CORPORATE HEDGING, EXECUTIVE COMPENSATION AND COMMODITY PRICE PREDICTION

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A DISSERTATION SUBMITTED TO THE FACULTY OF GRADUATE STUDIES IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

GRADUATE PROGRAM IN DEPARTMENT OF ECONOMICS
YORK UNIVERSITY
TORONTO, ONTARIO

APRIL 2021

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Abstract

This thesis examines the agency problem surrounding the corporate hedging decision. It gives insight on how managerial incentives impact corporate hedging decisions and on how executive compensation can be used to minimize the agency problem and factors determining the optimal compensation. The model predictions are then tested against empirical data. One of the factors affecting optimal executive compensation is volatility of commodity prices. To explore this, the last chapter develops an empirical model to forecast commodity prices.

Past theoretical and empirical studies found that risk-averse managers tend to overhedge, without analyzing how to align shareholders' and managers' hedging strategies. In this dissertation I develop a model aligning hedging strategies using executive compensation, incorporating a risk-averse manager's utility into the hedging decision. Consistent with standard theories, the model show managers hedge more of the expected production than shareholders. The model shows there is a decrease in corporate hedging with the presence of managerial equity-based incentive pay. It also shows managerial incentives can be used to impact corporate hedging to minimize agency problem. To align and optimize managerial hedging decisions, the optimal managerial incentive should comprise more of the equity-based portion when there

is a low risk tolerance, or low price volatility, or a low variable cost. In contrast, when there is high coefficient of absolute risk aversion, or low price volatility, or high variable cost, it is best to compensate the manager with a lower equity-based portion in order to optimally align hedging decisions. In other words, by determining and examining the primary factors affecting compensation scheme includes risk aversion, price volatility, and profit margin we can determine the optimal compensation scheme. When there is a low (high) coefficient of absolute risk aversion, low (high) price volatility, or low (high) variable cost, then optimal compensation should comprise more (less) equity-based incentives.

Next, using empirical data I test the model predictions from the theoretical framework; (i) when incentive pay increases, the optimal hedge ratio decreases, (ii) when price volatility increases, the optimal hedge ratio decreases, while price volatility have a negative relation with equity-based incentive, (iii) when risk aversion increases, the optimal hedge ratio decreases, while risk aversion have a negative relation with equity-based incentive, and (iv) when variable cost increases, the optimal hedge ratio decreases, while variable cost have a negative relation with equity-based incentive. The predictions are tested against data obtained from oil and gas firms using a standard regression approach. I find that the model predictions are further supported by empirical evidence from the oil and gas industry showing (i) a negative relationship between incentive pay and hedge ratio, (ii) a negative relationship between price volatility and hedge ratio/incentive pay, (iii) a negative relationship between risk aversion and hedge ratio/incentive pay, and (iv) a negative relationship between price volatility and hedge ratio/incentive pay. Overall, the first two chapters clarifies the optimal compensation scheme under varying economic environments in order to mitigate the agency problem associated with hedging decisions.

Last, a new model for the series of West Texas Intermediate (WTI) crude oil prices process is introduced, which accommodates spikes and local trends in its trajectory, as well as the multimodality of its sample distribution. The model relies on the convolution of two stationary processes, causal and noncausal processes, which allows for the estimation of the monthly WTI crude oil prices series. As an alternative specification, the mixed causal-noncausal autoregressive (MAR) models are estimated and used for oil price prediction. Two forecasting methods developed in the literature on MAR processes are applied to the data and compared. In addition, this chapter examines the long-term relationships between the WTI crude oil price, the Ontario Energy Price Index (OEP) and the Ontario Consumer Price Index (OCPI). These relationships are established using the cointegration analysis. The vector error correction (VEC) model allows us to predict the Ontario price indexes and the WTI crude oil prices. This chapter shows an alternative simple method of forecasting Ontario price indexes from stationary combinations of WTI crude oil price forecasts obtained from the mixed causal-noncausal autoregressive (MAR) models. This chapter shows that both method of prediction yields forecasts that are close approximation of the out of sample value.

Dedication

Dedicated to my parents, Katherine Pang and Wai Hung Tong, thank you for all your love and support.

Acknowledgements

The path to completing this thesis was not without challenges, however, I am tremendously fortunate to have the support of all the individuals, who I cherish, in playing a part in achieving this milestone.

I owe my deepest gratitude to my co-supervisors, Yisong Tian and Joann Jasiak. I am sincerely grateful for Yisong Tian's invaluable insight, guidance and recommendation/advice throughout my research. I am also greatly thankful for Joann Jasiak's words of encouragement, enthusiasm and her generous contribution of time despite her busy schedule. I am forever grateful for both their patience, understanding and immense knowledge. This thesis would not have been possible without them.

I would like to take this opportunity to thank my committee member, Mark Kamstra, for his positivity and feedback, and Robert McMillan's valuable comments.

My gratitude extends to the economics department at York University, especially to my colleagues and friends for making this process more enjoyable.

Finally, thank you to my friends and family. In particular, Katherine Pang and Wai Hung Tong, I am ineffably indebted to my incredible parents for bringing me up with all their hard work and love. I am also extremely thankful to my brother, Lindberg Tong, sister in law, Iris Lui and my husband Stanley Hung for their support.

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Chapter 1

Introduction

Commodity prices are volatile and difficult to predict. Hedging reduces the risk of sudden financial losses due to adverse market movements. The hedging decision does not always lie in the hands of the shareholder; instead it is often made by managers. Managers are risk-averse, and their hedging strategies do not always maximize firm value (Jin and Jorion, 2006). The first part of my thesis focuses on how executive compensation can be used to align the shareholders' and managers' hedging decision to maximize firm value. The last part of my thesis focuses on how commodity prices can be forecasted. In particular, I examine the bi-modality in the distribution of the West Texas Intermediate (WTI) crude oil prices, which appears to disregarded the existing literature and the co-movements between WTI crude oil prices, the Ontario Energy Price Index (OEP) and the Ontario Consumer Price Index (OCPI).

The second and third chapters examine how the hedging decisions between managers and shareholders can be aligned using executive compensation, as there is a tendency for risk-averse mangers to over-hedge (Holmstorm and Ricart i. Costa, 1986; Smith and Stulz, 1985). My research develops a model to solve this agency problem in an attempt to align hedging using executive compensation. First, factors used to determine the optimal executive compensation schemes are identified. Second, the role these factors play in aligning the conflicting incentives in hedging decisions is assessed, with the findings then applied in practice to minimize the agency problem.

The second chapter uses a standard principal-agent model and endogenously determines the hedging strategy and compensation. The one period model incorporates the risk-averse manager's utility into the hedging decision while keeping shareholder risk neutral; the model focuses on a linear derivative setting. Two scenarios are compared: first best, and second best. First best is fully observable; shareholder observes effort and makes the hedging decision, while second best is when managers make the hedging decision and have information of effort. The manger is paid a salary and equity incentive, with the latter tied to firm value. By comparing the two scenarios with changing parameters, I can determine [isolate] the factors that affect optimal hedging and optimal compensation. The model finds that there are three factors: risk aversion, price volatility and profit margin that determine optimal compensation. This is in line with Brown and Toft (2002) which find that optimal hedge depends strongly on price and quantity volatilities, and also correlation between prices and profit margin. In this chapter my model shows how managerial equity incentive pay impacts corporate hedging decisions as compared to the absence of incentive pay. With the presence of incentive pay the model shows a decrease in hedging. The model can also show the agency problem in hedging is more pronounced with high risk aversion, low price volatility and high profit margin. Using this model we can determine the optimal managerial compensation scheme that mitigates the agency problem. The optimal managerial compensation scheme should be comprised of less equity-based incentives when there is high risk aversion, high price volatility or low profit margin.

The third chapter examines the model prediction derived from the previous chapter. The predictions are: (i) when incentive pay increases, the optimal hedging ratio decreases, (ii) when price volatility increases, the optimal hedge ratio decreases, while price volatility exhibits a negative relation with equity-based incentive, (iii) when risk aversion increases, the optimal hedge ratio decreases, while risk aversion shows a negative relation with equity-based incentive, (iv) when variable cost increases, the optimal hedge ratio decreases, while variable cost displays a negative relation with equity-based incentive. The validity of these model predictions are then empirically test against a sample of U.S. oil and gas firm data from 2011-2014, which allows the focus to center on commodity hedges.

In January 1997 financial reporting release No. 48 was released which require firms to discloser of derivative instruments and quantitative information about market risks. This financial reporting release allows for the examination of hedging activities in each firm. Using that data we used a simple OLS model to test the model predictions. The companies in the sample use both linear and non-linear derivatives to hedge their price risk. The model prediction was based on a linear derivative setting, however, the empirical test includes both those derivatives. The empirical testing shows that equity-based executive compensation is negatively related to hedging which is consistent with prior research from Chen, Jin, and Wen (2011), and Tufano (1996). The chapter further shows evidence that price volatility is negatively related to hedging, while risk aversion is negatively related to hedging. Therefore, in general, the model predications are in line with empirical evidence.

The fourth chapter develops an econometric model to forecast the WTI crude oil prices, the OCPI, and the OEP. As illustrated in the previous chapters, one of the factors affecting optimal executive compensation is volatility of commodity prices. Being able to estimate commodity prices can give some insight to the economic environment, thus providing some indication to the optimal compensation scheme. Therefore, to explore how commodity prices can be forecasted, the next chapter develops an empirical model to predict future commodity prices.

The WTI crude oil price forecast is obtained by using the convolution approach of stationary causal and noncausal processes. To forecast the Ontario price index, this chapter estimates the long-term co-integrating relationships between the OCI, OEP and WTI crude oil prices. Then, given these estimated long-term relationships, the forecasts of both the Ontario price indexes can be obtained as functions of the

WTI crude oil price forecast, combined with the forecast of the stationary series of departures from the long-run equilibrium.

The WTI crude oil price forecast can be obtained by forecasting the convoluted series or from a simple mixed causal/noncausal model of WTI crude oil prices. The latter approach allows comparing two methods of forecasting for noncausal processes developed by Lanne, Luoto and Saikkonen (2012) and by Gourieroux and Jasiak (2016). While, the convolution of stationary causal and noncausal process is parsimonious and accommodates the local trends and spikes in the oil price, it also takes into account the bi-modality of its sample distribution which has not been studied in the existing research. The presented convolution approach shows a viable alternative approach for modelling the monthly WTI crude oil prices to accommodate the bi-model sample distribution.

As can be observed in my research, both the OCPI and OEP display a parallel global upward trend, while OEP exhibits local trends and spikes. The local trends and spikes in both the series of OEP and OCPI indexes appear to occur simultaneously as those observed in the trajectory of the WTI crude oil prices. Therefore, by estimating the long-term cointegrating relationships between the series, this chapter finds that Ontario price indexes can be forecasted using the forecasts of crude oil prices provided from the mixed causal-noncausal model and the co-movements between the series.

Chapter 2

Aligning Corporate Hedging Decisions with Executive Compensation

2.1 Introduction

Financial derivatives are widely used in corporations as a mean of risk management, with hedging strategies affecting firm value. While shareholders/principals hire managers/agents to manage the firm involving them and to increase the valuation of the firm, hedging strategies employed by managers do not always maximize firm value (Jin and Jorion, 2006). Both past theoretical and empirical studies indicate a tendency for risk-averse managers to overhedge (Holmstrom and Ricart i Costa, 1986; Smith and Stulz, 1985). Yet surprisingly, there is limited research examining how to align shareholders' and managers' incentives in terms of corporate hedging decisions.

To help fill the gap, this chapter develops a principal-agent model, and examines how executive compensation can be used to align such hedging decisions. The overall objective of the chapter is to identify factors used to determine the optimal executive compensation schemes, assess the role these factors play in aligning the conflicting incentives in hedging decisions, and apply these findings in practice to minimize the agency problem.

In this chapter, executive compensation is used to achieve a balance in the conflicting hedging incentives between shareholders and managers. Tufano (1996) suggests that executive compensation can influence how managers choose to hedge. Similarly, Prendergast (2002) and Core, Guay, and Larcker (2003), find that hedge risk affects the amount of annual executive compensation. The latter provides evidence that executive compensation includes a risk-premium for exposure to risk. Futhermore, empirical research (Rogers, 2002 and Rajgopal and Shevlin, 2002) indicates that there is a link between risk management and managerial incentive, suggesting that managerial compensation design plays a key role in optimizing hedging decisions. Building on these studies, this chapter shows that in addition to how certain managerial executive compensation schemes can be utilitzed to help align hedging decision, it also explains when to deploy a more (versus less) equity-based compensation scheme to yield the optimal hedging strategy. To examine how to align these incentives, I examine the optimal hedging strategy of shareholders and compare it to the optimal hedging strategy of managers. Similar to Smith and Stulz (1984), the model incorporates the risk-averse manager's expected utility into hedging decision. As noted, the hedging decision and executive compensation affect one another; therefore, unlike Smith and Stulz (1984), the hedging decision and executive compensation are endogenously determined. In my model, the risk-averse agent's expected utility increases with firm value, in line with Smith and Stulz (1985). Their paper however, does not examine how this compensation can solve the conflicting interest in hedging decisions.

To study the difference in optimal hedging strategies, I examine two scenarios. Under the first scenario, there is hidden information whereby only managers observe effort and have information on the hedging decision, and shareholders decide on the compensation scheme. The compensation received by managers can be bifurcated into fixed (i.e. salary) and equity-based incentive (i.e. performance-related bonus) components. Given the compensation scheme, managers will choose effort and a hedging strategy that maximizes their expected utility. The risk-averse managers exhibit a concave expected utility function, with the degree of concavity dependent on their coefficient of absolute risk aversion, while their compensation (i.e. wealth) is a convex function of the firm's value. It is noted in Smith and Stulz (1984) that the optimal hedging policy is to hedge the firm completely if the manager's end of period utility is a concave function of the end of period firm's value; in which case, managers will only take on risk if they are compensated to do so. If, on the other hand, the manager's end of period wealth is a convex function of the end of period firm value, then some of the hedging will be eliminated. If managers are not properly compensated, they may not manage risk in a way that maximizes firm value. The risk-averse managers will only maximize shareholder wealth if doing so will also maximize their own expected utility. In this case, the solution to the maximization problem yields a second best solution.

