Module: Probability Basics

Module Outline

- Introduction
- Basic Axioms of probability
- Rules of probability
- Independence and dependence of events

Probability: Introduction

- Uncertainty is ubiquitous: *weather, economy, health, jobs, sports, stocks*,.......

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- Lots of related decisions: *insurance, vaccinations, stocks vs bond, which stocks to buy,......*

Probability: Introduction

- Uncertainty is ubiquitous: *weather, economy, health, jobs, sports, stocks,.......*

- Lots of related decisions: *insurance, vaccinations, stocks vs bond, which stocks to buy,......*

- How to capture uncertainty/chance more formally?

Example 1: 6-sided Die

- A die is rolled; *possible outcomes*: $\{1, 2, 3, 4, 5, 6\}$

$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$

$\frac{1}{6}$

Example 1: 6-sided Die

- A die is rolled; *possible outcomes*: $\{1, \checkmark 2, 3, \checkmark 4, 5, \checkmark 6\}$

- What is the probability that an even number comes up?

$$\frac{\#\text{ favorable poss.}}{\#\text{ total possibilities}} = \frac{3}{6} = \frac{1}{2}$$

Example 1: 6-sided Die

- A die is rolled; *possible outcomes*: $\{1, 2, 3, 4, 5, 6\}$
- What is the probability that an even number comes up?
- What is the probability that the number exceeds 4?

$$\frac{2}{6} = \frac{1}{3}$$

Example 2: Office

- An office has 2 *secretaries*, 4 *interns*, 20 *data analysts*, 2 *accountants and* 2 *managers.*

  One of them is randomly picked to go to a convention.

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  One of them is randomly picked to go to a convention.

- What is the probability that one of the interns gets picked?

$$\frac{4}{30} = \frac{2}{15}$$

No intern

$$\text{Prob (data-analysts)} = \frac{20}{26} = \frac{10}{13} = \cdots$$

Example 2: Office

- An office has 2 *secretaries*, 4 *interns*, 20 *data analysts*, 2 *accountants and* 2 *managers.*

  One of them is randomly picked to go to a convention.

- What is the probability that one of the interns gets picked? $\frac{4}{30}$

- A family is expecting a child? Boy or girl?

$\frac{1}{2}$ $\frac{1}{2}$

$$\text{Prob.}(\text{boy}) = 52.5\% = 0.525$$

$$\text{Prob}(\text{girl}) = 47.5 = 0.475$$

Probability: Terminology

- *Sample space* S : Set of all possible outcomes

  e.g. for a die: $\{1, 2, 3, 4, 5, 6\}$

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- *Event*: a collection (subset) of outcomes

Evens $= \{2, 4, 6\}$ $\{5, 6\}$

Odds $= \{1, 3, 5\}$

Probability: Terminology

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  e.g. for a die: $\{1, 2, 3, 4, 5, 6\}$

- *Event*: a collection (subset) of outcomes

- Venn diagrams and sets

Example 3: Students and car-ownership

- 100 people were surveyed: 60 own a car

  30 were students, 15 of them own a car

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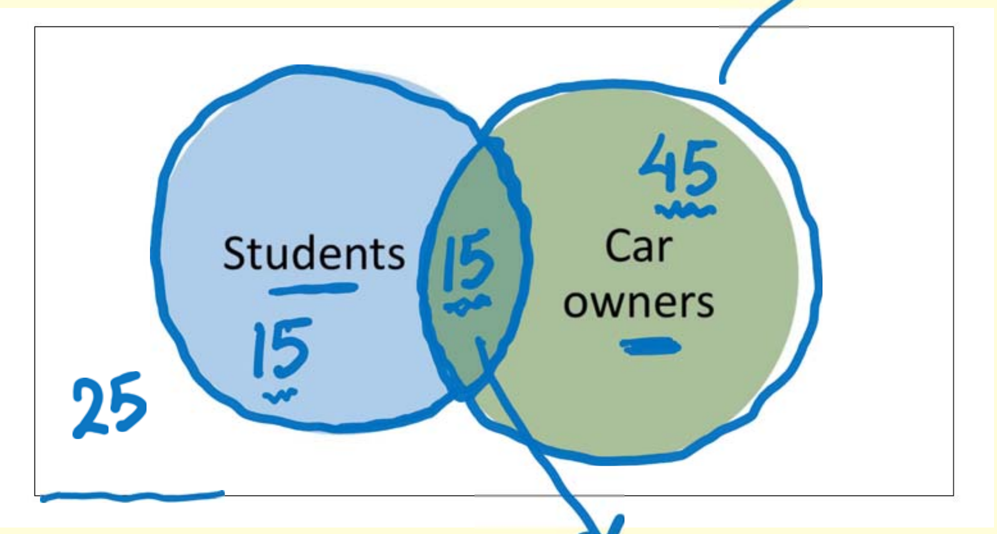
- Venn diagram:

St. ∪ C



$\frac{45}{100}$

St. ∩ C

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- $St.$ = set of students C = set of car owners

intersection
- $St \cap C$ = students AND car-owners

union
  $St \cup C$ = students OR car-owners

Example 3: Students and car-ownership

- 100 people were surveyed: 60 own a car

  30 were students, 15 of them own a car

- Venn diagram:

- $St.$ = set of students $\qquad$ C = set of car owners

- $St \cap C$ = students AND car-owners

  $St\ U\ C$ = students OR car-owners

- In the survey, what is the prob. of picking someone who is a car-owner, BUT not a student?

  $45/100 = 0 \cdot 45$

Example 3 extended: 100 surveyed

- 60 own cars; of them, 20 own 1+ cars

  30 students, 15 of them own cars, 5 own 1+ cars

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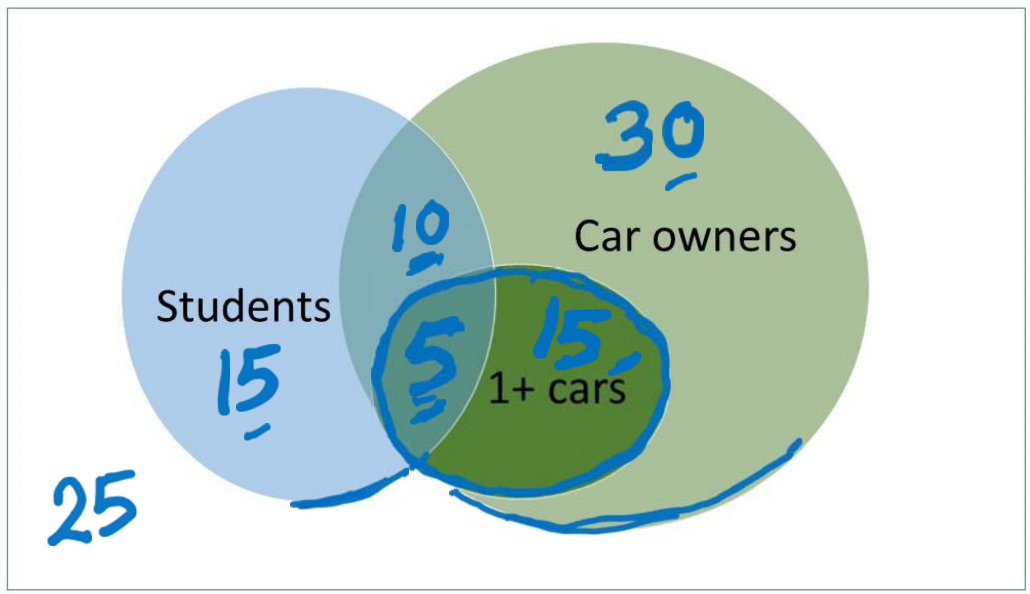
  30 students, 15 of them own cars, 5 own 1+ cars

