Radical Expressions I

SUMMARY KEYWORDS
square root, expression, equal, exponent rules, economics, number, cube root, rewrite, write, square root function, negative infinity, exponents, symbol, answer, radical, marginal returns, asked, interval notation, notice, simple

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Hi, everyone. Welcome back. It's ALEKS walkthrough number seven, and I am your host, Robert J. McKeown. I'm very pleased that you've decided to watch this video. As I mentioned in in video six, being able to manipulate an equation is very important. As an economist, and as an undergraduate student in economics, you're going to be asked to do that many, many times. And today's topic is about radical expressions. So that's like square roots, square roots show up a lot. And one reason why square root functions show up so much in economics, is because they display diminishing marginal returns, you're going to hear an awful lot about marginal. Everything's in economics, in economics, we say, almost all the excitement happens at the margin. And when we say margin, you can kind of think of that as like a border. And when we have diminishing marginal returns, well, that's just like, you know, if you have one ice cream cone, you know, one ice cream scoop, you're going to enjoy that a lot. If you have a second one, maybe you'll enjoy that a lot, too, but maybe not as much as the first one. And if you have three, well, maybe now you're going to get a stomachache. So you're not going to enjoy that as much as the second one or as much as you enjoyed the first one. So that's sort of the idea. So square roots show up a lot in economics, and it's very important to be able to manipulate them. Fortunately, as you'll see, it's a lot like any exponent. Now, before we jump into the video, and we start doing some work together, don't forget, you should have your pencil, you should have some paper, scrap paper, a notebook, what have you. And if you've got the money, invest in a tablet, because if you write it on this screen, you can save it, and then you can keep it in an electronic file forever and ever. And with that, let's do some problems together. Let's start off talking about what a radical expression is. a radical expression is any mathematical expression that contains this symbol. That includes the square root but not, but it's not exclusive to the square root. So it could also be a cube root, or any root to any number that you like. A few definitions, many roots create what we call an irrational number. And the definition of a number that's irrational is one that cannot be written as a simple fraction. So for example, the square root of two and pi are irrational numbers. When working with square roots, you always want to remember that it's not possible to take the square root of a negative number, the square root of a negative number is defined. And these are sometimes known as imaginary numbers. imaginary numbers are important if you're studying quantum mechanics, quantum physics. But they're not important, or at least I've never found them to be important. In economics, it's not something we use in economics. In economics, we focus on the real numbers, which I'll show you. In this next slide. We've been asked to graph a square root function. So it's a very, it's the most simple one we can possibly have. It's the function which is equal to the square root of x. And this function will look something like