In the second scenario, both effort and hedging are contractible. Shareholders observe the amount of effort, and devise both the hedging strategy and compensation scheme. In this case, there is perfect information and would be considered the first best solution. Here, shareholders maximize their utility, which is to maximize the firm value by choosing the effort exerted, the hedging strategy and the compensation scheme. However, this case is not achievable in practice, as information about effort is hidden, and managers are the ones managing the firm's operational risk.

In both cases, this chapter assumes that the policy of corporate risk management only allows for the use of linear contracts, which in turn, allows for the closed-form solution of the model to be interpreted. In this model, hedging decisions and executive compensation are studied together. The addition of a risk-averse manager's utility and executive compensation to the hedging decision yield a non-standard result.

The analysis support the view that managers will only take on risk when paid to do so. As observed in the numerical example, managers hedge more of the expected production than the shareholders. This means that shareholders take on more risk than managers.

This model also yields a set of intuitive outcomes. For example, when the equity-based incentive pay increases, the optimal hedge ratio decreases, because the manager is both compensated to undertake increased risk and a larger portion of their wealth is linked with firm value. This is consistent with Chen, Jin, and Wen (2011), who empirically show that higher managerial equity incentives result in lower degrees of hedging. Additionally, the model shows that when the firm's volatility in price increases, the optimal strategy is to hedge closer to the firm's average production (where the hedge ratio is closer to 1), and the compensation scheme should have a lower incentive portion. As a result of heightened uncertainty under increased price volatility, it is natural to lower expected losses by hedging close to average production. Moreover, the agency problem is less pronounced when price volatility is high, therefore the manager would require less equity based compensation to close the gap between their incentives.

The model also indicates that idiosyncratic aspects such as volatility in hedgeable price risk, variable cost, and executive risk aversion, in aggregate, influence optimal hedging and compensation decisions. These findings can help delineate an optimal compensation strategy that aligns the hedging decisions under different environments/firm characteristics. For instance, when variable costs are high, the shareholder can lower the equity incentive-pay portion to align the hedging decision. Given the model results are expressed in closed form, the role of how each factor affects the optimal compensation and hedging strategy can be interpreted and analyzed.

Overall, this chapter endeavors to bridge the gap in the past literature by using a principal-agent model to examine the optimal hedging strategies between risk-averse agents versus risk-neutral principals in a linear financial derivatives setting, and also proposes an optimal compensation structure. The model shows how incorporating managerial incentive pay will impact the hedging decision. By considering the managerial incentive decision, the model shows that the optimal hedge would decrease. In additionally, it also provide insights into how the optimal compensation scheme should be used when faced with different firm-specific characteristics to minimize the agency problem in the hedging decision. The model shows that optimal managerial incentive should comprise more of the equity-based portion when there is a low risk tolerance, or a low price volatility, or a low variable cost, to align and optimize managerial hedging decisions. In contrast, when there is high coefficient of absolute risk aversion, or high price volatility, or high variable cost, it is best to compensate the manager with a lower equity-based portion in order to optimally align hedging decisions.

The chapter is organized as follows: The next section includes a brief literature review of hedging. Section 2.3 describes the model and framework of the study by first examining the agent's optimization problem, followed by the principal's optimization problem. Section 2.4 compares the optimal hedging strategy under different scenarios and changes in parameters. Section 2.5 displays the model findings and prediction. Concluding remarks are provided in Section 2.6.

2.2 Literature Review

Past research focuses on why firms hedge, and how firms should hedge, while others have examined the agency problem in the risk management area. Up to now, these studies do not provide a solution on how exactly to align the corporate hedging decision using executive compensation, and do not provide insight on how executive compensation could be adjusted under certain economic environments. The following are some of the past literature that relates to this topic.

Numerous papers focus on why firms hedge, for example, Smith and Stulz (1984) studies why some firms hedge and others do not, and they show that a value maximizing firm would hedge for three reasons: (i) reduction in expected taxes, (ii) lower costs of financial distress, and (iii) managerial risk aversion. Others argue that firms hedge: (i) to increase debt capacity (Ross 1997 and Leland 1998), (ii) to ensure sufficient internal funds for investment when external funds are costly (Froot, Scharfstein, and Stein 1993), or (iii) to reduce probability of downside risk (Stulz 1996). Thus, the reasons why firms hedge can be generalized into two categories: (i) increase firm value, and (ii) managerial risk aversion.

There are many theories that suggest managerial risk aversion plays a prominent role in affecting hedging behavior. Bodnar, et al. (2004) document why firms hedge by questioning the managers and find that personal risk aversion plays an important role. While Tufano (1996) and Rogers (2002) find empirical evidence that manager's risk aversion is the key determinant to hedging strategy; in particular they find that, managers will hedge less when they are compensated more with risk taking incentives. Rajgopal and Shevlin (2002) also show that risk incentive compensation is crucial in determining hedging policy, while Chen, Jin, and Wen (2011) provide evidence that hedging may be motivated by managerial risk aversion in particular. Jin and Jorion (2006) meanwhile, show that hedging does not increase firm value for U.S. oil and gas producers, but instead serve as a function of managerial risk aversion. Overall, these studies all indicate that risk-averse managers directly affect firm hedging strategy.

On the other hand, there are studies documenting how firms should hedge. Brown and Toft (2002) find that optimal hedge depends strongly on price and quantity volatilities, and also correlation between prices and profit margin. They examined environments where firms should use a linear vs. non-linear hedging strategy, and find that when price risks are high, quantity risk is low, or when there are no correlations between price and quantity, it is better to use a linear hedging strategy. Though Brown and Toft (2002) examine how firms hedge, they do not examine the impact of a risk-averse agent on hedging strategy. In addition to examining hedge strategy, this chapter also looks at the role of the risk-averse agent under different economic environments, similar to Brown and Toft (2002). Smith and Stulz (1985) meanwhile, note that the optimal hedging policy is to hedge the firm completely, and the manager will only take on risk if they are paid to do so. Lastly, Kuwornu, et al (2005) use a classical principal-agent model to derive and provide a tool to determine the optimal hedge ratio, but neglect to provide a link between hedging ratio and optimal compensation structure. Therefore, when determining optimal hedge strategy under the optimal compensation, the role of the compensation structure and risk-averse agent has yet to be explored.

There are studies that attempt to solve the agency problem in the risk management area. The model Smith and Stulz (1984) devises takes the manager's compensation as given, despite the fact that manager's compensation and hedging strategy are often endogenously determined. Holmstrom and Ricart i Costa (1986) formulate a model that examines optimal contracting when the risk-averse manager makes the investment decision. They find that managers accept few investments. However, it does not specifically study the risk-averse manager's role in hedging decision nor does it examine how the compensation can be used in difference environments to align the investments. Chang (1997) use a simple model to look at what the factors are in determining both hedging and compensation policies. In it, they examine the scenarios where the manager can either hedge or not hedge, and gives zero or 100 percent effort. They find that optimal hedging policy and stock-based compensation contract are determined by the firms' abandonment value, profitability, and the extent of its risk exposure. However, their model is not done in a principal-agent framework and does not include non-hedgeable risks. They also do not examine the difference in hedging strategy and the optimal compensation scheme when managers versus shareholders make the decision. Both of those are dually important as Chao et al (2011) provide empirical evidence on the impact of hedging and executive compensation on firm value. This chapter takes it a step further whereby the model endogenously determines the optimal manager's compensation and hedging strategy. In addition, the empirical testing examines the hedge ratio and factors affecting the hedging decision, and how to align the hedging decision with managerial equity-based incentives.

Studies also examine Moral Hazard problems in which the manager has more information of risk exposure then the shareholders (DeMarzo and Duffie 1991). However, all these previous studies do not closely examine the role of the risk-averse manager when determining the optimal hedge strategy along with the optimal compensation scheme to align conflict in hedging. This chapter addresses how executive compensation can be use to align the hedging strategies under varying economic environments and finds empirical support for the model predictions.

2.3 Framework and Model

Given that managers are known to have non-diversified human capital investment in the firm, and that risk neutral shareholders can diversify risk across investments, differences in risk tolerance arises resulting in conflicting optimal hedging strategies. Here, two different risk tolerance cases are modelled, and the effects they have on optimal hedging strategy and optimal compensation scheme. Taking the standard agency theory approach, this model is a one period model with a risk-averse agent, the manager, and a risk neutral principle, the shareholder. The optimal hedging strategy and optimal compensation scheme are endogenously determined in the model.

In the first case, there is information asymmetry regarding the agent's effort; the principal can only observe the outcome, but not the level of effort. The risk-averse agent chooses to maximize the expected utility by choosing the level of effort and the amount to hedge, given the compensation schedule is determined by the principle. The agent will receive a fixed salary and an incentive portion (equity-based incentive). This incentive portion is given as a percentage of the firm value. The agent is subjected to a participation constraint where the agent will only take on employment opportunity if the compensation package meets a minimum expected utility, reservation utility. As opposed to the agent, the risk neutral principal can diversify their investment across multiple assets, and thus, is only concerned about expected payoff. The principal maximizes their expected utility by selecting the compensation method and amount. The asymmetry of information yields a ′second best solution′.

The second case is one with perfect information resulting in a ′first best solution′. The principal contracts directly on effort and decides the amount to hedge along with the compensation scheme. In practice, the effort of the manager is unobserved and the hedging decisions are in the hands of the risk-averse managers. However, modelling this case can serve as a benchmark when comparing ′second best solution′ against the ′first best solution′.

Similar to other risk management studies, the Value of the firm $'V_t'$ is determined by revenue minus expense. Revenue is calculated by taking the number of hedged contracts $'h'$ at the Forward Price $'F_o'$ [1] subtracted by the Spot price $'S_t'$ [2] plus

[1] The forward price is given by:

$$F_o = S_o e^{t(r-q)}$$

r is the risk free rate and q is the carry cost

[2] Given the Black-Scholes Return:

the product of quantity $'q_t(a)'$ produced multiplied by the Spot rate. The quantity produced is a function of the agent's effort.

$$V_t = h(F_o - S_t) + q(a)S_t - c_1(q(a)) - c_2 \tag{2.3.1}$$

Agent's Payoff and Utility:

The agent's return/payoff $'\pi_t'$ is given by, equation 2.3.2, adding the fixed salary $'s'$, with the incentive component $'b'$ (percentage of firm value) multiplied by the firm value $'V_t'$, before subtracting the disutility of effort $'g(.)'$. Next substitute equation 2.3.1 to equation 2.3.2 to get the payoff function in terms of all the variables.

$$\pi_t = s + bV_t - g(a) \tag{2.3.2}$$

$$\pi_t = s + b[h(F_o - S_t) + q(a)S_t - c_1(q(a)) - c_2] - g(a)$$

Note, the quantity produced '$q(a)$' is equal to the production function $f(a)$ plus white noise ε with zero mean and variance σ_ε^2.

$$q(a) = f(a) + \varepsilon$$

$$c_1(q(a)) = c_1(f(a) + \varepsilon)$$

$\frac{ds}{s} = \mu dt + \sigma\sqrt{\partial W}$, where $W \sim N(0,t)$ and $\partial W = \sqrt{dt}\varepsilon$, the spot price at time t $'S_t'$ is:

$$S_t = S_o e^{\mu t - \frac{\sigma_s^2 t}{2} + \sigma_s W_t}$$
$$E(S_t) = S_o e^{\mu t}$$

The agent is risk-averse and has a negative exponential utility; stated in certainty equivalent terms their expected utility can be written as:

$$CEA_t = E_t(\pi_t) - \frac{1}{2}\gamma Var_t(\pi_t) \qquad (2.3.3)$$

See Appendix 1 for Calculation for $E_t(\pi_t)$ and $Var_t(\pi_t)$

$$CEA_t = s + b[h(F_o - S_o e^{\mu t}) + f(a)S_t - c_1(f(a)) - c_2] - g(a) -$$

$$\frac{\gamma}{2}[S_o^2 b^2(e^{\sigma_s^2 t} - 1)e^{2\mu t}(h - f(a)^2 + b^2\sigma_\varepsilon^2[S_o^2 e^{\sigma_s^2 t + 2\mu t} - 2S_o c_1 e^{\mu t} + c_1^2]]$$

To examine the cases closely and to deduct a close form solution, both $'f(a)'$ and $'g(a)'$ will take on a simple functional form; the agent exhibits linear productivity and quadratic effort ($f(a) = ka$ and $g(a) = \frac{1}{2}\eta a^2$, where k is the marginal productivity, η is the effort aversion).

2.3.1 Case 1 - Hidden Information (Second Best)

Agent's problem

In this case, the agent's problem is solved by maximizing the expected utility, this is given in terms of certainty-equivalent approximation, CEA. Here, the agent chooses effort $'a'$ and hedge amount $'h'$, then the principal chooses salary $'s'$ and incentive pay $'b'$:

By taking the First Order Condition, FOC, with respect to $'a'$ to maximize the effort it gives the following solution:

$$\gamma b^2 Var(S)[h - ka]k - \eta a + bk(S_o e^{\mu t} - c_1) \quad = \quad 0 \tag{2.3.4}$$

By taking the FOC with respect to hedge amount $'h'$ gives:

$$h - ka = \tfrac{F_o - S_o e^{\mu t}}{\gamma b Var(S)} \tag{2.3.5}$$

As $'h'$ is the amount hedge and $'ka'$ is the average production, $'h - ka'$ shows the difference between the hedge amount and average production. Substitute $'(h - ka)'$ from equation 2.3.5 into 2.3.4 to solve for $'a'$

$$a = \frac{bk[F_o - c_1]}{\eta} \tag{2.3.6}$$

$$h = b\frac{k^2[F_o - c_1]}{\eta} + \frac{1}{b}\frac{F_o - S_o e^{\mu t}}{\gamma Var(S)} \tag{2.3.7}$$

Equation 2.3.6 shows that effort (a) depends on the incentive based pay (b); the higher the incentive pay the more effort the manager is willing to exert. With higher incentive pay the manager will receive a higher return by increasing the value of the firm through production. Therefore, they will demonstrate more effort to increase production in order to increase their return. The equation also shows that if the marginal productivity (k) increases, then effort (a) will also increase. As marginal productivity (k) increases, the manager can produce more with the same level of effort. Consequently, the higher the marginal productivity, the greater the effort the manager will exert as there is a higher return per effort.