- Venn diagram:



$Pr(\text{car owner}) = \frac{60}{100}$

$Pr\left(\begin{array}{l}\text{car owners} \\ \text{who are not} \\ \text{students}\end{array}\right) = \frac{45}{100}$

Basic Axioms of Probability

- **[i]** $0 \leq P(A) \leq 1$

$40\% = 0.4$

Basic Axioms of Probability

- **[i]** $0 \leq P(A) \leq 1$
- **[ii]** $P(\underbrace{Sample\ Space}_{\text{all possible outcomes}}) = 1$

$P\{1, 2, \ldots, 6\} = 1$

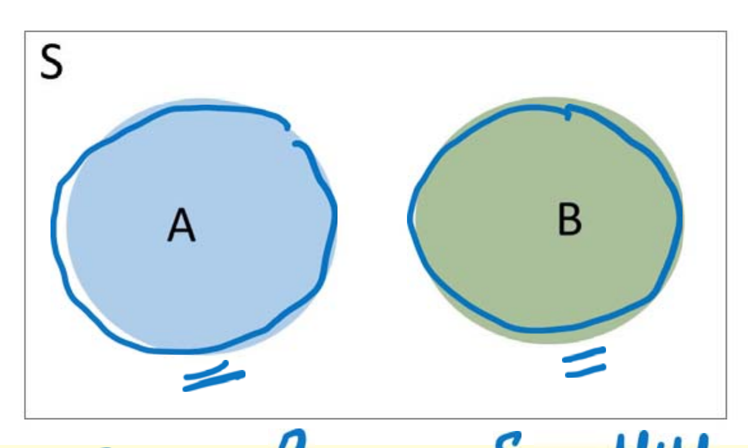
Basic Axioms of Probability

- **[i]** $0 \leq P(A) \leq 1$
- **[ii]** $P(\underline{Sample\ Space}) = 1$

nothing in common

- **[iii]** If A and B are disjoint (*mutually exclusive*),

  $P(A \cup B) = P(A) + P(B)$



$$P(\{2,4,6\} \cup \{5\}) = \frac{3}{6} + \frac{1}{6} = \frac{4}{6}$$

die: {evens} = {2,4,6}; {multiple of 5} = 5

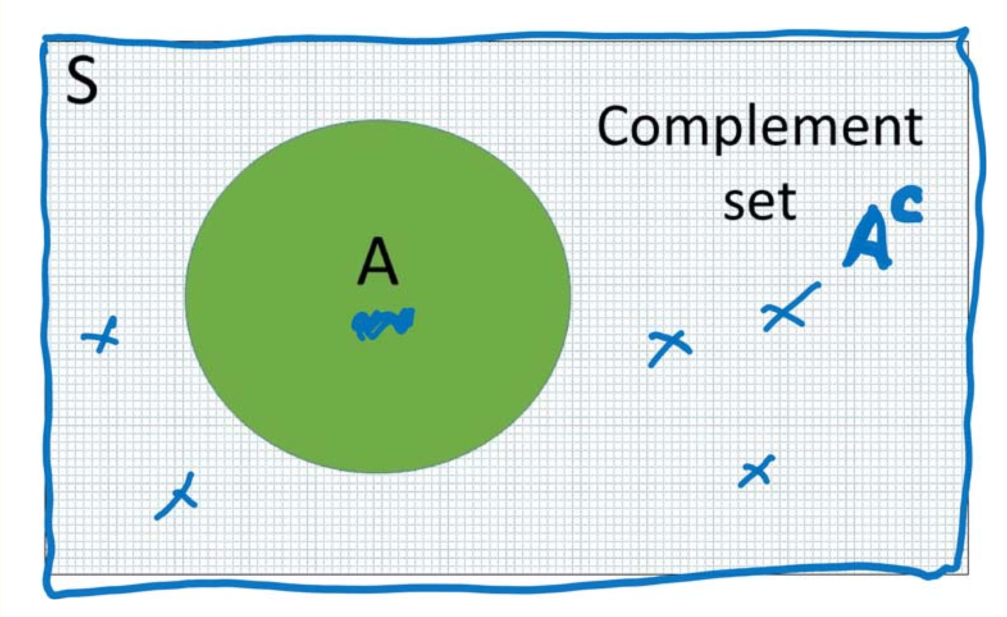
Probability rules

$A \cup A^c = S$

$P(A \cup A^c) = P(S)$

$P(A) + P(A^c) = 1 \Rightarrow P(A^c) = 1 - P(A)$

- **[Complement rule]** $P(A^c) = 1 - P(A)$



$P(\text{Rain}) = 0.4$

$P(\text{Sunny}) = 1 - 0.4 = 0.6$

Example 4: Lottery

- A lottery has numbers $1, 2, \ldots\ldots, 50$. What is the prob. that the number picked is not a multiple of 5?

