, so we can suppose that we're, I'm looking over because I want to do the underline with something that's going to show up a little bit better. Okay? So imagine that we're in a circumstance, we're going to have two vectors. So I'm going to make one of them green. And I'm going to decide to make another one orange. I'm going to make another one W, or red. Okay, so then this is going to equal, so there's, there's, there's two kind of ways to look at this. So one is it's going to equal, where it's going to equal the length, you're going to take the product of the lengths of these vectors, and then multiply it by cosine of theta, where, okay, so I've got to stick a number of things in here. So we're going to take the length of V times the length of W. Right? And then cosine theta where we have so what is kind of the picture here. So we have this vector V. Right? So this is equal to so this vector is V1 V2. And we have this other vector W. So we're getting here W1, W2. Okay, and theta is the angle between them. So this is kind of one formula for it, but one that maybe is a little easier to work with, is that we're going to take V1 times W1, and multiply that by, or sorry and add that, it was supposed to be a 1. Let's just okay. W1 and then add to that V2 W2.

Okay, and that's what the dot product of vectors is. And it's really important to keep straight kind of, a lot of times students get a little confused. It's like, what are the inputs? And what are the outputs? This is like, really, this is something you should always ask yourself, it's not just students, it's everybody gets confused about this, like, what kind of object am I even talking about? And part of that is understanding that the input here is two vectors. So we're going to input two vectors. Right? Specifically, the two vectors that I'm inputting are V and W. This looks, it looks like I have a point there. So V and W are the two vectors that I'm going to be inputting. And then my output is going to be a number, is going to be a number, right? What number is it? It's the number V1, W1 plus V2 W2, right? Because these are numbers W1. I did it again. I'm not quite sure why I want to do that. I think it's because I'm thinking ahead of the the two part, okay, W1 plus V2 W2.

Right, and right, because I'm multiplying and adding numbers, so I get a number but my input are two vectors. Okay, and then let's like kind of look at some key circumstances to kind of make sure we understand what's going on. So here are some key circumstances. Or at least to give, you know, a feel for something. It's hard to understand what an object is often until you've played with it to some extent, and at least understood some kind of special circumstances and what happens there. And then you can kind of build from that, okay? So, here we go. So here our picture could be, I have, I use, I'm going to use colors a little bit differently than I did over there. So I'm going to take, right, because we're so used to this. So, right, this is, I have here, my two axes. Right? And then I have my I on here, by I, I mean this, right, this length one vector here. So this is I, which we can often think of is 1, 0, right? And then I have my J, which we can, like right here. So this is J equals 0,1. Right? So then my A, right, and then I multiply this, you know, I have this, this A here. Okay, so I'm going to put it, so A here. So now if I multiply this, right, so what is this here? This is AI, and then we multiply there. So if I have a B here, then this here, right, is going to be B, J, right? And then if I kind of add those together, so I take this and I add to it that, I get here a vector that's going to go all the way out to there, which is going to be, I know I'm out of colors, so we're repeating color. Sorry for the disgrace of that. So this is going to be, this is a vector. Okay, and it's going to be a B.

Right? And what happens if I take so now let's kind of take this vector and play with it a little bit. Okay, so I'm going to take A, B. So A and then, A, B, like this, okay, and what happens if I dot this with I? So I need to take the dot product, these are two vectors. So this is a vector and this is a vector. So what would I get out is my dot product. So this is actually going to equal, right, so again, so this is like A, B, oops I wanted that red there. This is A, B. And then the next one we can actually think of so I'm dotting that with, well, this is this is 1, 0, right? Okay, well, what happens when I dot these together? I'm going to end up getting, well I have A times 1, so I have A times 1. And then I get plus B times 0. Right? Well, that just equals A.

Okay, and this was an arbitrary vector. Let's make sure this has enough of a tail that you don't think I'm writing zero, that that's A. Okay, and this was an arbitrary vector, I dotted it with I, what did I get out? I got out the first coordinate. Okay? So that's what happens when I dot with I, I get out the first coordinate. Now what happens if I dot with J? So now if I want to go and take the same vector, and I want to dot it with J. Well, what do I get now? So this is the same thing as right, so I have A times B. And then I'm dotting it with, so now I'm dotting it with 0, 1. Well, what happens here, so now I have this is going to equal so I have A times 0. And then I'm adding to that B times 1.

Okay, and this actually picks out so this one picks out the second coordinate, it picks out B. Okay? So if I dot with I, I pick out the first coordinate, if I dot with J, I pick out the second coordinate. Let's look at one more circumstance. What if they're perpendicular? Okay, so what if my V and my, like my V and my W are perpendicular? So what if V and W are perpendicular? Okay, well, so this is this kind of situation where I have, so here's an example. So maybe my V, so I have my V here. So V maybe is equal to A, B, right, And then my W, well, we kind of know what perpendicular lines like in the plane or something look like. And it's the same kind of idea here. So my W here, what if my W, so these are perpendicular, and this doesn't have to be length one. So maybe it's something like minus 2B, 2A. So minus sign there, just to make that extra especially clear. And it's okay that this is not length one. So maybe I'll even note that, okay that not of length one. Nobody says when you dot two vectors, they have to be of length one, I want to kind of, right, I want to emphasize that because sometimes you don't know whether something's like, you know, just because of the example we're doing or whether it's always true. So we have A, B.

And we're going to dot that with, so we have here, here's an example of two vectors that are perpendicular, which you, you know, is something that maybe you saw when you were in earlier mathematics. That, that's kind of how you make two vectors, or two, yeah, two vectors perpendicular like that. Okay, then these are going to equal, what you're going to get out is going to be, well, we have A times, so we have A times minus 2B. And then we're adding to that B times 2A. And so what we get is, is that this is going to equal so we have minus 2AB plus 2AB. So what do we get out? We get out 0. Okay? So anytime that you have two vectors that are perpendicular, that's actually you're going to get the same thing, which is that you're actually going to get zero. So the dot product is something well, actually, these circumstances kind of tell you what the dot product is, in some sense, right? When you dot with I you pick out the first coordinate, when you dot with J you pick out the second coordinate, when two vectors are perpendicular, you get zero. And so in some sense, what it's going to allow you to do is it's actually allowing you so, like maybe let me give this kind of interpretation here, which is that when we dotted here, what did we get? We got, right, this was, you can think of this as projection onto I. So this is projection onto I, or that, the the the vector in that, you know, in that direction. Projection into that direction. And then in this one here, you have projection onto, so this one, you could think of here as projection onto J, or the kind of the second coordinate. But you know, in general, this there is this sense in which is this is the kind of projection, okay?

Okay, so here's the dot product. It's going to be very useful in kind of projecting a vector onto another vector. It's also actually going to be the definition of what it means to be perpendicular. Okay, so, I hope that makes some sense, and I will see you in the next lecture.