To interpret equation 2.3.5, as $'ka'$ gives the average production, when the coefficient of absolute risk aversion (γ), equity incentive pay (b), volatility in price ($Var(S)$) or spot price (S_o) increases, it would decrease the difference between the number of hedge and average output. If the parameters γ, $Var(S)$ and spot price (S_o) increases, the agent would want to hedge closer to average production. It seems intuitive that if the agent is more risk averse, they would limit risk and hedge close to the output. Similarly, when price volatility is high, risk-averse managers face increased price uncertainty and would want to hedge closer to average production.

Equation 2.3.5 also illustrates that if forward price at time 0 is greater than the expected spot price at time t, $F > E(S)$, the optimal amount hedged would be closer to the average production (as the prices are expected to decrease). If the forward price today is higher, selling outputs at the forward price would yield a higher return. On the other hand, if $F < E(S)$ then it would make sense to hedge less than average production and sell the output at the expected spot price(as the prices are expected to increase).

Principal's problem

In certainty equivalent terms, the principal's expected payoff/utility, CEP, is given by $E_t((1-b)V-s)$, which is the left over firm value after compensating the agent. Since the principal is risk neutral they only care about maximizing firm value. By substituting V with equation 2.3.1 the expected utility becomes:

$$CEP = (1-b)[h(F_o - S_o e^{\mu t}) + f(a)S_o e^{\mu t} - c_1 f(a) - c_2] - s \quad (2.3.8)$$

The principal's optimization problem is to maximize the following equation:

$$max_{s,b}((1-b)[h(F_o - S_o e^{\mu t}) + f(a)S_o e^{\mu t} - c_1 f(a) - c_2] - s) \quad (2.3.9)$$

The maximization problem is also subjected to the following constrains on effort $'a'$, hedge amount $'h'$ and participation constrains:

$$a = \frac{bk[F_o - c_1]}{\eta}$$

$$h = b\frac{k^2[F_o - c_1]}{\eta} + \frac{1}{b}\frac{F - S_o e^{\mu t}}{\gamma Var(S)}$$

The participation constrain is given by the following formula:

$$s + b[h(F - S_o e^{\mu t}) + kaS_o e^{\mu t} - c_1 ka - c_2] - g(a) -$$

$$\frac{\gamma}{2}[b^2 S_o^2 e^{2\mu t}(e^{\sigma_s^2 t} - 1)(h - ka)^2 + b^2 \sigma_\varepsilon^2 [S_o^2 e^{\sigma_s^2 t + 2\mu t} - 2S_o e^{\mu t} c_1 + c_1^2] \geqslant W_o$$

W_o is certainty equivalent value that defines the agent's reservation utility. [3]

Below is the principal's expected payoff, CEP, after including the participation constrain:

$$CEP = h(F_o - E(S_t)) + f(a)E(S_t) - c_1 f(a) - c_2 - W_o + g(a) + \frac{\gamma}{2}b^2 Var(S)[h - f(a)]^2 + \frac{\gamma}{2}b^2 G$$

$$CEP = bkAF_o + B(F_o - E(S_t))b^{-1} - c_1 bkA - c_2 - W_o - g(a) - \frac{\gamma}{2}Var(S)B^2 + \frac{\gamma}{2}b^2 G$$

To solve the principal's optimization problem, take the first order condition, FOC, with respect to salary $'s'$

$$s = -b[h(F_o - S_o e^{\mu t}) + kaS_o e^{\mu t} - c_1 ka - c_2] + g(a) +$$

$$\frac{\gamma}{2}[b^2 S_o^2 e^{2\mu t}(e^{\sigma_s^2 t} - 1)(h - ka)^2 + b^2 \sigma_\varepsilon^2 [S_o^2 e^{\sigma_s^2 t + 2\mu t} - 2S_o e^{\mu t} c_1 + c_1^2] + W_o$$

and take the FOC with respect to agent's incentive value 'b', which is given as a

[3] Given that :

$$A = k(F_o - c_1)\eta$$

$$B = \frac{F_o - S_o e^{\mu t}}{\gamma Var(S)}$$

$$G = \sigma_\varepsilon^2 [S_o^2 e^{\sigma_s^2 t + 2\mu t} - 2S_o e^{\mu t} c_1 + c_1^2]$$

$$h = bkA\frac{B}{b}$$

$$a = Ab$$

portion of the firm's value.

$$kAF_o - B(F_o - E(S))b^{-2} - c_1Ak - \gamma bG = 0$$

Finally, multiply by b^2 gives:

$$\psi(b) = -\gamma Gb^3 + [kA(F_o - c_1)]b^2 - B(F_o - E(S)) = 0 \tag{2.3.10}$$

$$\psi^{'}(b) = 2[kA(F_o - c_1)]b - 3\gamma Gb^2$$

The equation cannot be explicitly interpreted. Figure 2.1 is the graphical solution of 'b' under the base case parameters, it examines the incentive portion in the agent's optimization problem, The base case numerical parameter values are detailed and analyzed in 2.4.

[Insert Figure 2.1:Graphical solution of the equity base incentive, b, that optimizes $\psi(b)$ under the base case detailed in session 2.4]

2.3.2 Case 2 - Observable (First Best)

Alternatively, in the second case ('first best solution'), the principal is able to observe effort $'a'$ and chooses amount hedge $'h'$, incentive pay $'b'$ and salary $'s'$.

The principal would maximize their payout subject to the agent's participation constraint.[4]

$$max_{a,h,s,b}(1-b)V - s$$

$$max_{a,h,s,b}((1-b)[h(F_o - S_o e^{\mu t}) + f(a)S_o e^{\mu t} - c_1 f(a) - c_2] - s)$$

Take the FOC with respect 'h' to maximize the hedging decision :

$$[F_o - S_o e^{\mu t}] - \gamma Var(S_t) b^2 [h - f(a)] = 0$$

$$[h - ka] = \frac{1}{b^2} \frac{(F_o - S_o e^{\mu t})}{\gamma Var(S)} \tag{2.3.11}$$

Take the FOC with respect to 'a' [5] and substituting in $[h - ka]$ from equation 2.3.11 into 2.3.12 to solve for 'a':

$$S_o e^{\mu t} - c_1 k - \eta a - \gamma [b^2 Var(S_t)[h - ka]k] = 0 \tag{2.3.12}$$

$$a = \frac{k(F_o - c_1)}{\eta} \tag{2.3.13}$$

[4]Subjected to the following participation constraint

$$s = b[h(F_o - S_o e^{\mu t}) + f(a)S_o e^{\mu t} - c_1 f(a) - c_2] + g(a) +$$

$$\frac{\gamma}{2}[b^2 S_o^2 e^{2\mu t}(e^{\sigma_s^2 t} - 1)(h - f(a))^2 + b^2 \sigma_\varepsilon^2 [S_o^2 e^{\sigma_s^2 t + 2\mu t} + 2S_o e^{\mu t} c_1 + c_1^2] + W_o$$

[5]This gives the First Order Condition with respect to effort, 'a' : $f'(a)S_o e^{\mu t} - c_1 f'(a) - g'(a) - \gamma[b^2 Var(S)[h - f(a)]f'(a)] = 0$

Take the FOC with respect to $'b'$ then substitute $[h - ka]$ from equation 2.3.11 to solve for $'b'$ [6]:

$$b = \frac{|F_o - E(S_t)|}{\gamma\sqrt{GVar(S_t)}}, where 0 \leq b \leq 1 \tag{2.3.14}$$

The optimal soluiton for incentive pay, $'b'$, can result in a corner solution where there is no incentive pay, $b = 0$. This solution is reasonable as the shareholders are making all the decisions and observe effort. Comparing the case with no incentive pay (i.e. Brown and Toft, 2002) versus the presence of incentive pay shows how managerial equity incentives change the optimal hedge decisions. It indicates how hedging decision would be affected in the absence of managerial equity incentives along with the effort exerted by the managers.

$$\frac{h}{ka} = 1 + \frac{1}{b^2}\frac{(F_o - S_o e^{\mu t})}{\gamma Var(S)ka}$$

In the special case where there is no incentive pay, b=0 , the hedging ratio would be higher as compared to the case with some incentive pay involved in the

[6] This gives the FOC solution to incentive pay $'b'$: $-\gamma Var(S_t)(h - ka)^2 b - \gamma G b = 0$. Which can result in a corner solution where $b = 0$ (which is a reasonable soluation given that shareholders are chosing all variables), or after further substitution the equation becomes $\frac{1}{b^2}\frac{(F_o - E(S_t))^2}{\gamma Var(S_t)} - \gamma G = 0$. The optimal hedging decision can be further examined by substituting optimal 'b' from equation 2.3.14 and optimal 'a' from equation 2.3.13 back into 2.3.11.

$$h = \sqrt{\frac{\sigma_\varepsilon^2[S_o^2 e^{\sigma_s^2 t + 2\mu t} + 2S_o e^{\mu t}c_1 + c_1^2]}{Var(S_t)}} + \frac{k^2(F_o - c_1)}{\eta}$$

compensation scheme. That is consistent with the fact that managers are risk averse and will only take on risk when compensated. In other words, when incentive pay is taken in consideration it would encourage managers to take on additional risk and to hedge less.

2.3.3 Summary Results Case 1 and Case 2

To compare Case 1 (second best scenario) and Case 2 (first best scenario), the optimization solutions from both the cases are summarized in Table 2.1.

[Insert Table 2.1: Results from Case 1 and Case 2]

It compares the (i) principal-agent's problem where the agent chooses both effort and the amount to hedge, while the principal chooses the amount of fixed salary and equity-base pay, with (ii) the case where the principal chooses all the aforementioned choice variables. First, as seen from the table, when comparing the level of effort to exert, $'a'$, the only difference between the two cases is the incentive portion the agent receives, $'b'$. This outcome is intuitive as the agent will increase effort only if given the incentive to do so, as there is a disutility of effort. As compared to case 2, the effort is not determined by the incentive pay since principal observes and chooses effort, thus the agent will have to deliver full effort.

Second, in both the scenarios the hedge amount depends on the average production (k) and an adjustment term. This adjustment term includes the firm/market specific parameters. The solution in the second case provides insight into which parameters affect equity incentive pay. However, the difference in the amount of hedge, $'h'$, between the two cases is not easily interpreted. This difference in hedge amount is also due to the equity incentive portion, which is consistent with previous studies (Tufano, 1996). The first and second terms both have incentive pay $'b'$ embedded in it, but this is endogenously determined and cannot be meaningfully interpreted here. Though the variables are endogenous, some insight can be gain by rearranging the equations:

$$\text{Case 1} : h - ka = \frac{1}{b}\frac{F_o - E(S_t)}{\gamma Var(S_t)}$$

$$\text{Case 2} : h - ka = \frac{1}{b^2}\frac{F_o - E(S_t)}{\gamma Var(S_t)}$$

The term $'h - ka$' is the difference in the amount hedge and average production, thus, in the 'Case 2: first best case', the principal will hedge $\frac{1}{b}$ (where 'b' between 0 and 1) of what the agent would hedge. However the average production, $'ka'$, is different in both cases as it depends on effort, which in turn is dependent on equity incentives, $'b'$ as well. The incentive portion, $'b'$, cannot be easily interpreted, because in the first case a closed form solution is not prevalent.

Since incentive pay, $'b'$, is endogenously determined we cannot directly compare/interpreted the difference in amount hedge and incentive pay, but using this model, in the next section I will compare different economic environments numerically, showing the difference between the 'first best solution', where the principal makes all the decisions, and the 'second best solution', where the manager makes some decisions.

2.4 Compare optimal hedging strategies

This section examines the solutions derived in the model by analyzing the factors that determine the optimal hedge. The solutions can be analyzed by assigning numerical values to the parameters in the equations from Table 2.2. It examines the effect of risk aversion, price volatility and variable costs on the optimal hedging strategy and compensation scheme under the 'first best solution' and 'second best solution'. The spot price S_o is normalized to 1. The base case includes reasonable parameters estimates that are taken from past literature (i.e. Brown and Toft, 2002) where:

- variable costs are 0.25 and fixed costs are 0.3
- agent's marginal productivity k = 2
- effort aversion $\eta = 3$
- ρ=1.03, where $\rho = e^{t(r-q)}$
- μ is 0.05

- $\sigma_S^2 = 0.55$, quantity volatility = 0.15, thus Variance in $S_t = 0.3904$

- Base case coefficient of absolute risk aversion $\gamma = 5$

Table 2.2 shows the difference between the two optimal hedging strategies and incentive pay under the base case scenario.

[Insert Table 2.2: Base Case of Numerical Example]

In the base case, the agent would choose to hedge closer to average production of the firm, while the principal is willing to take on additional risk by hedging further away from average production. As expected, the effort is higher in the first best solution because there is no hidden information, and the principal can observe and choose effort. The optimal incentive is higher when managers are the ones making hedging decisions; this aligns with previous research where principal will have to pay the agent to take on risk. This incentive portion can be lower when the principal makes all the decisions.

2.4.1 Change in coefficient of absolute risk aversion

Next, the effect on change in risk aversion to the hedging strategy and optimal compensation is examined. To study this effect, the base case is examined where γ =5, which is then adjusted both upwards and downwards to γ = 7 and γ =3, respectively, to compare the results.

[Insert Table 2.3: Change in coefficient of absolute risk aversion]

Table 2.3 shows the change in coefficient of absolute risk from 3 to 7. An increased coefficient of absolute risk aversion means that the agent is more risk averse, which leads to a reduction in effort. The change in effort is intuitive, when the agent's risk tolerance decreases, they would not be willing to work as hard for unforeseeable results. Under the first best scenario, changes in risk aversion does not affect effort or the hedge ratio, as effort is observed and the shareholders are the ones making the hedging decisions. Surprisingly, an agent's hedge ratio decreases with risk aversion, while in both cases, an increase in risk aversion shows a decrease in the incentive portion for optimal compensation. One would expect that as risk aversion increases, the optimal hedge ratio would increase, and the incentive pay would be higher when risk aversion increases. However, the solutions from the model shows the opposite which could be due to a higher level of optimal incentive portion in compensation when coefficient of absolute risk aversion is lower.

$$\frac{h}{ka} = 1 + \frac{\eta(F_o - E(S_t))}{k^2 b^2 \gamma Var(S)(F_o - c_1)}$$

In the base case scenerio, the assumption is that the spot price at time t is greater than forward price at time 0. This means that the price is expected to increase,

$F_o < E(S_t)$. Holding all parameters the same, if risk averison increases the hedge ratio is expected to also increase. However, when incentive pay, $'b'$, is endogenously determined this relationship is not observed. The solutions from the model illustrates that the optimal incentive pay is lower with an increase in risk aversion. When the agent is more risk-averse they would rather receive a compensation schedule with a higher portion in fixed amount and a lower portion in incentive based pay. This can be due to the fact that when managers are highly risk averse, they prefer a fixed salary rather than a higher incentive base pay, since the latter is not optimal/efficient. The increase in risk aversion is met with a decrease in incentive pay and the net effect (when the prices are expected to increase, $F_o < E(S_t)$) results in a decrease in hedge ratio.