$\overline{}$
50

Example 4: Lottery

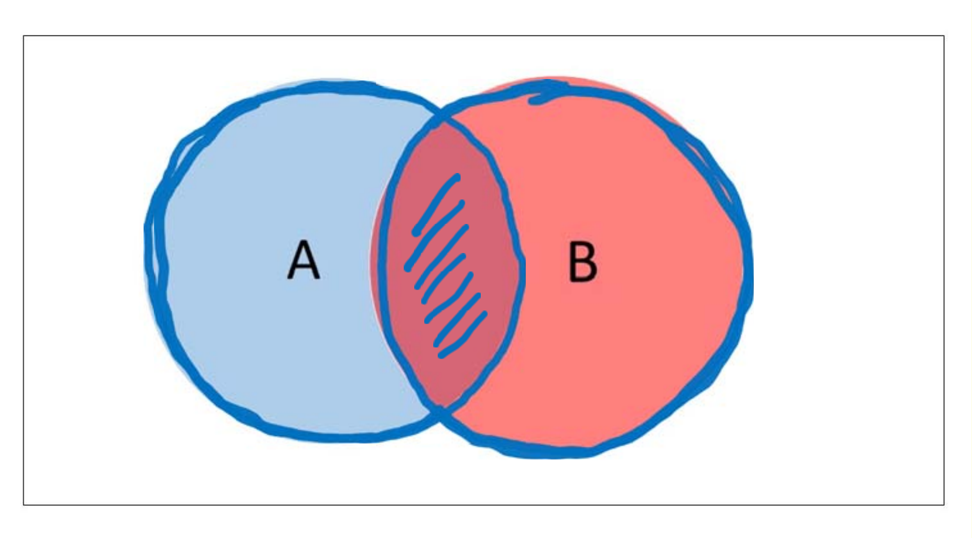
- A lottery has numbers $1, 2, \ldots\ldots, 50$. What is the prob. that the number picked is <u>not</u> a multiple of 5?
- Prob(multiple of 5) $=$ $\dfrac{10}{50} = \dfrac{1}{5} = 0.2$

Example 4: Lottery

- A lottery has numbers $1, 2, \ldots\ldots, 50$. What is the prob. that the number picked is not a multiple of 5?
- Prob(multiple of 5) $=$ 0.2
- $P(A^c) = 1 - P(A)$: Prob(not a multiple of 5) $=$ 1 − 0.2
  = 0.8

Probability rules: Addition rule

- **[Addition rule]** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

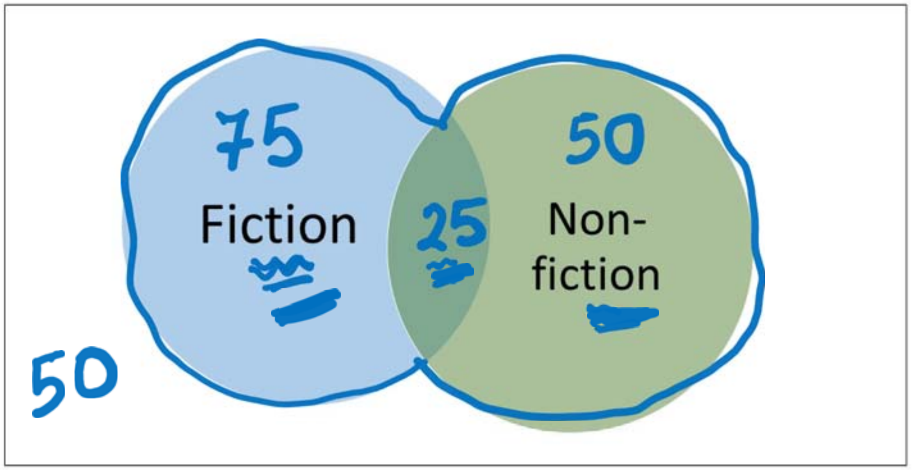


Example 5: Bookstore

- 200 customers visit a bookstore

  75 bought a non-fiction book, 100 bought a fiction

  25 bought both



$\frac{150}{200}$

Example 5: Bookstore

- 200 customers visit a bookstore

  75 bought a non-fiction book, 100 bought a fiction

  25 bought both

- What is the prob. of a customer buying a book?

  [Addition rule] $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\underset{\text{fiction}}{A} \qquad \underset{\text{non-fiction}}{B}$$

$$= \frac{100}{200} + \frac{75}{200} - \frac{25}{200} = \frac{150}{200}$$

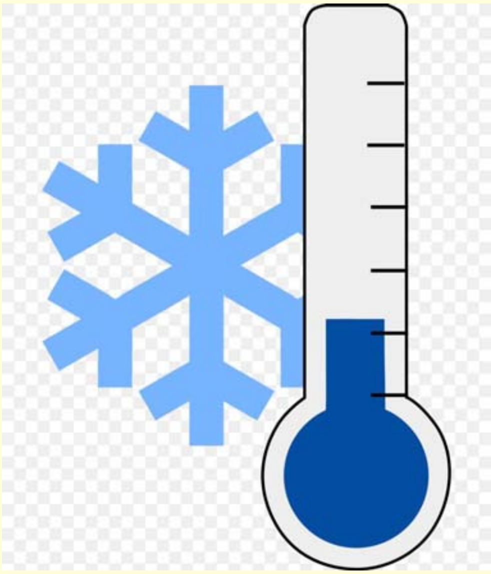
$$= 0.75$$

Example 6: Winterfest in Ottawa

- Organizing a festival in Ottawa on Feb. 5th.

  Past 10 yrs: $\texttt{snowed:}\ \frac{2}{10}$, $\texttt{very cold}: \frac{5}{10}$,

  $\texttt{snow AND very cold}: \frac{1}{10}$



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- What is the prob. of there being snow OR being very cold on the day of fest?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

snow v.cold

$$= \frac{2}{10} + \frac{5}{10} - \frac{1}{10} = \frac{7-1}{10} = \frac{6}{10} = 0.6$$

Example 6: Winterfest in Ottawa

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  Past 10 yrs: `snowed:` $\frac{2}{10}$, `very cold :` $\frac{5}{10}$,

  `snow AND very cold :` $\frac{1}{10}$

- What is the prob. of there being snow OR being very cold on the day of fest?

  $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Prob. of good weather? = 1 − P(snow or v. cold)

  no snow AND temp. > -10 C = 1 − 0.6 = 0.4

Clicker question 1

$$P(A \cup B) = P(A) + \underline{P(B) - P}(A \cap B)$$

- Toronto, Dec. 25th, *past* 10 *yrs*:

  `snowed:` $\frac{2}{10}$, `cold:` $\frac{4}{10}$, `snow AND cold:` $\frac{1}{10}$

  Q: What is the prob. of there being neither snow NOR being cold on Dec. 25th?

  (a) $\frac{2}{10}$ **(b)** $\frac{4}{10}$

  (c) $\frac{5}{10}$ **(d)** $\frac{6}{10}$

Solution to Clicker question 1

- Toronto, Dec. 25th, *past* 10 *yrs*:

  snowed: $\frac{2}{10}$, cold: $\frac{4}{10}$, snow AND cold: $\frac{1}{10}$

  Q: What is the prob. of there being neither snow NOR being cold on Dec. 25th?