this. Maybe I'll label it f of x. And if x were equal to four, and has I haven't drawn it to scale very well, y is going to be equal to two. And if y is equal to three, x must be equal to nine. Now everything in this space, these are real numbers over here, and then anything over here at least on the x axis, focusing on the x axis here. We don't know these are undefined. For our purposes, the square root of a negative number is undefined. We don't know what it is. We don't know what it is. If you want to learn more about them, by all means, take a course on complex numbers taken in Dance mathematics course, and knock yourself out. Here's our first ALEKS question. We're asked to find the domain. What has been described or given to us is a radical expression to radical expression because it has a square root. And we're supposed to write our answer using interval notation. So this is a more complicated square root function than the one we did together on the previous slide. Now, we don't need to draw anything, we don't have to graph anything, we just have to figure out for which values of x is there an answer for y. So rewrite our function as y is equal to negative x, negative eight x plus 24. And the first thing we can notice here is that if x is equal to three, y is equal to zero. Or, well, I'm getting ahead of myself. Y is equal to the square root of zero. And the square root of zero is just equal to zero. If x is greater than three, then y will be equal to a negative number. Which means y is on defined. So we won't know what y is if x is larger, strictly greater than three. If x is less than three, then y is equal to the square root of a positive number. And y is defined, it's going to be equal to y is equal to a real number. So by just sort of looking at the expression, and making sure that I'm not taking the square root of a negative number, I know what the domain of this function is. And an interval notation I might write x is in this space, well, any number less than three will give us a positive value for y. So I can go all the way to negative infinity. Let me try and write that again, negative infinity. And the number can be x can be as large as three. And I'm going to make sure I put a square bracket, because x is allowed to be equal to three or less than three. So here I am on ALEKS. And it's asking me to write my answer in interval notation. So I guess maybe I'll start off by picking the correct brackets that I want. And it's this one right here, where it's a rounded bracket on the left, because there it's an open open space. And it's closed on the right, it can only go up to three. And I'll just click this negative infinity symbol. And then I'll click into the right side box, and I'll put in the number three. And I'll click on Check. And we got the right answer. Notice it didn't have to do the axes in this interval, or axes in this set, we just had to put in the set itself. That's all ALEKS wanted. Let's do two things with this question. First, I'm going to show you some alternative expressions for this thing up here, this radical expression, and I'm going to then simplify it, I'll put it into a simple expression as possible. So there's a connection between square the square roots, cube root expressions and exponents. So I could rewrite. I could rewrite this as an exponent like that. And we already learned some exponent rules. So this is good for us because it turns out square symbols, square roots, cube roots, they all Follow the same exponent rules. And often in economics, we just choose to express everything and exponents. Because it's a little bit maybe a little bit easier that way. But you need to be able to go back and forth between exponents and square root symbols, effortlessly, effortlessly. And this is going to be really great practice for you to do that. So according to the exponent rules, I could rewrite this again as 250 to the power of one third, multiplied by x to the power of one third. And if I wanted to, I could rewrite this as the cube root of 250 times the cube root of x. So I'm still I'm using these exponent rules that we learned previously. Now, let's simplify. So let's simplify this expression up here. I can, there's not too much I can do with it. But I'll notice that if I have two times 125, times x, so I'm sort of factoring out that 250 a little bit, I can write this in a more simplified version by noting that five times five times five, which is equal to five, three is equal to 125. And so I can take that 125, outside of the square root, so I'm going to have to be careful with my writing, I've got the cube

root of 125 times the cube root of two x. And this thing on the outside is just going to be equal to five. And I've got five times the cube root of two x. And that's as simple as I can make this expression. Okay, let's tackle a more challenging problem. You can read the expression in front of you, you can see it on the slides and the video. How are we going to go about simplifying this thing? Well, there's a few different ways to go about it, I suppose. And I've been sitting on the fence how I'm going to show it, but let's do it this way. So let's notice that the square root of 24 can be written as the square root of four times six. That would be one way to do it. Or maybe, yeah, okay, I guess that's fine. But we could go, we could go a little, we could go further, we could say, well, this is also equal to the square root of two times two, that's four, times two times three, two times three is six. So I've now written that in its lowest possible factors. Similarly, I can look at the square root of 98. Now while the square root of 98 has no particular significance for me, I know that anything, that's a pot, it's an even number divided by two is going to produce a whole number. So if I rewrite this as two times 49. I'm a little bit closer to something useful because then when I see 49, and I know my multiplication table, I know that seven times seven is equal to 49. So I'm going to go a little bit further here. I've got two times seven times seven. And so why don't I bring this all together? And Alright, I've got the square root of two times two times two, times three. multiplied by three, and multiplied by two. Why should be careful here? Square root of two, seven and seven. Now, I can do a few things. So I noticed that maybe I want to look at the fact I've got four twos, I've got the square root of two, four times, so I'll rewrite this as the square root of two to the power of four, and I've got two threes. And I'll rewrite that out note, excuse me, I've got a square root of three, and I've got a three itself. So I'll just put the three there. I'll put the square root of three there. And I've got two sevens. And so I've got the square root of seven squared. Now, you should see that if I've got the square root, and I've got the square, they're going to cancel each other out. So I've got two square to the power of four, that's going to give me four. And I've got the three that's still sitting there. And I've got the seven. And I've got the square root of three. And now I'm going to have 12 times seven times the square root of 312 times seven. Oh, my multiplication table up what the 12 is a little shaky, but I know that seven times three is 21. And I know that for 20 ones is equal to 84. So notice that I'm doing this I'm trying to do this without using a calculator. If you use it with a calculator, there wouldn't be much of a challenge to it, but it's handy. Now we've got our answer. Let's put our answer into ALEKS. So I'll start by typing 84. And then I'll click this button right here. So the first little box represents the 84 and then I've got the square root symbol that's going to be put in for me, and I type in three. And if I hit the check button we've got the right answer.