From the FOC, we see the first term is 1. When $\frac{h}{ka} = 1$ the firm is hedging 100% of average production. Here, when the prices are increasing, the second term is negative, therefore when the second term is higher it means a lower amount of the average production is being hedged. Since the prices are expected to increase, hedging less of the average production is in turn less risky. As an agent is risk averse, they would want to take less risk and therefore would want to hedge less of the average prodcution when prices are expected to increase. This explains why a decrease is observed in hedging activities even though the risk aversion is increasing.

On the contrary, when prices are decreasing the agent is already hedging close to average production. The second term from the FOC would be positive when prices are expected to decrease, therefore when the second term is higher it means a higher amount of average production is being hedged. Hedging more of the average production translates into taking on less risk since the prices are expected to decrease. In the case where, $F_o > E(S_t)$, if the manager's risk aversion increases it will cause the hedge ratio to also increase. This would be in line with standard theory whereby when the agent is more risk averse they have a tendency to overhedge.

[Insert Figures 2.2 and 2.3: Graphical solution to optimal equity-base incentive, $'b'$ when γ =3 and γ =5 and graphical solution to optimal equity-base incentive, $'b'$ when γ =7]

Figures 2.2 and 2.3 represents the case 1 graphical solution to the FOC with respect to equity base incentive, $'b'$, as the coefficient of absolute risk aversion changes from 3 to 7. This is when the manager decides the level of effort and chooses the amount to hedge and the shareholder pays them a fixed and equity base incentive portion. In general, as risk aversion increases, the equity base incentive portion should correspondingly decrease.

2.4.2 Change in price volatility

The change in economic environment would change the optimal hedging strategy. For example, when prices are volatile, the firm would likely want to hedge closer to average production. The effect of change in price volatility in hedging strategy between the 'first best solution' and 'second best solution' can be determined by comparing the results when $\sigma_s^2 = 0.55$ changes to $\sigma_s^2 = 0.20$ (i.e. The price volatility/variance S_t changes from 0.39 and 0.045). Empirical data shows crude oil price volatility ranged from 28% to 46% between the years 2011 and 2015.

[Insert Table 2.4: Change in price volatility]

Table 2.4 shows the change in price volatility. The difference in hedge ratio between the managers and shareholders decreases when volatility increases. For example when $\sigma_s^2 = 0.55$ decrease to $\sigma_s^2 = 0.2$, the difference between managers' and shareholders' hedge ratio increases from 0.9610 (= (0.9610) − (0.0378)) to 1.1174 (= (0.7626) − (−0.3548)). In both cases when volatility increases, the optimal hedge ratio moves closer to average production. In a high price volatility environment, both the agent and principal would hedge closer to average production as both the manager and the shareholder would want to reduce the increase risk due to increase in volatility. Thus, their hedging strategy would be closer to each other, as compared to when price volatility is low. Therefore optimal compensation schedule in low price volatility environments should comprise more of the incentive portion to close the gap between the differing hedging incentives. When volatility in prices are high, an agent would need a lower incentive portion to make the optimal hedging decision.

2.4.3 Change in Variable Cost

Next, the change in variable cost on the model is examined. In both cases, when variable cost is high the manager would put in less effort, as it would take more effort to produce the same amount of product. The solution also shows the optimal compensation scheme consist of a lower incentive pay portion when variable cost is high. In both cases, the hedge ratio would decrease with increase in variable cost. When variable cost is low the agent's hedge ratio is further away from the principal's hedge ratio, in other words, the agency problem is more pronounced when the variable is low.

[Insert Table 2.5: Change in Variable Cost]

The above could be due the fact that low variable cost translates into a higher gross margin, which means more funds are generated per sale. The firm will make more with each additional sale, therefore managers will want to hedge more to manage their risk exposure. This is becasue both average production and effort will increase when variable cost is low.

2.5 Summary of Model Findings and Prediction

The above model shows that the agent's hedge ratio is further away from the optimal hedge ratio for the principal when (i) the agent's coefficient of absolute risk aversion is high, (ii) volatility in spot price is low and (iii) variable cost is low under the optimal compensation scheme devised in each scenario. This optimal compensation scheme is determined by the firm specific economic environments for example, when the agent's coefficient of absolute risk aversion is high, when volatility in spot price is high, or when variable cost is high the equity-based incentive portion should be lower.

$$\frac{h}{ka} = 1 + \frac{\eta(F_o - S_o e^{\mu t})}{k^2 b^2 \gamma Var(S)(F_o - c_1)}$$

Model Prediction 1: When incentive pay increases, the optimal hedge ratio decreases

Model Prediction 2: When price volatility increases, the optimal hedge ratio increases, while price volatility has a negative relation with equity-based incentive

Model Prediction 3: When risk aversion increases, the optimal hedge ratio decreases, while risk aversion has a negative relation with equity-based incentive

Model Prediction 4: When variable cost increases, the optimal hedge ratio decreases, while variable cost has a negative relation with equity-based incentive

2.6 Conclusion

This chapter develops a theoretical model to examine the moral hazard problem in corporate hedging decisions between risk neutral shareholders and risk-averse managers. It examines how this conflict of interest can be aligned through the use of optimal executive compensation scheme by comparing the 'first best solution' (shareholders decide on hedging strategy) to the 'second best solution' (managers decide on hedging). Depending on the different economic states the firm faces, the shareholders should deploy different incentive structures to align manager's hedging decisions with theirs. More specifically, the optimal compensation should comprise of more(less) equity-based incentive if the coefficient of absolute risk aversion is low (high), price volatility is low(high), or variable cost is low (high).

Past research examined why and how firms should hedge; however, it does not closely examine the role of the risk-averse agent when determining optimal hedge strategy and the optimal compensation scheme. Moreover, it does not provide answers to how the shareholders' and managers' hedging strategies can be aligned. This chapter bridges this gap by integrating the risk-averse manager's expected utility and executive compensation into the corporate hedging decision.

The theoretical model find that managers have the tendency to overhedge, which is consistent with other researchers have found. Under the optimal compensation scheme, managers will hedge closer to the expected production of the firm as compared to the shareholders. This chapter also gives practical insights to how to compensate managers in different states.

The theoretical model focuses on the linear financial derivatives setting a possible extension is to develop a formal theoretical model studying the effects of different types of derivatives while considering the role of the risk-averse managers. Developing such a model can provide insight into which derivative is most beneficial for different types of firms when the managers make the hedging decision under varying environments. Future research in this area can also consider extending this one-period model to a continuous model to examine its effects.

Figure 2.1: Graphical solution of the equity base incentive, b, that optimizes $\psi(b)$ under the base case detailed in section 2.4

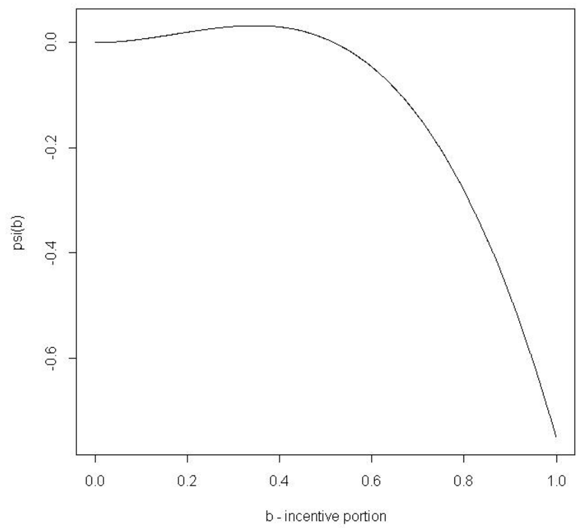

Table 2.1: Results from Case 1 and Case 2

•	**Case 1**: Agent choose a,h & Principal choose b,s	**Case 2**: Principal choose a,h,b,s
'a' =	$b\frac{k(F_0-c_1)}{\eta}$	$\frac{k(F_0-c_1)}{\eta}$
'h' =	$b\frac{k^2[F_o-c_1]}{\eta}+\frac{1}{b}\frac{F-E(S_t)}{\gamma Var(S_t)}$	$\frac{k^2(F_o-c_1)}{\eta}+\frac{1}{b^2}\frac{F_o-E(S_t)}{\gamma Var(S_t)}$
'b' =	$\gamma Gb^3+[kA(F_o-c_1)]b^2-B(F_o-E(S_t))=0$	$\frac{\lvert F_o-E(S_t)\rvert}{\gamma\sqrt{GVar(S_t)}}$ or $=0$

In case 1, the manager chooses the effort 'a' to exert and the amount to hedge 'h', while the shareholder chooses the fixed salary 's' and the equity based incentive 'b'. In case 2, the shareholder observes effort 'a' and choose amount to hedge and the compensation structure.
Note: $G=\sigma_\varepsilon^2[S_o^2e^{\sigma_s^2t+2\mu t}+2S_oe^{\mu t}c_1+c_1^2]$ and $B=\frac{F_o-S_oe^{\mu t}}{\gamma Var(S)}$
Note: The following gives the equation to the optimal 'h' in case 2 if the optimal 'a' and 'b' was substituted in $h=\sqrt{\frac{\sigma_\varepsilon^2[S_o^2e^{\sigma_s^2t+2\mu t}+2S_oe^{\mu t}c_1+c_1^2]}{Var(S_t)}}+\frac{k^2(F_o-c_1)}{\eta}$

Figure 2.2: Graphical solution to optimal equity-base incentive, 'b' when γ =3 and γ =5

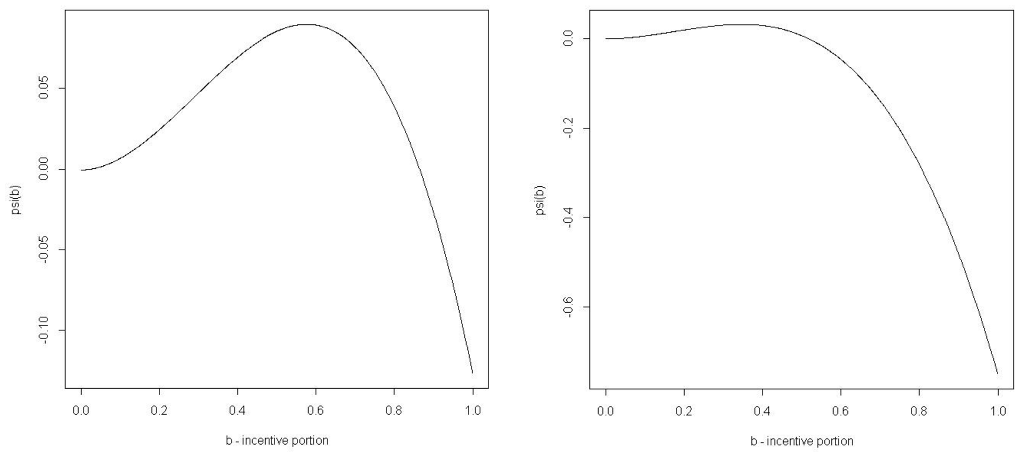

Figure 2.3: Graphical solution to optimal equity-base incentive, 'b' when γ =7

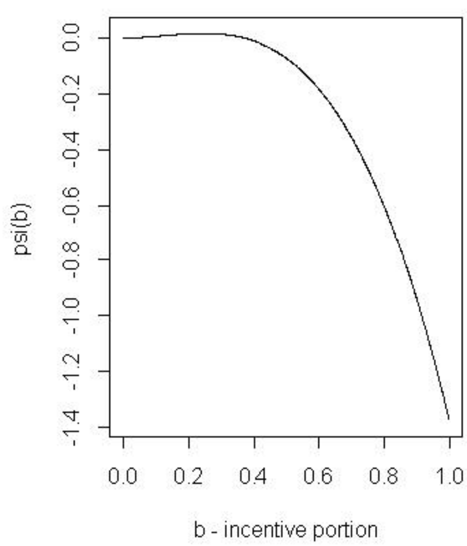

Table 2.2: Base Case of Numerical Example

•	Case 1: Second Best	Case 2: First Best
Effort $'a'$	0.2696	0.5200
Hedge Amount $'h'$	0.5183	0.1452
Incentive $'b'$	0.5185	0.1104
Average Quantity $'f(a)'$	0.5392	1.0400
Hedge Ratio $'h/f(a)'$	0.9610	0.0378

The table presents the base case results. It shows the optimal level of effect $'a'$, amount hedge $'h'$, equity-base incentive portion $'b'$, expected production $'f(a)'$ and hedge ratio $'h/f(a)'$ that maximizes utility in the both cases (Case 1: hidden information and Case 2: perfect information.).

Table 2.3: Change in coefficient of absolute risk aversion

γ	3		5 (Base Case)		7	
	2^{nd}Best	1^{st} Best	2^{nd}Best	1^{st} Best	2^{nd}Best	$1^{st} Best$
a	0.4496	0.5200	0.2696	0.5200	0.1925	0.5200
h	0.8782	0.1452	0.5183	0.1452	0.3640	0.1452
b	0.8646	0.1425	0.5185	0.1104	0.3702	0.0932
$f(a)$	0.8992	1.0400	0.5392	1.0400	0.3850	1.0400
$h/f(a)$	0.9766	0.0378	0.9610	0.0378	0.9454	0.0378

As the coefficient of absolute risk aversion increases the manager's optimal level of effort decreases while they hedge less and the optimal compensation should comprise of an increase in the equity incentive portion. In first best scenario, the manager effort is observed, thus the effort exerted does not change with coefficient of absolute risk aversion. The hedge amount increases along with the equity incentive portion.