  (a) $\frac{2}{10}$ **(b)** $\frac{4}{10}$

  (c) $\frac{5}{10}$ **(d)** $\frac{6}{10}$

Pr. (SNOW OR COLD)

$$= \frac{2}{10} + \frac{4}{10} - \frac{1}{10}$$

$$= \frac{6-1}{10} = \frac{5}{10}$$

$$\frac{5}{10} = 1 - \frac{5}{10} = \text{Pr. (NO SNOW NOR COLD)}$$

Addition rule: Application

- *Example 7*: Candn. immigrants in 2017:

  Total: $300,000$; *Economic class:* $160,000$

  Ontario: $110,000$; *Economic class in ON:* $50,000$

Addition rule: Application

- *Example 7*: Candn. immigrants in 2017:

  Total: 300,000; *Economic class:* 160,000

  Ontario: 110,000; *Economic class in ON:* 50,000 $A \cap B$

- What is the prob. that a randomly selected immigrant is

  (a) *of economic class*, **(b)** *of economic class or in ON?* $A \cup B$

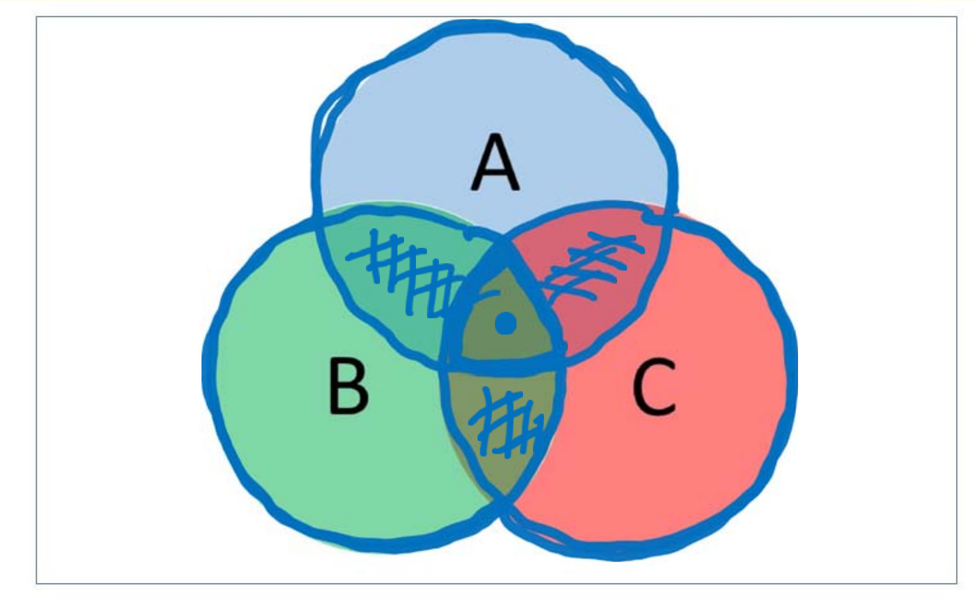
$$\frac{160,000}{300,000} = 0.533$$

$$\frac{160000}{300000} + \frac{110000}{300000} - \frac{50000}{300000} = \frac{220000}{300000} = 0.733$$

Probability rules

- **[Addition rule extension]**

$$
\begin{aligned}
P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap B) \\
&\quad - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)
\end{aligned}
$$



Addition Rule Extension: Application

- *Example 8*: 100 students in a class did projects $A/B/C$

A	B	C	A and B	B and C	A and C	all 3
30	40	20	10	12	15	5

Q: What is the prob. that a random student did at least one of the assignments?

$P(A \cup B \cup C)$

Addition Rule Extension: Application

- *Example 8*: 100 students in a class did projects $A/B/C$

A	B	C	A and B	B and C	A and C	all 3
30	40	20	10	12	15	5

What is the prob. that a random student did at least one of the assignments?

$$\frac{30}{100} + \frac{40}{100} + \frac{20}{100} - \frac{10}{100} - \frac{12}{100} - \frac{15}{100} + \frac{5}{100}$$

-

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap B) \\ &\quad - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C) \end{aligned}$$

$$= \frac{90 - 37 + 5}{100} = \frac{58}{100} = 0.58$$

Clicker question 2

- In the above example, what is the prob. that a random student did not submit any of the projects?

(a) 0.10 **(b)** 0.42

(c) 0.55 **(d)** 0.62

Solution to Clicker question 2

- In the above example, what is the prob. that a random student did not submit any of the projects?

$1 - P(A \cup B \cup C)$

$\parallel$

0.58

(a) 0.10 **(b)** 0.42

(c) 0.55 **(d)** 0.62

Interpretation of probability

- *Frequentist*:

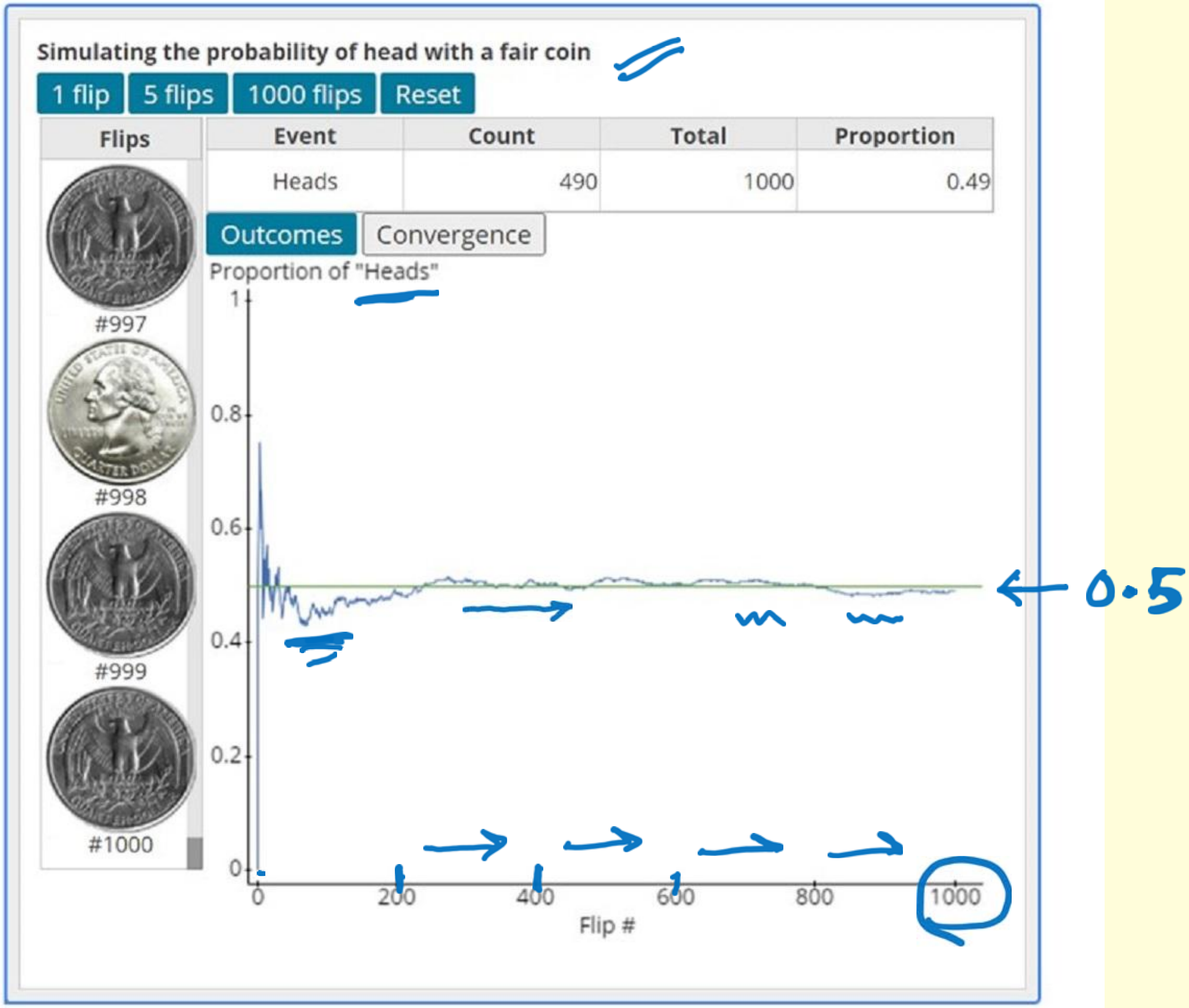
  If you repeat the experiment many, many times, the fraction will go to probability