Table 2.4: Change in price volatility

σ_s^2	0.55 (Base Case)		0.3		0.25		0.2	
	2^{nd}Best	1^{st} Best	2^{nd}Best	1^{st} Best	2^{nd}Best	1^{st} Best	2^{nd}Best	1^{st} Best
$'a'$	0.2696	0.5200	0.3120	0.5200	0.3174	0.5200	0.3214	0.5200
$'h'$	0.5183	0.1452	0.5558	-0.5695	0.5370	-0.8870	0.4903	-1.3645
$'b'$	0.5185	0.1104	0.6000	0.1594	0.6104	0.1760	0.6182	0.1981
$f(a)$	0.5392	1.0400	0.6240	1.0400	0.6348	1.0400	0.6429	1.0400
$h/f(a)$	0.9610	0.0378	0.8908	-0.1481	0.8460	-0.2306	0.7626	-0.3548

As the price volatility decreases the manager's optimal level of effort increases, while they hedge less and the optimal compensation should comprise of a decrease in equity incentive portion. In Case 2: First Best Scenario, the manager effort is observed; thus the effort exerted remains constant with price volatility. The hedged amount increases, while the optimal compensation increases in the equity incentive portion.

Table 2.5: Change in Variable Cost

c_1	0.45		0.25 (Base Case)	
	2^{nd}Best	1^{st} Best	2^{nd}Best	1^{st} Best
Effort $'a'$	0.0866	0.3867	0.2696	0.5200
Hedge Amount $'h'$	0.1244	-0.2346	0.5183	0.1452
Incentive $'b'$	0.2238	0.1040	0.5185	0.1104
Average Quantity $'f(a')$	0.1731	0.7733	0.5393	1.0400
Hedge Raio $'h/f(a)'$	0.7188	-0.0454	0.9610	0.0376

Variable Cost, c_1, changes from 0.25 to 0.45. As c_1 increases the manager's optimal level of effort decreases while they hedge less and the optimal compensation should comprise of a decrease in equity incentive portion under the Case 1: 2nd Best Scenario.

Chapter 3

Empirical Application in aligning Corporate Hedging Decisions with Executive Compensation

3.1 Introduction

Risk averse managers tends to overhedge (Holmstrom and Ricart i Costa, 1986; Smith and Stulz, 1985). To examine how the hedging decision can be aligned between shareholders and managers, I develop a theoretical model in the previous chapter that examines the factors that optimizes both executive compensation and hedging strategy. The model also shows how the divergent corporate hedging incentives can be mitigated in practice by using executive compensation. In this chapter I empirically test the validity of the theoretical model predictions obtained from the last chapter with US oil and gas firm data and find in general, the model predictions are supported by empirical data.

The relationship between hedge ratio and executive compensation has previously been documented in literatures from Tufano (1996), Prendergast (2002), and Core, Guay and Larcker (2003). Empirical research such as Rogers (2002) and Rajgopal and Shevlin (2002) indicates that there is a relationship between risk management and managerial incentive. While, Chen, Jin, and Wen (2011) extend on past literature and examine empirically the endogenous relationship between executive compensation and hedging decisions, and the effects on firm value. However, the use of executive compensation to reduce the managers' overhedging tendency has not been studied. As suggested by the theoretical model in the previous chapter, this agency problem can be mitigated by deploying different compensation schemes under varying economic environments.

To examine the validity of the various model predictions in practice, this chapter goes beyond the theoretical model in the previous chapter and empirically tests model predictions: (i) hedging decision is negatively correlated with executive compensation, which is consistent with studies from Tufano (1996) and Chen, Jin, and Wen (2011), (ii) price volatility is negatively related to hedging, (iii) variable cost is negatively related to hedging, and (iv) risk aversion is negatively related to hedging.

The first model prediction from the previous chapter suggests that as equity-based incentive pay increases, the optimal hedge ratio decreases. This is supported by Chen, Jin, and Wen (2011) which shows evidence supporting this model finding. Their study focuses on the oil and gas industry, while taking into account the endogeneity between hedging and executive risk-taking incentives; they find significant negative correlation between the hedging incentives and executive compensation. While, Tufano (1996) also finds equity-based executive compensation is negatively related to hedging.

The other model predictions suggest that price volatility and variable cost (gross margin) are negatively (positively) related to hedging. Although their relationship is not directly examined in Brown and Toft (2002), these factors have been documented to affect the hedging decision. Their study shows that the decision on how firms should hedge is affected by the price volatility the firm faces and also the correlation between price and profit margin.

Furthermore, there is evidence in past literature supporting the last model prediction, which predicts that while risk aversion increases, the optimal hedge ratio decreases. For example, Tufano (1996) finds that hedging is generally negatively associated with the tenure of firm executives. While Tenure is often seen as a proxy for executive risk aversion, as executive wealth is more invested in the firm with increased tenure.

Beginning in January 1997, annual 10K filings are required to include risk-management activities, including the nominal amount of hedge that was not previously available nor widely used in many related research studies. Thus, the model predictions can be empirically tested against a sample of U.S. oil and gas firm data. The use of the oil and gas sector allows for the control of market risk faced by each firm, and to focus hedging activity on commodity hedges. Firms are required to disclose both the nominal amount hedged and the annual production, which collectively allow for the examination of the percentage of production hedged.

Consistent with Core and Guay (2002), the executive risk-incentive compensation is measured by delta following value-increasing incentives. The theoretical model in the previous chapter is based on linear derivatives, while the empirical test is based on the general case which includes both the aforementioned derivatives to examine if the model prediction is valid. As the companies in the sample use both linear and non-linear derivatives to hedge their price risk.

In this chapter, to test the validity of the previous chapter's results, OLS regressions are used to show the linear relationships between the economic factors and the hedging decision while taking into account the executive's incentive pay. The empirical testing reveals that in general, the model predications are in line with empirical evidence.

The chapter is organized as follows: The next section includes a brief description of the model prediction. Section 3.3 describes the data. Section 3.4 shows the empirical analysis of model prediction and results. Concluding remarks are provided in Section 3.5.

3.2 Model Findings and Prediction

The previous chapter develops a theoretical model to examine the agency problem surrounding the corporate hedging decision between shareholders and managers, where the managers have been documented to over-hedge. The model examines two hedging scenarios; the observable case and the hidden information case. In the observable case the shareholder makes the hedging decision, while, in the hidden information case, the manager makes the hedging decision. The model compares the two cases and shows that the agency problem is more pronoun when (i) the manger's risk aversion is higher, (ii) the price volatility is low, and (iii) the variable cost is high. The model further suggests that the optimal compensation scheme should be comprised of a lower portion of equity-based incentive when Manager's risk aversion is high, when price volatility in high, or when variable cost is high. In this chapter, I preform empirical testing of the following model predictions from the previous chapter against firm data to analyse the model validity.

Model Prediction 1: When incentive pay increases, the optimal hedge ratio decreases

Model Prediction 2: When price volatility increases, the optimal hedge ratio decreases, while price volatility have a negative relation with equity-based incentive

Model Prediction 3: When risk aversion increases, the optimal hedge ratio decreases, while risk aversion have a negative relation with equity-based incentive

Model Prediction 4: When variable cost increases, the optimal hedge ratio decreases, while variable cost have a negative relation with equity-based incentive

These predictions are tested against empirical data in the following section.

3.3 Data

Standard and Poor's ExecuComp data was used to test the theoretical model findings. This set of data provides information on executive compensation on over 2,500 companies and more than 24,000 executives. Focusing on the oil and gas industry in particular, only companies with NAICS code 211 from years 2011 and 2014 in the sample are included. The advantages in using the oil and gas industry are (i) the oil and gas price exposure can be identified, (ii) the reserves and productions are disclosed in the financial statements, and (ii) the commodity risk is easily hedged with derivatives sold on the exchange.

3.3.1 Incentive Pay

Consistent with past empirical research, delta is used to measure the incentive pay. The estimation of *delta* follows Core and Guay's (2002) approach. *Delta* measures the dollar change in wealth associated with a 1 percent change in a firm's stock price. It calculates the executives' value-increasing based on their stock and option holdings. *Delta* is comprised of two components, (i) share delta and (ii) option delta. Share delta represents the number of shares multiplied with the share price, which is then multiplied by '0.01'. Option delta uses the Black-Scholes Model to calculate option price, which includes the use of data on exercise price, ex-date, volatility, dividend yield, and risk-free rate. It also examines the amount of options vested and unvested (in the money unexercised exercisable/unexercisable options). Adding the two components gives the value of delta. Refer to Core and Guay (2002) for complete description on delta calculation.

$$Delta = Share\ delta + Option\ delta$$

$$Deltamean = \frac{\sum delta_i}{\#executives_i}$$

3.3.2 Risk Aversion

Risk aversion is difficult to model. In past literature, tenure, age, gender and wealth were often associated with risk aversion. Thus, CEO's tenure and age data collected from ExecuComp Data were used (iii) it as proxies for absolute risk aversion for the model testing. Risk taking has commonly been negatively associated CEO tenure, age, gender and wealth. This risk-averse arises because of the non-diversified human capital investment. Two proxies are used for risk-aversion in the testing, CEO tenure and age. The CEO tenure is estimated by taking the difference between the year-end date, and the date they become CEO. Due to the low amount of executives and CEO's in opposite genders, risk aversion cannot be proxy by gender and the information on wealth of the executives are linked to compensation.

3.3.3 Hedge Ratio

In January 1997 the Financial Reporting Release No. 48 (FRR 48) was released which requires firms to disclose Derivative Instrument and Quantitative Information about market risk. As a result, this allows for the examination of hedging activities on the firm. The sampled firms typically have commodity hedges on Crude Oil, Natural Gas Liquid, and Natural Gas. This data is manually collected through the firm's 10-K that is available for download on Edgar. The 10-K's provide the types of derivatives used, the notion amount hedge and the production for the year. The 10-Ks separate different risks that are being hedged; to test the model, focus is placed on the commodity risk hedges that are measured in various units (Bcf, Mbbl, MMbtu, MMgal, Boe) that are manually converted to Millions of Barrel Equivalent(MMBoe). Crude Oil, Natural Gas Liquid, and Natural Gas are then combined before samples are taken thereafter. The samples are then winsorized to 1th percentile and 95th percentiles, resulting in 34 companies and 685 observations. The majority of the companies use both forwards and options, however, if companies were to choose only one method of hedging, it would be the linear contract (i.e. forwards). Tables 3.1 to Table 3.4 summarizes the number of firms that uses linear, non-linear, or both derivatives. The combined notional amount of different types of hedging is used to estimate the hedge ratio. The hedge ratio is estimated by taking the total volume hedged divided by the total volume produced.

$$Hedge\_ratio = \frac{total\_vol\_hedged}{total\_production}$$

Table 3.1: 2011

		Options		Total •
		No	Yes	
Forwards •	No	2	3	5
	Yes	7	22	29
Total		9	25	34

Table 3.3: 2013

		Options		Total •
		No	Yes	
Forwards •	No	1	1	2
	Yes	6	26	32
Total		7	27	34

Table 3.2: 2012

		Options		Total •
		No	Yes	
Forwards •	No	3	1	4
	Yes	6	24	30
Total		9	25	34

Table 3.4: 2014

		Options		Total •
		No	Yes	
Forwards •	No	4	1	5
	Yes	6	22	28
Total		10	23	33

3.3.4 Volatility

Price volatility is the simple average of crude oil and natural gas, as most companies in the sample produces both in various portions. This annualized price volatility was calculated for each year (2011 to 2014) by taking the standard deviation of the change in percentage daily spot prices multiplied by the square root of the trading days in any given year (assumed 252 days) for crude oil and natural gas. The price volatility in crude oil alone ranged from 23% to 35% between 2011 to 2014.

$$\% \triangle S = \frac{S_t - S_{t-1}}{S_{t-1}}$$

$$PriceVol = (\sigma_{\% \triangle S})(252^{1/2})$$

3.3.5 Controls and Variable Cost

S&P's Compustat North America Annual Fundamental Data contains company financial, statistical, and marketing information. The variable cost is estimated using the one minus gross margin of the company. The companies report total revenue and cost of good sold. The gross margin is calculated by taking the difference between the total revene and cost of good sold, this is then divided by the total revenue. The gross margin is a percentage of each dollar of revenue that the company retains as gross profit, while one minus gross margin is the variable cost. Therefore, the proxy for variable cost ('VC') was obtained by taking one minus gross margin.

$$VC = 1 - [\frac{Total\ Revenue - Cost\ of\ Good\ Sold}{Total\ Revenue}]$$

The data on location, firm size, leverage and capital expenditure 'Capexp' was extracted for each company, these were then used as control variables in the empirical model. The firm size would affect hedging as larger firms tend to hedge than more than small firms. This size effect is proxy by log total asset. Leverage affects firm's capital structure, which in turn may be related to its value. Leverage would be measured by debt in current liabilities plus long-term debt, scaled by total assets. Capex is the net capital expenditure scaled by total assets. The location of the company might also play a role in hedging ratio due to transportation costs, where a city variable would need to be included to control for this.

3.3.6 Summary Statistics

Below is the summary statistics of the variables.

Table 3.5: Summary Statistics:

Variables	Obs	Mean	Std. Dev.	Min	Max
hedge_ratio	111	0.4842	0.2657	0	1.0240
deltamean	115	202.2711	269.2205	6.4668	1720.6930
VarS	120	0.3345	0.0522	0.2471	0.3842
VC	112	0.4379	0.4134	0.1275	4.1980

3.4 Empirical Analysis of Model Prediction and Results

To test model predictions, the following empirical models was examined. The first prediction suggests that 'when incentive pay increases the optimal hedge ratio decreases'. This prediction is in line with previous studies. When incentive pay increases, the manager's pay is more linked to equity and options. With increase risk the optionvalue increases and it also gives an upside potential/benefit for equity holders. As a result, with more equity-based pay, the manager is more motivated to increase the risk of firm (because doing so will increase the manager's equity-based pay). To increase the risk of the firm, the manager will reduce the hedge ratio in the firm because less hedging will increase firm risk. If the prediction is accurate β_1 should be negative and significant in equation 3.4.1.

$$Hedge\_ratio_{it} = \alpha_i + \beta_1 \frac{\sum delta_{it}}{\#executives_{it}} + \beta_2 Controls_{it} + \varepsilon_{it} \tag{3.4.1}$$

Table 3.6: Empirical testing of Prediction 1

	Hedge_ratio	
Deltamean	-0.000165*	
	(-2.39)	
Delta		-0.0000457
		(-0.62)
City	0.0225**	0.0212*
	-2.71	-2.37
B2M	-0.140*	-0.107
	(-2.21)	(-1.62)
Size	-0.00496	-0.00433
	(-0.21)	(-0.18)
Leverage	1.426*	1.237
	(-2.14)	(-1.77)
Capexp	0.427	0.547
	(-1.35)	(-1.7)
Constant	0.378	0.333
	(-1.36)	(-1.14)
# Obs	95	86
R^2	0.201	0.161

t statistic in parentheses, *p<0.05,**p<0.01,***p<0.001

From Table 3.6 , deltamean is negative and significant. Therefore, the results from the empirical testing were in-line with the model prediction (i.e. when incentive pay increases the optimal hedge ratio decreases). This is because executives are often paid for the risk they take, and when some of the risks are hedged, the incentive pay would decrease. This is consistent with findings in Tufano (1996), Rogers (2002), Rajgopal and Shevlin (2002) Supanvanij and Strauss (2006) Chen, Jin, and Wen (2011).

The size effect and capital expenditure was not significant. The model did not produce the same result when the CEO's delta was examined in isolation. This could be because the CEO pay is very noisy as compared to the average delta across executives at the same firm, which would otherwise produce a much more reliable result. Consistent with Graham and Rogers (2002), the empirical model result show that firms with higher leverage will hedge more.

The second model prediction is: 'when price volatility increases the optimal hedge ratio decreases, and the compensation scheme will have a lower incentive portion'. Applying the empirical model below, if the prediction stands β_1 will be significant and negative.

$$Hedge\_ratio_{it} = \alpha_i + \beta_1 Volatility_t + \beta_2 Controls_{it} + \varepsilon_{it} \tag{3.4.2}$$

$$Hedge\_ratio_{it} = \alpha_i + \beta_1 \frac{\sum delta_{it}}{\#executives_{ti}} + \beta_2 Volatility_t + \beta_3 Controls_{it} + \varepsilon_{it} \tag{3.4.3}$$

From Table 3.7, it is observed that both the coefficients for delta and price volatility was significant and negative suggesting that our model prediction holds. When price volatility increases, the hedge ratio decreases. This is surprising as the hedging ratio would be expected to increase when the price is volatile. A possible explanation might be that the relationship between price volatility and average incentive pay (deltamean) is negative, suggesting that in a volatile price environment the company

Table 3.7: Empirical testing of Prediction 2

	Hedge_ratio			
Deltamean	-0.000169*	-0.0001695*		
	(-2.05)	(-2.04)		
PriceVol	-0.980*	-0.960*	-1.015*	-0.969
	(-2.18)	(-2.06)	(-2.27)	(-1.92)
Size	-0.00482	0.00486	-0.0105	-0.00982
	(-0.20)	(0.21)	(-0.52)	(-0.51)
Capexp	0.609*	0.678*	0.750**	0.727*
	(2.1)	(2.42)	(-2.89)	(-2.63)
Leverage		0.669	0.230*	1.524*
		(1.92)	(-2.26)	(-2.3)
City				0.0231**
				(-2.94)
B2M		-0.0230		-0.0804
		(-0.48)		(-1.40)
constants	0.744*	0.614*	0.711**	0.584*
	(2.55)	(2.08)	(-2.63)	(-2.11)
# Obs	106	102	107	96
R^2	0.136	0.183	0.143	0.224

t statistic in parentheses, *p<0.05,**p<0.01,***p<0.001

compensates the executive with less incentive pay. On the other hand, the relationship between hedge ratio and incentive pay is also negative, this therefore counteracts the effect of price volatility on hedge ratio. Price volatility varied significantly between 2011 and 2014, and the results may have been stronger if the data spanned over a longer period of time. In additionally, the relationship between price volatility and average incentive pay (deltamean) was negative but not significant in the empirical data.

Model prediction three postulated: 'when risk aversion increases, the optimal hedge ratio decreases and the compensation scheme will have a lower incentive portion'. The risk aversion of the executive cannot be observed as it is hard to proxy. However, as explained in prior sections, a CEO's tenure and age is used to proxy for this risk aversion. If the model prediction is correct, β_1 and β_2 and β_3 would be negative and significant.

$$Hedge\_ratio_{it} = \alpha_i + \beta_1 \frac{\sum delta_{it}}{\#executives_{ti}} + \beta_2 CEOtenure_{it} + \beta_3 age_{it} + \beta_4 Controls_{it} + \varepsilon_{it} \tag{3.4.4}$$

$$Hedge\_ratio_{it} = \alpha_i + \beta_1 delta_{it} + \beta_2 CEOtenure_{it} + \beta_3 age_{it} + \beta_4 Controls_{it} + \varepsilon_{it} \tag{3.4.5}$$

Measuring risk-aversion is difficult, here CEO tenure and age were used to proxy risk-aversion. Even though both proxies for risk aversion, CEO tenure and age, are significant, but the coefficient for CEO tenure is not as predicted by the model. According to the model prediction with increased risk aversion, there is a tendency for the hedge ratio to decrease, and compensation should have a lower incentive portion, thus the coefficient CEO tenure should be negative. This result might be due to the poor proxy for risk-aversion and minimum data available. In addition to CEO tenure, the mean tenure across executives in each companies were tested but yielded the same result. There is a negative relationship between 'Deltamean' and CEO tenure which suggests when risk aversion is high, there would be a lower incentive pay portion, this relationship was not statistically significant from the empirical data.

Additional to tenure, risk-aversion can also be proxy by wealth, however the wealth of the manager cannot be measured and part of the wealth was captured in the variable 'Deltamean' in the form of executive compensation.

Table 3.8: Empirical testing of Prediction 3

	Hedge_ratio	
Deltamean	-0.000207**	
	(-3.20)	
Delta		-0.000117*
		(-2.14)
Ceotenure	0.0151***	0.0180***
	(-3.45)	(-3.73)
Age	-0.0125*	-0.0134*
	(-2.18)	(-2.46)
City	0.00787	0.00761
	(-0.72)	(-0.69)
B2M	-0.124	-0.128
	(-1.92)	(-1.83)
Size	0.0457	0.0445
	(-1.68)	(-1.61)
Leverage	0.849	0.863
	(-1.15)	(-1.15)
Capexp	0.377	0.384
	(-1.2)	(-1.19)
Constant	0.676	0.721
	(-1.47)	(-1.64)
# Obs	86	86
R^2	0.277	0.268

t statistic in parentheses, *p<0.05,**p<0.01,***p<0.001

Finally, for the last model prediction:'when price variable cost increases, the optimal hedge ratio decreases, and the compensation scheme will have a higher incentive portion'. If this is the case, then β_1 should be significant and negative and β_2 should

Table 3.9: Empirical testing of Prediction 4

	Hedge_ratio		
Deltamean	-0.000231**		
	(-3.31)		
Delta		-0.0000508	
		(-0.69)	
VC	-0.186	-0.128	-0.0409
	(-1.16)	(-0.77)	(-0.30)
City	0.0139	0.0155	0.0191*
	(1.69)	(1.92)	(2.44)
B2M	-0.0882	-0.0681	-0.0529
	(-1.83)	(-1.25)	(-1.03)
Size	-0.0486	-0.0675*	-0.0447
	(-1.42)	(-2.01)	(-1.55)
Capexp	-0.458	-0.409	0.143
	(-1.12)	(-0.97)	(0.37)
Leverage	0.670	0.770	1.456*
	(1.00)	(1.15)	(2.19)
Constant	1.145**	1.216**	0.744*
	(2.79)	(2.92)	(2.01)
# Obs	82	82	92
R^2	0.2257	0.1866	0.1803

t statistic in parentheses, *p<0.05,**p<0.01,***p<0.001

be significant and positive.

$$Hedge\_ratio_{it} = \alpha_i + \beta_1 \frac{\sum Delta_{it}}{\#executives_{ti}} + \beta_2 VC_{it} + \beta_3 Controls_{it} + \varepsilon_{it} \tag{3.4.6}$$

From Table 3.9, the coefficient of the variable cost, $'VC'$, is negative which follows the model prediction that: 'the hedge ratio decreases when variable cost increases'. The coefficient of $'VC'$ is not significant in the regressions, however the relationship in general is in the correct direction and the significance could possiblity improve with additional years of data. An alternative measure of variable cost can also be explored, such as variable cost as a proportion of total cost, which should produce the same relationship with hedge ratio. The relationship between variable cost and average incentive pay (deltamean) is negative, which suggests that when variable cost is high, there would be a lower incentive pay portion (this relationship is also insignificant in the empirical data).

3.5 Conclusion

The theoretical model developed in Chapter 2 was empirically tested to validate model results. The theoretical model focuses on the linear financial derivatives setting, while the model prediction holds empirically with both linear and non-linear financial derivatives. In practice, managers select their level of effort and make the hedging decisions; therefore the theoretical model results comparing the observable case (best solution), to the hidden information case (second best solution), cannot be tested empirically due to the absence of a control group. With consideration of empirical data from oil and gas companies, the testable results of the theoretical model suggest that (i) when incentive pay increases, the optimal hedge ratio decreases, (ii) when price volatility increases, the optimal hedge ratio decreases, with a negative relationship between price volatility and equity-based incentive, (iii) when risk aversion increases, the optimal hedge ratio decreases, with a negative relationship between risk aversion and equity-based incentive, and (iv) when variable cost increases, the optimal hedge ratio decreases, with a negative relationship between variable cost and equity-based incentive.

This chapter finds empirical evidence supporting these predictions. First, the empirical test shows that there is a negative relationship between hedging and incentive pay, which supports the prediction that when incentive pay increases, the optimal hedge ratio decreases. Second, the paper finds a negative relationship between hedging and volatility, with a negative relationship between incentive pay and volatility. This is in line with the second prediction when price volatility increases, the optimal hedge ratio decreases. Third, the empirical results shows a negative relationship between hedging and risk aversion when using age as a proxy, but a positive relationship when using tenure as a proxy. In the previous literature, age and tenure has been used as a proxy, however the low evidence on the relationship between hedging and risk aversion can be attributed to poor proxy. There is also a negative relationship between incentive pay and risk aversion that supports the theoretical prediction. Lastly, the results show a negative relationship between hedging and variable cost, and a negative relationship between incentive pay and variable cost. The empricial model presents endogeneity issues as all regressions are performed with contemporaneously observed variables. To reduce the impact of endogeneity, independent variables can be lagged. However, the empirical models in this chapter were mainly used to test the theoretical model and the relationship between the variables.

Chapter 4

Ontario Energy Prices Analysis: A Convolution Approach

4.1 Introduction

This chapter introduces a new approach for modelling and forecasting the West Texas Intermediate (WTI) crude oil prices. We estimate the WTI crude oil prices using a convolution model of two stationary processes and, alternatively, a mixed causal-noncausal autoregressive (MAR) model. This approach allows us to accommodate the multimodality, heavy tails and various asymmetric local trends observed in the WTI data distribution, which was not explored in previous literatures. The long-run relationships between the Ontario consumer price index (OCPI), Ontario energy price index (OEP) and the WTI series are established by using the cointegration analysis. Then, we can use the WTI crude oil price forecasts to predict the Ontario price indexes (OCPI and OEP) by exploiting the long-run relationships between the Ontario consumer and energy price indexes.

To illustrate the dynamics of comovements between the OEP, OCPI and WTI, Figure 4.1 displays each monthly series recorded between January 2000 to December 2018[7], where the WTI series is expressed in US Dollars and the price indexes are expreseed in Canadian Dollars with the base year 2002:

[Insert Figure 4.1: Dynamics of OCPI, OEP and WTI crude oil price]

In Figure 4.1 we observe that the OCPI series is upward trending and smooth, while

[7]https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1810000413

the OEP series shows both a global trend and local trends and spikes. In general, the global trend of the OEP series is parallel to that of the OCPI and the local trends and spikes in both the series of OEP and OCPI indexes seem to occur at the same time as those observed in the trajectory of the WTI crude oil prices (WTI). The WTI series does not display a global trend, but is characterized by local trends and spikes.

The first contribution of this chapter is to model and forecast the WTI crude oil prices and estimate the long-run relationships between the Ontario price indexes (OCPI and OEP), adjusted for the exchange rate, and the WTI crude oil prices. The cointegration analysis provides the forecasts of the Ontario price indexes and WTI crude oil prices. Alternatively, the forecasts of Ontario price indexes can be obtained as functions of the WTI crude oil price forecasts combined with the forecast of the stationary series of departures from the long run equilibrium.

In addition, the WTI crude oil price process is modelled as a convolution of stationary causal and noncausal autoregressive processes. This model is parsimonious and accommodates the local trends and spikes in the trajectory of the oil price process as well as the bi-modality of its sample distribution. The bi-modality in the distribution of the WTI crude oil price data evidenced in this paper seems to be disregarded in the existing literature.

The forecast of the WTI crude oil prices can also be obtained from the mixed causal -noncausal model of WTI crude oil prices. We explore this approach, which allows us for comparing two methods of forecasting for noncausal processes developed by Lanne, Luoto and Saikkonen (2012) and Gourieroux and Jasiak (2016). A comparison of these two methods in an empirical application is an additional contribution of this paper. A simulation-based comparison of the two forecasting methods for noncausal processes is given in Voisin, Hecq (2019). Given the WTI crude oil price forecast and the estimated long-term relationships, the Ontario price indexes can be forecast as well, as functions of the WTI crude oil price forecast.

There exists a large body of literature on oil price forecasting. A commonly used heuristic model for oil price prediction is the no-change model, which is also used in the literature as a benchmark for comparison of forecast performance.

For example, the existing oil price forecasting literature provides evidence that the structural models of real oil prices outperform the no-change forecast at short horizons (Alquist, Kilian, and Vigfusson, 2013 and Baumeister and Kilian, 2012, 2014, 2015, among others). The forecast combination model proposed by Baumeister and Kilian (2015), includes six models, in which only four models appears to be essential to price prediction: global oil market VAR model, model based on non-oil commodity prices, model based on oil future spreads and time-varying product spread model. They construct an inverse mean squared prediction error MSPE weight based on recent forecasting performance of each model. They then allocate larger weight to models in the combination forecasts that have smaller MSPE date t. The model use a predictor's lags of the real price of oil, current oil spot prices and oil future prices, current spot prices in the market for refined products and current and lagged data on economic fundamentals. They find placing equal weights on all forecasting models produces the most accurate forecasts. In particular, forecasts obtain by the combination forecasts from (i) global oil market VAR model, (ii) model based on non-oil commodity prices, (iii) model based on oil future spreads, and (iv) time-varying product spread model are systemically more accurate than the no change forecast at horizon 1 month to 18 months.