Toss a coin :

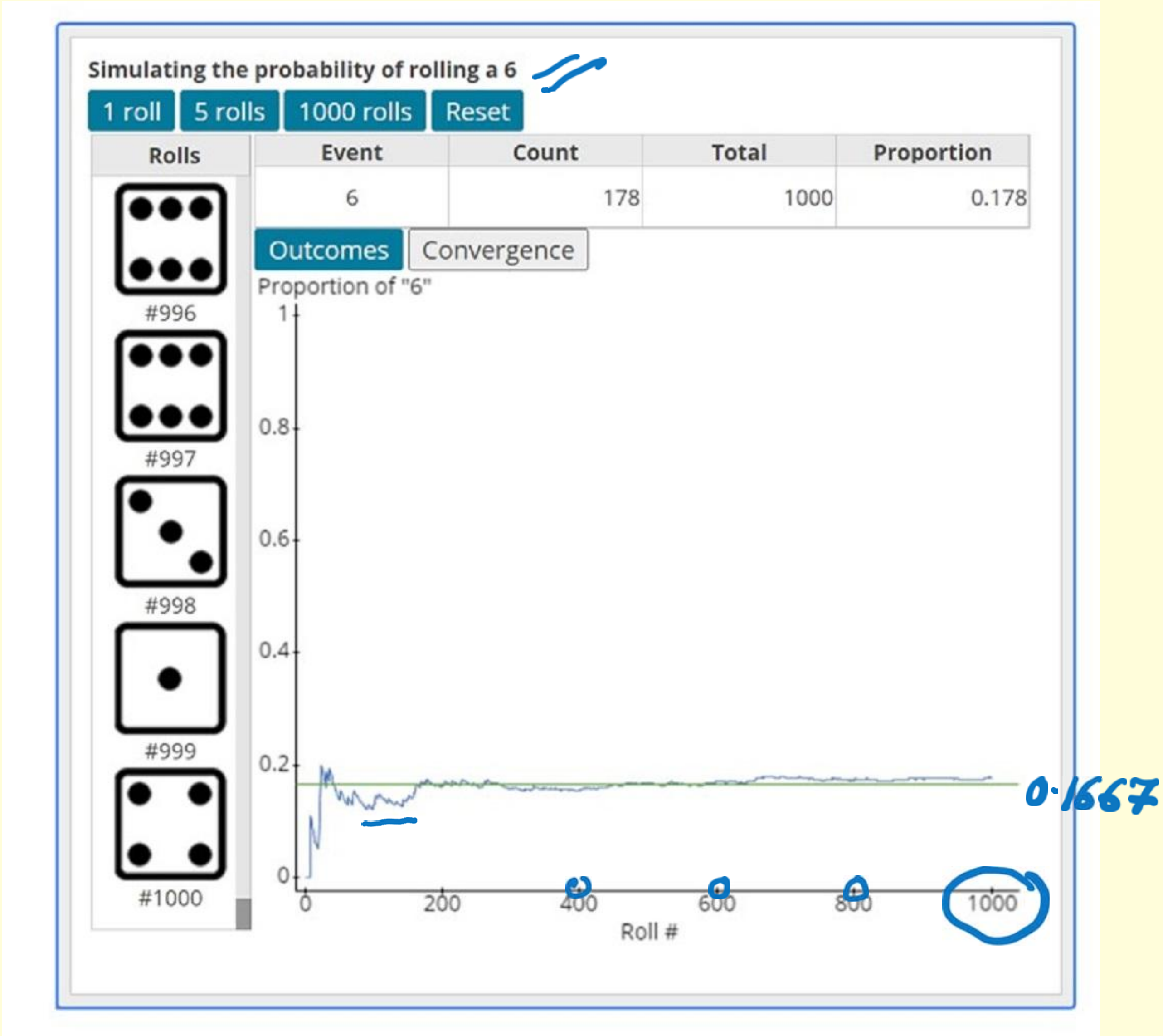
$P(H) = \frac{1}{2} = 0.5$?

$P(T) = \frac{1}{2} = 0.5$?

Frequentist: Tossing a coin 1000 times (statcrunch.com)



Frequentist: Rolling a die 1000 times (statcrunch.com)



Die: $\frac{1}{6}$

$= 0.1667$

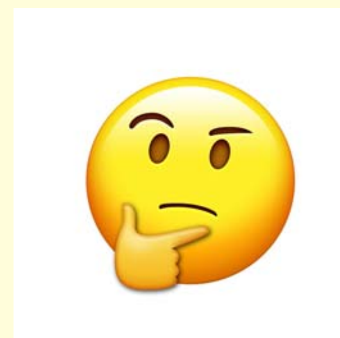
Interpretation of probability

- *Frequentist*:

  If you repeat the experiment many, many times, the fraction will go to probability