Li, Xu and Tang (2016) introduce sentiment analysis, a useful big data analysis tool, to understand the relevant information of on-line news and formulate an oil price trend prediction method with sentiment.

Gao and Lei (2017) propose a novel approach for crude oil price prediction based on a machine learning paradigm called stream learning. The main advantage of the stream learning approach is that the prediction model can capture the changing pattern of oil prices since the model is continuously updated whenever new oil price data are available. In the literature, a model trained with artificial neural networks

(ANN), is a classical machine learning model for oil price prediction (Yu et al., 2008; Kulkarni and Haidar, 2009). Li, Shang and Wang (2019) apply deep learning techniques to crude oil forecasting, and to extract hidden patterns within online news media using a convolutional neural network (CNN). Chen, He and Tso (2017) use the deep learning model to capture the unknown complex nonlinear characteristics of the crude oil price

Snudden (2018) uses a high-order VAR (p=24) and proposes the method of targeted growth rate filtering using spectral analysis , which is a modification of the standard forecasting method. The lags in growth rate transformations are chosen in order to target lower frequencies. The method removes high frequencies and emphasizes certain low frequencies which correspond to particular forecast horizons. Li, Xu and Tang (2016) introduce sentiment analysis, a useful big data analysis tool, to understand the relevant information of on-line news and formulate an oil price trend prediction method with sentiment.

This paper is organized as follows. Section 4.2 describes the data. Section 4.3 examines the comovements between the Ontario price indexes and the cointegrating relationships between the WTI crude oil prices and the Ontario price indexes. Section 4.4 presents the convolution model and the alternative mixed causal-noncausal autoregressive (MAR) specifications. Section 4.5 discusses the forecasting from MAR models and the application of the available methods. Section 4.6 concludes the paper. Additional results are provided in Appendices 1 and 2.

4.2 Data Description

This section describes the three series of 228 monthly observations on the OCPI, OEP and the WTI crude oil prices observed over the period January 2000 to December 2018.

Ontario CPI and Ontario Energy Price Index

The data on OCPI and OEP are provided by Statistics Canada [8]. Our sample contains monthly data from January 2000 to December 2018, which is seasonally non-adjusted and consists of 228 observations.

The OCPI compares the cost of a static or equivalent quantity and quality fixed basket of goods and services purchased by consumers, therefore the OCPI reflects a pure price change indicating the general level of inflation. The goods and services' price movements in the OCPI are weighted according to the relative importance to the total expenditures of consumers. Each good or service is an element in the basket, and price movements of these elements are assigned a share in the basket that is proportional to the total consumption expenditure. Amongst the elements in

[8] The source of monthly data is the Statistics Canada website: https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1810000401 from table 18-10-0004-01 (formerly CANSIM 326-0020)

the 'basket of goods' is energy. The OCPI index measures the inflation based on the price increases of goods in a predetermined 'basket of goods'. Consumers typically switch between products as the relative prices of goods change. Thus, the price index is based on a fixed-basket as opposed to the cost-of-living.

The base period of the CPI was chosen as 2002, which means the CPI in 2002 is set equal to $100. The change allows the percentage changes between any two periods to remain the same as it is an arithmetic conversion, which alters the index levels.

The OCPI series shows a global upward sloping trend in Figure 4.1.

The aggregate Ontario Energy Price Index is one of the components of OCPI which includes: 'electricity', 'natural gas', 'fuel oil and other fuels', 'gasoline', and 'fuel, parts and accessories for recreational vehicles'. All the prices listed within energy are also consumer prices. The fuel oil data is collected at least once a year. The energy does not seem to include transportation cost, as it is listed as a separate element of OCPI. The base year for Ontario Energy Price Index is 2002, which is consistent with the base year for OCPI.

The dynamic of the OEP series shows a global upward sloping trend and local trends and spikes in Figure 4.1.

WTI Crude Oil Prices

The monthly WTI crude oil price data from January 2000 to December 2018 (in US dollars per barrel) are provided by the U.S. Energy Administration (EIA) [9]. The WTI crude oil is produced in Texas and Southern Oklahoma. It is often used as a benchmark for oil pricing and serves as a reference point for pricing a number of other crude streams traded in the Cushing, Oklahoma spot market. The price of oil is the market spot price, i.e. the price of a one time open market transaction for immediate delivery of a specific quantity of product at a specific location where the commodity is purchased "on the spot" at a current market rate. The monthly data provided by EIA are computed as the unweighted average of the daily closing spot prices over a specific month.

The oil series shows two significant troughs due to considerable crude oil price drops in years 2008 and 2014. The first price drop in 2008 coincides with the beginning of the economic recession tiggered in part by the Lehman Brothers bankruptcy filing on September 15,2018. This economic recession caused strain in the banking system and the demand for oil fell. The Organization of the Petroleum Exporting Countries (OPEC) subsequently announced a 16 percent reduction in production over 8 months with the intent to help stabilize oil prices.

The second price drop occured after Thanksgiving 2014, following the period

[9] https://www.eia.gov

between January 2011 and June 2014 when oil prices were relatively stable. A decline in oil prices started in June 2014 due to an increased supply of oil prompted by a technological advancement that enabled the "shale" oil production. However, the increased supply was not matched by the anticipated increase in oil demand. At the same time, OPEC countries maintained their output at a constant level, thus increasing the overall supply. In the second quarter of 2014, OPEC changed its policy and decreased the production, causing a drop in oil prices, which is observed in the dataset after October 2014.

The monthly OCPI and OEP Index data from January 2000 to December 2018 are expressed in Canadian dollars. In order to establish the relation between the series and the WTI crude oil expressed in US Dollars, we need to use a common currency of reference. Therefore, we adjust the OEP and OCPI indexes for the CAD/USD exchange rate to make them comparable.

Exchange Rate

The exchange rate is obtained through the Bloomberg terminal "USDCAD BGN Curncy" series. The Bloomberg series of exchange rates is displayed in Figure 4.12, Appendix 2.

The monthly exchange rates are representative of the Bloomberg Generic Composite rate (BGN). It is a representation based on indicative rates contributed by market participants. The data is not based on any actual market trades. BGN is a pricing algorithm that produces indications of bid and ask quotes that are derived from hundreds of quality sources, including indicative and executable price quotes from money-center and regional banks, broker-dealers, inter-dealer brokers, and trading platforms. BGN is designed to track executable bid/ask rates and to be resistant to manipulation by market participants. To adjust for the difference in currency between OCPI, OEP and WTI crude oil price data we used the monthly mid price exchange rate from Bloomberg BGN rates between January 2000 to December 2018 . The Mid Price is the average of the Bid Price and Ask Price. If there is no bid value or ask value provided, the mid price will simply be the value provided.

The exchange rate series does not have a global trend. It shows a local downward sloping trend in the first part of the sampling period and a local upward trend in the second part of the sampling period. It also shows a spike around the crisis of 2008.

Exchange Rate Adjusted Series

The exchange rate adjusted series of OCPI and OEP indexes are displayed in Figure 4.2 below along with the WTI crude oil prices:

[Insert Figure 4.2: Dynamics of transformed OCPI, OEP and WTI crude oil price]

After adjusting for the exchange rate, the dynamics of OCPI and OEP series change. The global upward trends in these series have been flattened because of the patterns revealed in the exchange rate dynamics. The most noticeable changes are in the OCPI dynamics. The OCPI series no longer has a smooth trajectory resembling a linear trend and becomes more volatile with local trends. Moreover, the local trends in the OCPI and the OEP series occur at the same time as those in the oil price series. The global trends in OEP and OCPI series remain parallel, as before the adjustment for exchange rates.

The QQ-plot in Figure 4.13 shows that the sample quantiles of OCPI differ from those of a normally distributed stationary variable. This could be due to the fact that the series is either non-normally distributed, or non-stationary, or both. More specifically, either the sample quantiles have not converged to the quantiles of the limiting distribution, causing deviations from the line, or they have converged and the deviations are caused by the non-normal distribution of the series, such as the multimodality and thin or thick tails. The QQ-plot in Figure 4.14 shows that a similar conclusion can be drawn for the OEP series as well. The next section examines the stationarity of the series.

4.3 Dynamic analysis

Let us now examine the dynamics of the adjusted series. The OCPI and OEP series display parallel global trends, which suggest nonstationarity due to a possible presence of a unit root. The WTI series does not have a global trend and features local trends and spikes. Nevertheless, in the early time series literature this behaviour was called "meandering" and attributed to unit root dynamics as well.

A simple method of testing for stationarity can be linked to the test of normality, as suggested in the previous Section. This method is used in the natural sciences. For example, the stationarity of encephalographic signals is often inferred from the results of the Shapiro–Wilk test for Gaussianity [see Bender et al. 1992].

Let us now test the variables of interest for normality by using the Shapiro-Wilk test and its simplified version the Shapiro-Francia test. The *implicit* null hypothesis is that each series is normally distributed and stationary while the *implicit* alternative hypothesis is that each series is either non-normally distributed or non-stationary. Tables 4.1 and 4.2 below show the outcomes of the tests.

Table 4.1: Shapiro-Wilk W' Test

	Obs	W'	V'	z	Prob $> z$
OCPI	228	0.92486	12.568	5.862	0.00000
OEP	228	0.94285	9.560	5.229	0.00000
Oil	228	0.95488	7.547	4.681	0.00000

Table 4.2: Shapiro-Francia W' Test

	Obs	W'	V'	z	Prob $> z$
OCPI	228	0.92930	12.878	5.329	0.00001
OEP	228	0.94715	9.625	4.722	0.00001
Oil	228	0.95770	7.704	4.258	0.00001

Both tests reject the null hypothesis in the OCPI, OEP and WTI crude oil prices at the significance level 1%. This outcome is explored further to determine whether the series are non-stationary.

Unit Root Tests

We proceed to test each series for unit root, using the DF (Dickey-Fuller) and ADF (Augmented Dickey Fuller) tests. The null hypothesis H_0 is that the given series has a unit root and the alternative H_A is that the series is stationary. Tables 4.3, 4.4 and 4.8 below test the null hypothesis $H_0 : \gamma = 0$ against $H_A : \gamma < 0$ in the three following specifications:

$$\begin{aligned} \Delta p_t &= \gamma p_{t-1} + v_t \\ \Delta p_t &= \mu + \gamma p_{t-1} + v_t \\ \Delta p_t &= \mu + \delta t + \gamma p_{t-1} + v_t \end{aligned}$$

where t is a sequence of time indexes, $t = 1, ..., 228$, p_t is the series of price indexes or prices considered, and process v_t is a strong White Noise, with finite moments of order 4, which may not necessarily be Normally distributed.

The results of the Dickey Fuller test with no constant for the three series are given in Table 4.3 below:

Table 4.3: Dickey Fuller Test with no constant

	Obs	coefficient	τ
OCPI	227	0.0009	0.487
OEP	227	0.0002	0.057
Oil	227	-0.0025	-0.459

Note: All three series exhibit unit roots

The results of the Dickey Fuller test with a constant for the three series are given in Table 4.4 below:

Table 4.4: Dickey Fuller Test with Constant

	Obs	constant	coefficient	τ
OCPI	227	1.73285	-0.0163793	-1.732
OEP	227	2.80832	-0.0214907	-1.876
Oil	227	1.605374	-0.0242862	-1.788

All three series exhibit unit roots

Under the null hypothesis, the t-ratio is non-standard and its asymptotic distribution is non-normal. Therefore, it is denoted by symbol τ. It is compared with the critical values of the unit root test. We find that the unit root hypothesis cannot be rejected in all three series at the significance level 5%. The results of the ADF with constant are given in Tables 4.5, 4.6, 4.7 below:

Table 4.5: Augmented Dickey Fuller OEP

Constant	Coefficient L1	Coefficient LD	Coefficient L2D	Coefficient L3D	τ	Observations
2.704	-0.022	•	•	•	-1.876	227
(1.96)	(-1.88)	•	•	•		
2.760	-0.022	0.119	•	•	-1.932	226
(1.99)	(-1.93)	(1.79)	•	•		
2.807	-0.023	0.110	0.079	•	-1.966	225
(2.00)	(-1.97)	(1.65)	(1.18)	•		
2.987	-0.024	0.115	0.085	-0.031	-2.052	224
(2.11)	(-2.05)	(1.72)	(1.26)	(-0.46)		

τ-statistic is reported in the parenthesis.

Table 4.6: Augmented Dickey Fuller OCPI

Constant	Coefficient L1	Coefficient LD	Coefficient L2D	Coefficient L3D	τ	Observations
1.733	-0.016	•	•	•	-1.732	227
(1.86)	(-1.73)	•	•	•		
1.730	-0.016	-0.061	•	•	-1.701	226
(1.84)	(-0.92)	(-1.70)	•	•		
1.739	-0.016	-0.058	0.053	•	-1.705	225
(1.83)	(-1.71)	(-0.86)	(0.80)	•		
1.831	-0.017	-0.054	0.050	-0.057	-1.767	224
(1.91)	(-1.77)	(-0.81)	(0.75)	(-0.84)		

τ-statistic is reported in the parenthesis.

Table 4.7: Augmented Dickey Fuller WTI Crude Oil Prices

Constant	Coefficient L1	Coefficient LD	Coefficient L2D	Coefficient L3D	τ	Observations
1.605	-0.024	•	•	•	-1.788	227
(1.75)	(-1.79)	•	•	•		
1.957	-0.030	0.394	•	•	-2.443	226
(2.29)	(-2.44)	(6.39)	•	•		
2.128	-0.034	0.359	0.099	•	-2.637	225
(2.46)	(-2.64)	(5.43)	(1.46)	•		
2.111	-0.033	0.364	0.124	-0.070	-2.535	224
(2.41)	(-2.54)	(5.49)	(1.74)	(-1.03)		

τ-statistic is reported in the parenthesis.