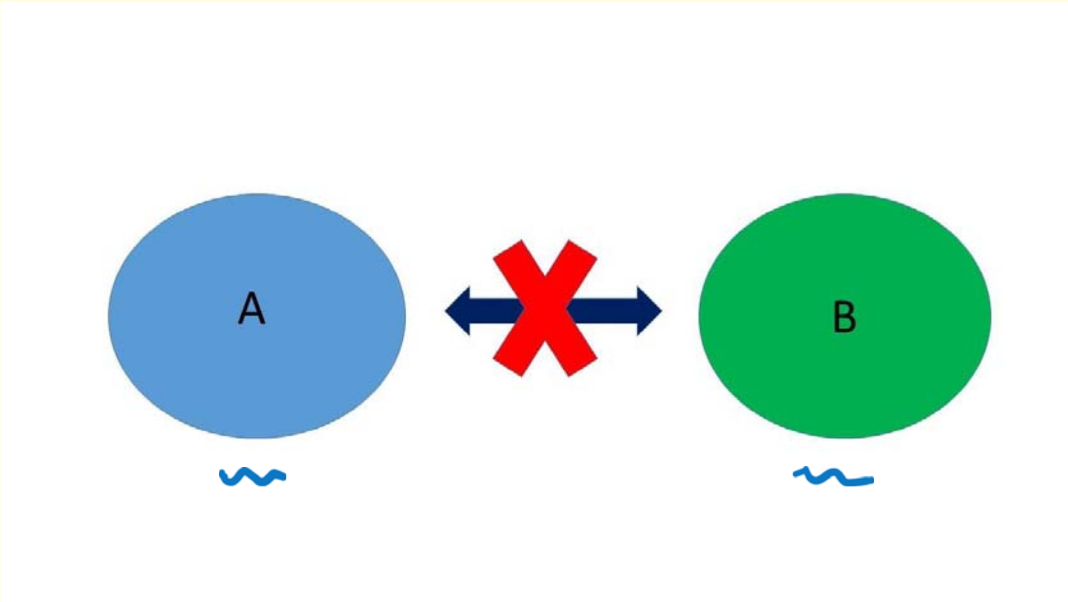
- *Personal prob.*:

  Beliefs around rare events (e.g. will I do well in the course?, will Trump get re-elected?.....)



Relationship between events

- *Independence*: Two events A and B are independent if the occurrence of one **does not influence** the prob. of the other occurring.



Relationship between events

- *Independence*: Two events A and B are independent if the occurrence of one **does not influence** the prob. of the other occurring.

- Example: coin tosses, gender of children,.....

1st : H/T

2nd : $P(H) = \frac{1}{2}$

$P(T) = \frac{1}{2}$

1st : M/F

2nd : M/F

Relationship between events

- *Independence*: Two events A and B are independent if the occurrence of one **does not influence** the prob. of the other occurring.

- Example: coin tosses, gender of children,.....

- *Example*: draw a card from a deck of 52 cards

  Clubs
  D
  H
  S

  1^{st} one is *King* of *Hearts*: **Replace** it back or **not**

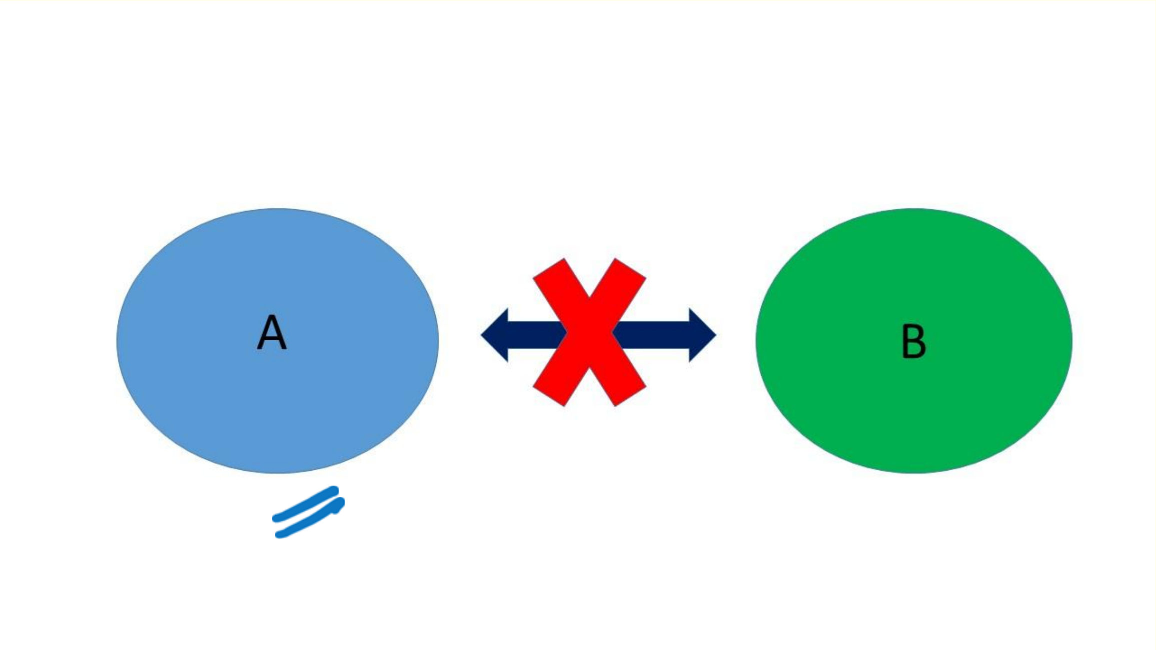
  Effect on 2^{nd} card drawn?

  No impact (indep.)

  dependent

Independent and Dependent events

- **Independent**: No relationship between the events.



Independent and Dependent events

- **Independent**: No relationship between the events.
- *Examples*: *has a pet* and *owns a car*, *wins a lottery ticket* and *finds \$10 on the street*, *rainy in NY* and *rainy in TO*,.....

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- **Dependent**: Some relationship between the events.
- *Examples*: *has a pet* and *owns a vaccum*, *drives over the speed limit* and *gets a ticket*, *is snowy* and *flights are delayed*, ...
- *Random walk theory* of stock prices

Movements in stock prices are indep.

Independence

- If events A and B are independent:

$$\boxed{P(A \text{ and } B) = P(A \cap B) = P(A)P(B)}$$

Independence

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$$P(A \text{ and } B) = P(A \cap B) = P(A)P(B)$$

C (13)
D (13)
H (13)
S (13)

- **Example 9:** draw ~~a~~ 2 cards from a deck of 52 cards

indep: With replacement: Prob. that first two are *Hearts*?

$$P(1^{st}\ H \cap 2^{nd}\ H)$$

$$= P(1^{st}\ H)\,P(2^{nd}\ H)$$

$$= \frac{\cancel{13}\,1}{\cancel{52}\,4} \times \frac{\cancel{13}\,1}{\cancel{52}\,4} = \frac{1}{16} = 0.0625$$

$$= 6.25\%$$

Independence

- If events A and B are independent:

$$P(A \text{ and } B) = P(A \cap B) = P(A)P(B)$$

- **Example 9:** draw a card from a deck of 52 cards

  With replacement: Prob. that first two are *Hearts*?

- Without replacement: Prob. that first two are *Hearts*?

dependent events: $\underset{1^{st}\,H}{\frac{13}{52}} \times \underset{2^{nd}\,H}{\frac{12}{51}} = 0.0588$

Independence Rule: Extension

- If events $A_1, A_2, ..., A_n$ are independent:

$$P(A_1 \cap A_2 \cap .... \cap A_n) = P(A_1)P(A_2)....P(A_n)$$

A₁ and A₂ and A₃ . . . . . occur

Independence Rule: Extension

- If events $A_1, A_2, ..., A_n$ are independent:

$$P(A_1 \cap A_2 \cap .... \cap A_n) = P(A_1)P(A_2)....P(A_n)$$

- **Example 10:** A family has 3 children. What is the prob. that all 3 are boys?

$\frac{1}{2}$ B

$\frac{1}{2}$ G

Independence Rule: Extension

- If events $A_1, A_2, ..., A_n$ are independent:

$$P(A_1 \cap A_2 \cap .... \cap A_n) = P(A_1)P(A_2)....P(A_n)$$

- **Example 10:** A family has 3 children. What is the prob. that all 3 are boys?

- $P(\text{all boys}) = P(BBB) \overset{\text{b/c these are indep.}}{=} P(B)P(B)P(B)$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

Independence Rule: Extension

- If events $A_1, A_2, ..., A_n$ are independent:

$$P(A_1 \cap A_2 \cap .... \cap A_n) = P(A_1)P(A_2)....P(A_n)$$

- **Example 10:** A family has 3 children. What is the prob. that all 3 are boys?

- $P(\text{all boys}) = P(BBB)$

- $\underline{P(BBB)} = \left(\frac{1}{2}\right) * \left(\frac{1}{2}\right) * \left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

Example 10 continued

- **Q:** A family has 3 children. What is the prob. that at least one of them is a girl?

Example 10 continued

- **Q:** A family has 3 children. What is the prob. that at least one of them is a girl?

- $1 - P(\text{all boys}) = 1 - P(\underbrace{BBB}_{1/8}) = 1 - \frac{1}{8} = 7/8$

Complement rule

$P(A) = 1 - P(A^c)$

Example 10 continued

- **Q:** A family has 3 children. What is the prob. that at least one of them is a girl?
- $1 - P(\text{all boys}) = 1 - P(BBB) =$ 7/8
- $P(\text{at least one girl}) = 1 - \left(\frac{1}{8}\right) = 0.875$

Example 10 continued

- **Q:** A family has 3 children. What is the prob. that exactly two of them are girls?

indep.

$P(GGB)$

$= P(G)P(G)P(B)$

$= \frac{1}{2}\cdot\frac{1}{2}\frac{1}{2} = \frac{1}{8}$

$P(GGB) = \frac{1}{8}$ $\quad P(GBG) = \frac{1}{8}$ $\quad P(BGG) = \frac{1}{8}$

$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$

Example 10 continued

- **Q:** A family has 3 children. What is the prob. that exactly two of them are girls?
- $P(GGB) = \left(\frac{1}{2}\right) * \left(\frac{1}{2}\right) * \left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

Example 10 continued

- **Q:** A family has 3 children. What is the prob. that exactly two of them are girls? $= P(GGB \cup GBG \cup BGG)$
- $P(GGB) = \left(\frac{1}{2}\right) * \left(\frac{1}{2}\right) * \left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$
- *GGB*, *GBG*, *BGG*...... Total prob. $= \frac{3}{8}$

$= P(GGB) = \frac{1}{8}$

$+ P(GBG) = \frac{1}{8}$

$+ P(BGG) = \frac{1}{8}$

$= 3/8 = 0.375$

Clicker question 3

- Which of the following events are independent?

  (a) *Parking illegally* and *getting a ticket*

  (b) *Owning a phone* and *being a hockey fan*

  (c) *A dry summer* and *forest fires*

  (d) *Buying more lottery tickets* and *winning the lottery*

Solution to Clicker question 3

- Which of the following events are independent?

  (a) *Parking illegally* and *getting a ticket*

  (b) *Owning a phone* and *being a hockey fan*

  (c) *A dry summer* and *forest fires*

  (d) *Buying more lottery tickets* and *winning the lottery*

Independence: Implications

- **Gambler's fallacy**: "The last ten flips were tails. I am now surely due for a head."   ✗

Independence: Implications

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- Random sampling

Independence: Application

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$p = 0.15$ smoker $\quad p' = 0.85$ non-smoker

- Suppose you survey 10 persons at random. What is the prob. that:

  (a) none is a smoker:

$$P(\underbrace{NNN \cdots N}_{\text{indep.}})$$

$$= \underbrace{P(N)\,P(N) \cdots P(N)}_{10 \text{ terms}}$$

$$= 0.85 \times 0.85 \times \cdots \times 0.85$$

$$= (0.85)^{10} = 0.19687 \approx 0.2$$

Independence: Application

- **Example 11**: 15% of Canadians are smokers (2017 UWaterloo study)

- Suppose you survey 10 persons at random. What is the prob. that:

  (a) none is a smoker:

- (b) $\boxed{\text{exactly one of them is a smoker?}}$

0.0347

SNNN . . . . . N

NSN . . . . . N

NNSN . . . . N

⋮

$$P(SNNN\ldots\ldots N) \overset{\text{indep}}{=} P(S)\underbrace{P(N)P(N)\ldots\ldots P(N)}^{9}$$

$$=(0.15)(0.85)^9 = \underline{\underline{0.0347}}$$

$$P(NSN\ldots\ldots N) = P(N)P(S)P(N)\ldots\ldots P(N)$$

$$(0.15)(0.85)^9 = 0.\underline{\underline{03}}47$$

(i) $P(\underline{\underline{\text{exactly } 1}} \text{ smoker}) = P(SNNN\ldots N \cup NSN\ldots N \cup NNSN\ldots\ldots \cup \ldots\ldots)$

$$= P(SNN\ldots N) + P(NSN\ldots N) + \ldots\ldots$$

$$= 10 \times 0.0347 = \underline{0.347}$$

Independence: Application

- **Example 11**: 15% of Canadians are smokers (2017 UWaterloo study)
- Suppose you survey 10 persons at random. What is the prob. that:

(a) none is a smoker:

(b) exactly one of them is a smoker?

- (c) at least two of them are smokers

At least 2 smokers

Complementary event — no smoker prob. $= 0.2$

— 1 smoker prob. $= 0.347$

$$\begin{array}{r} 0.2 \\ +\ 0.347 \\ \hline 0.547 \end{array}$$

$$\Pr(\text{at least 2 smokers}) = 1 - 0.547$$

$$= 0.453$$

Clicker question 4

- A family has 3 children. What is the prob. that exactly one of them is a boy?

(a) $\frac{1}{8}$ **(b)** $\frac{1}{2}$

(c) $\frac{3}{8}$ **(d)** $\frac{1}{3}$

Solution to Clicker question 4

- A family has 3 children. What is the prob. that exactly one of them is a boy?

(a) $\frac{1}{8}$ **(b)** $\frac{1}{2}$

(c) $\frac{3}{8}$ **(d)** $\frac{1}{3}$

BGG $\rightarrow \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

GBG $\rightarrow \frac{1}{8}$

GGB $\rightarrow \frac{1}{8}$

Add : $3/8$

Checking for independence

- **Example 12**: In a university sample, 70% of the students reported living off-campus. 40% reported belonging to some club, 20% reported living off-campus and belonging to a club.

Checking for independence

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  $O \equiv$ lives off-campus

  Q: Are C and O independent?

Checking for independence

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- Suppose: $C \equiv$ belongs to a club

  $O \equiv$ lives off-campus

  Q: Are C and O independent?

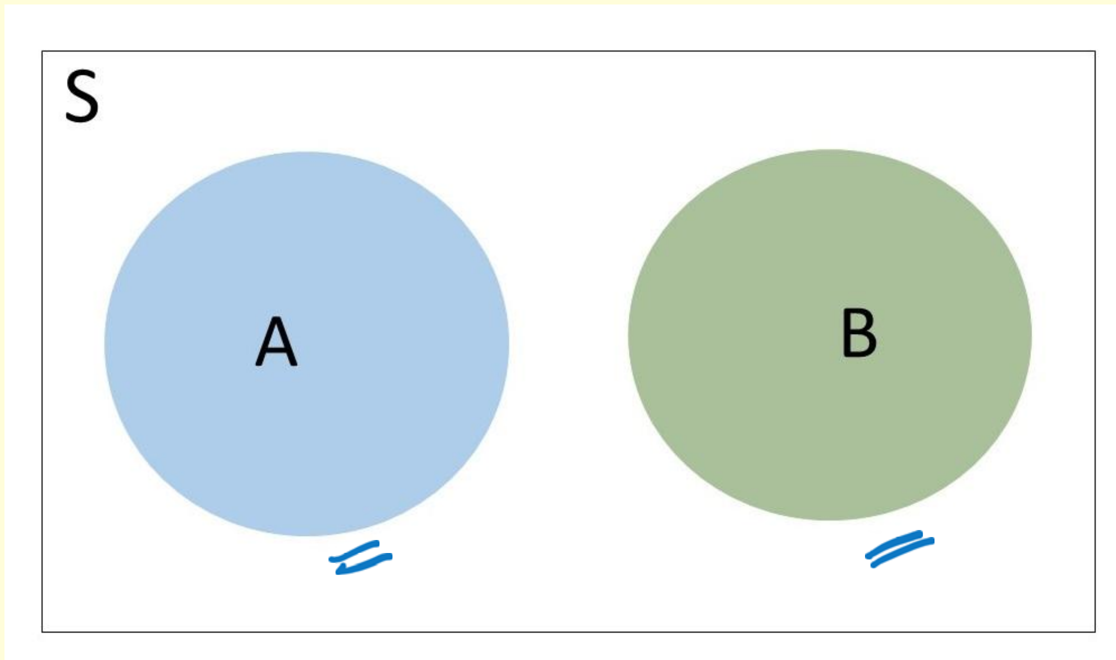
- If C and O are independent:

$$\rightarrow P(C \cap O) = P(C)P(O) \quad \Rightarrow C \text{ \& } O \text{ are NOT indep.}$$

$$0.2 \neq 0.4 \times 0.7$$

Independence vs Mutually Exclusive

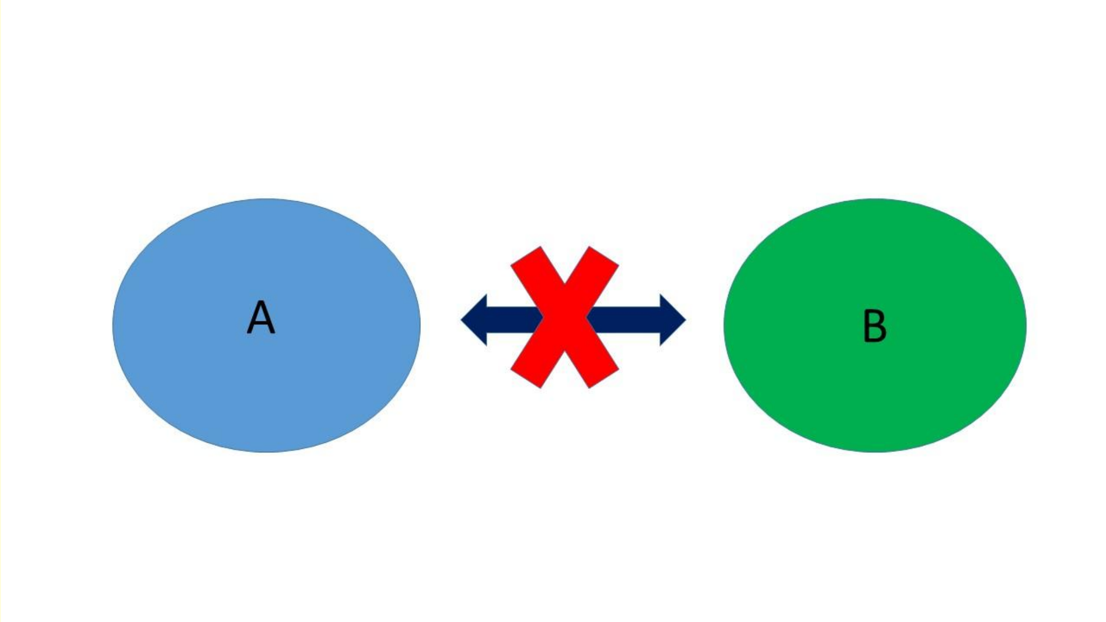
- *Mutually exclusive*: two events cannot occur at the same time



Disjoint sets

Independence vs Mutually Exclusive

- *Mutually exclusive*: two events cannot occur at the same time
- *Independence*: one event does not influence the other



Independence vs Mutually Exclusive

- *Mutually exclusive*: two events cannot occur at the same time
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- Example: $P \equiv$ Jim passes the course

  $F \equiv$ Jim fails the course

mutually exclusive

$Pr(P \cap F) = 0$

Independence vs Mutually Exclusive

- *Mutually exclusive*: two events cannot occur at the same time
- *Independence*: one event does not influence the other
- Example: $P \equiv$ Jim passes the course

  $F \equiv$ Jim fails the course
- P and F are disjoint, but not independent

If Jim Passes $\Rightarrow$ Pr. (F) = 0

Jim doesn't Pass $\Rightarrow$ Pr. (F) = 1