The test results suggest that the null hypothesis of a unit root cannot be rejected at the significance level 5%. The outcomes of the DF tests with a constant and trend are given in Table 4.8:

Table 4.8: Dickey Fuller Test with Trend

	constant	trend	coefficient	τ	Obs
OCPI	1.754093	0.0001633	-0.0167924	-1.193	227
	(1.63)	(0.04)	(-1.19)		
OEP	2.806635	-0.0000289	-0.0214495	-1.315	227
	(1.86)	(-0.00)	(-1.32)		
Oil	1.656705	-0.0011774	-0.0229507	-1.494	227
	(1.73)	(-0.19)	(-1.49)		

t-statistic is reported in the parenthesis. All three series exhibit unit roots

Table 4.9 shows the outcomes of the ADF test with a trend.

Table 4.9: Augmented Dickey Fuller Test with Trend

	constant	trend	coefficient L1	coefficient LD	coefficient L2D	τ	Obs
OCPI	1.745804	0.0000498	-0.0165538	-0.0575078	0.0535956	-1.151	225
	(1.59)	(0.01)	(-1.15)	(-0.85)	(0.79)		
OEP	3.035848	0.0039103	-0.0286789	0.1151561	0.0843935	-1.719	225
	(2.05)	(0.49)	(-1.72)	(1.70)	(1.24)		
WTI Oil	1.96958	0.0036452	-0.0379302	0.3617519	0.1025899	-2.617	225
	(2.18)	(0.62)	(-2.62)	(5.45)	(1.51)		

t-statistic is reported in the parenthesis. All three series exhibit unit roots

The results shows that we cannot reject the null hypothesis at the significance level 5% either. In the WTI series, the unit root hypothesis can be rejected at level 10%.

Common Trends Analysis

The evidence of unit root tests suggests that all series have a stochastic global trend.

For the OEP and OCPI series, this result is consistent with the patterns displayed in Figure 4.2. The OEP and OCPI series have a parallel upward sloping global trend, resembling the dynamics of a pair of cointegrated variables. The presence of the common trend follows from the construction of the price indexes and the fact that the energy prices are included in the OCPI. Technically, the presence of unit roots in both price indexes suggests that we can test for the common stochastic trend. If the presence of the common stochastic trend is not rejected, we can establish a long term relations between the Ontario price indexes using the cointegration approach.

To estimate the long-run relation between the OEP series and the consumer price index OCPI series, we regress the OEP on OCPI.

$$OEP_t = \beta_0 + \beta_1 OCPI_t + e_t$$

$$\widehat{OEP}_t = -39.047 + 1.610 OCPI_t \tag{4.3.1}$$

The regression coefficient β_1 has a standard error of 0.021 with a t-ratio of 75.82. To check if the above regression represents a long-term relation rather than a spurious regression, we examine the residual series , displayed in Figure 4.3.

[Insert Figure 4.3 : Departures from long-term relation of OCPI and OEP]

The residual seems stationary over time. To confirm this observation, we use the cointegration test. The cointegration test performed on the residual, using the Engle-Granger approach consists in testing the null hypothesis of the residuals being non-stationary i.e. $H_0 : \gamma = 0$ against $H_A : \gamma < 0$ that the residuals are stationary. The test is based on the model:

$$\Delta e_t = \gamma e_{t-1} + v_t$$

where v_t is a assumed to be a strong White Noise and the residuals $\hat{e}_t$ of equation (4.3.1) are used as the proxies of the true errors e_t. The Engle-Granger critical value at the significance level 5% is -3.35, while the τ statistic for the γ coefficient, is

-4.005. The results in Table 4.10 show that the regression of OEP on OCPI is valid and represents a long-run relation.

Table 4.10: Cointegration Test

	Coefficient	τ
Regression	-0.1412	-4.016

Residuals are stationary at probability level 5%

Let us now examine the relations between the Ontario price indexes and WTI series. The evidence on the presence of a unit root in the WTI series is more difficult to interpret. As shown in Figure 4.2 , the WTI series does not display a global trend. Instead, it has short-lived local trends. As the local trends are short-lived, the dynamics of the WTI does not resemble the behavior of stock prices or interest rates, which are known to be non-stationary random walk processes. The WTI resembles rather the behavior of the commodity price processes, which are modelled in the recent literature as stationary processes with non-causal components.

We know that both Ontario price indexes are based on baskets that include oil and depend on oil prices. As the weight of oil in the basket varies over time and is consumer rather than producer price based, the econometric relationship in the long run will reflect a complex accounting relationship. The evidence of a unit root suggests that we can regress the price indexes on the WTI, and search for the presence of a common trend. The problem is how to reconcile the lack of a global trend in the WTI with the upward global trends in the price indexes.

We first consider the relationship between the exchange rate adjusted OEP and the WTI crude oil prices and regress the OEP on WTI. Due to the difference between the global trends of OEP and WTI this relationship is invalid as the residuals of the regression are non-stationary. More specifically, the residuals display an upward sloping trend, displayed in Figure . The tests does not reject the unit root in residuals, given the value of $\tau = -1.80$.

In order to "lift up" the WTI series, we can add a deterministic trend to the linear relationship, by analogy to adding a deterministic trend to a cointegrating relation. The regression estimated with a linear (deterministic) trend is:

$$OEP_t = \beta_0 + \beta_1 t + \beta_2 WTI_t + u_t$$

$$\widehat{OEP}_t = 41.6163 + 0.1700t + 0.8905WTI_t \tag{4.3.2}$$

The regression coefficient β_1 has a standard error of 0.007907 and t-ratio of 21.51. The regression coefficient β_2 has a standard error of 0.019468 and t-ratio of 45.74. We perform the cointegration test on the residuals from equation (4.3.2) by estimating

$$\Delta u_t = \gamma u_{t-1} + v_t$$

and testing $H_0 : \gamma = 0$ with residuals $\hat{u}_t$ used as the proxies of errors u_t. Table 4.11 below shows the results. We find that the τ statistic exceeds the Engle-Granger adjusted critical value at 5%. Accordingly, we can reject the unit root in the residuals at level 5%. The residuals of regression (3.2) displayed in Figure 4.4 below are stationary and the regression can be considered as a long-run relation between the OEP and WTI series with a deterministic trend.

Table 4.11: Cointegration Test

	Coefficient	τ
Regression	-0.19732	-4.818

Residuals are stationary at probability level 10%

[Insert Figure 4.4 : Rgression of OEP on oil prices - Residuals]

To compare the differences in global trends between the price indexes and the WTI, we also regress the OCPI series on the WTI. Without a deterministic trend, this regression is not valid either. The residuals of the regression of OCPI on WTI without a deterministic trend displayed in Figure 4.6 show an upward sloping trend.[10] The cointegration tests does not reject the unit root in the residuals with $\tau = -2.32$.

The residuals of equation (4.3.1) $\hat{e}_t$ [resp. residuals of equation (4.3.2) $\hat{u}_t$] can be interpreted as a series of departures of the OEP and OCPI [resp. OEP and WTI] series from the long run relations given in equation (4.3.1) [resp.(4.3.2)]. The sample densities of these residuals are estimated and plotted in Figures 4.15, 4.17, Appendix 2. The qq-plots are given in Figures 4.16 and 4.18, Appendix 2. We observe that the

[10]The normality of errors is not required for the validity of cointegration analysis.

residuals have distributions rather close to the Normal, although residual $\hat{e}_t$ seems to display some departures from Normality due to the thick right tail. Residual $\hat{u}$ shows slight asymmetry in its distribution and a thin right tail.

The absence of a global trend in the WTI series and the presence of short-lived local trends suggests that this series may indeed be stationary with a noncausal dynamic, which is explored later in the text. The forecast of the stationary noncausal WTI series would then be available and could be exploited to forecast the Ontario price indexes.

Forecasting

The stationary residual series $\hat{e}_t$ and $\hat{u}_t$ are autocorrelated and can be estimated by an ARMA model. These series represent stationary combinations of components displaying various types of trend: global and local in terms of duration and stochastic, and deterministic in terms of nature.

Tables 4.17 and 4.18 in Appendix 3 provide the estimates of simple ARMA(p,q) models fitted to the series of residuals from equations (4.3.1) and (4.3.2). The autoregressive AR(2) models are found to provide the best fit for both series. Hence, both series of residuals can be easily forecast using standard software, such as SAS or STATA.

Then, the forecast of the WTI crude oil prices combined with the forecast of residual (4.3.2) provides a forecast of OEP as follows:

Let $\widehat{WTI}_{T+1}$ denote the one step ahead out of sample forecast of the WTI crude oil price and $\hat{u}_{T+1}$ denote the one step ahead forecast of the error term provided by the AR(2) model. We can use equation (4.3.2) to obtain the forecast of the energy index based on the forecast of WTI crude oil price:

$$\widehat{OEP}_{T+1} = 41.6163 + 0.1700(T+1) + 0.8905\widehat{WTI}_{T+1} + \hat{u}_{T+1}$$

where et $\widehat{OEP}_{T+1}$ denotes the one step ahead out of sample forecast of the Ontario energy index. In practice, the regression parameters can be estimated locally by rolling to capture the potential changes in the slope of the trend function:

$$\widehat{OEP}_{T+1} = \theta_{0,T} + \theta_{1,T}(T+1) + \theta_{2,T}\widehat{WTI}_{T+1} + \hat{u}_{T+1}$$

where $\theta_{i,T}, i = 0, 1, 2$ denote the coefficients updated by rolling. This method can be used for short term forecasting.

Next, we can forecast the OCPI series from equation (4.3.2) one step ahead out of sample using the predicted energy index and residual $\hat{u}_{T+1}$ as follows:

$$\widehat{OCPI}_{T+1} = 24.2528 + 0.621118\widehat{OEP}_{T+1} - 0.621118\hat{e}_{T+1}$$

.

The above forecasts are not optimal in the sense that $\widehat{OEP}_{T+1}$ and $\widehat{OCPI}_{T+1}$ are not equal to their conditional expectations, given the past values of the right hand side series. However, the forecast is unbiased as the expected forecast error is 0. The forecast error variance depends on the quality of the component forecasts and needs to be examined in a simulation study.

The proposed approach resembles a simple neural network that combines two layers of regressions. It is a simplified alternative to the Vector Error Correction (VEC) model presented below.

The VEC Model

The specifications of the VEC model examined in this study are based on the insights on common trends given earlier in this Section. The bivariate model of price indexes with the current value of WTI as an explanatory variable provides the best forecasts of Ontario price indexes. The VECX(2) model is specified as follows:

$$\Delta p_t = const + \Pi p_{t-1} + \Phi \Delta p_{t-1} + WTI_t + \epsilon_t$$

where p_t contains the series of OEP_t and $OCPI_t$, matrix $\Pi = \alpha\beta'$ is of rank 1 and the errors are a white noise process with finite moments up to order 4. The parameters $\alpha_1 = 0.513, \alpha_2 = 0.157$ are both significant at 5%. The cointegrating vector $\beta' = [-0.927, 1]$ under the normalization with respect to OCPI. The coefficients on the

WTI, as well as the lagged first difference of OEP, are statistically significant. The detailed estimation results are provided in Appendix 3.

The out-of sample forecasts of Ontario price indexes at horizons from 1 up to 5 months are as follows:

Var.	Obs	Forecast	St. Error	95% Conf. Int.	Actual	Residual
oep	224	128.553	4.641	119.455 , 137.650	128.139	-0.413
	225	128.363	5.929	116.742, 139.985	127.514	-0.848
	226	128.404	6.591	115.484, 141.323	122.140	-6.263
	227	125.888	7.037	112.094, 139.682	114.654	-11.233
	228	122.549	7.425	107.994, 137.104	108.089	-14.460
ocpi	224	104.166	2.612	99.046, 109.286	104.213	0.047
	225	104.220	3.398	97.560, 110.881	104.738	0.518
	226	104.289	4.049	96.353, 112.225	103.063	-1.226
	227	103.655	4.612	94.614, 112.697	101.639	-2.016
	228	102.933	5.124	92.890, 112.976	98.996	-3.937

The presence of WTI_t improves the forecasts of price indexes. We observe that the forecasts of OEP_t worsen quickly with the forecast horizon, while the forecast of $OCPI_t$ remains quite reliable.

Next, we model jointly the two Ontario price indexes and the WTI series. Under this approach the three series satisfy a cointegrating relation with a linear trend.

The model is the following VEC(2):

$$\Delta pw_t = const + \alpha(\beta', \beta_1)(pw'_{t-1}, t)' + \Phi \Delta pw_{t-1} + \epsilon_{wt},$$

where pw_t denotes the vector containing OEP_t, $OCPI_t$ and WTI_t, t denotes the linear trend, matrix Π is of rank 1 and the errors are a white noise process with finite moments up to order 4. The coefficients in vector α are 0.333, 0.089 and 0.461. The α coefficients on OEP and WTI are statistically significant at 5%, while coefficient α_{OCPI} is not, due to its p-value of 0.06. The coefficients in the cointegrating vector β are -0.483, 1, -0.122, under the normalization with respect to OCPI. The parameter estimates are given in Appendix 3.

This trivariate VEC(2) specification provides the best forecasts of the WTI series up to horizon 3, as compared with the VEC(2) model with matrix Π of rank 2 and a linear trend, which provides better forecasts of WTI at horizons 4 and 5 and is the second best WTI forecast provider. The forecasts of the Ontario prices obtained from both trivariate VEC(2) models are worse than those obtained from the VEXC(2) model with the WTI as an explanatory variable. However, the price indexes help forecast the WTI.

The out-of-sample WTI forecasts at horizons 1 up to 5 from the trivariate VEC model with 1 cointegrating vector and a linear trend are as follows:

Obs	Forecast	St. Error	95% Conf. Int.	Actual	Residual	er(NC)
224	70.946	4.575	61.978, 79.913	68.060	-2.886	-2.92
225	69.549	7.502	54.845, 84.253	70.230	0.680	-0.75
226	67.820	9.691	48.826, 86.815	70.750	2.929	-0.23
227	66.223	11.375	43.927, 88.519	56.960	-9.263	-14.02
228	64.909	12.759	39.901, 89.917	49.520	-15.389	-21.46

The last column shows the errors of a forecast based on the "no-change" (NC) method, which assumes that the best forecast of a future value is the last observed one. We find that the VEC-based forecasts outperform the NC forecast one-step ahead and in terms of the forecast MSE up to horizon 5. The MSE of the VEC forecast is 67.915, while the MSE of the NC forecast is 133.246.

4.4 Estimation of Dynamic Oil Price Models

The unit root analysis suggests potential nonstationarity of the WTI crude oil price series. However, the WTI series does not have an explosive behaviour, although its "meandering" can be interpreted as a nonstationary pattern due to a unit root, according to the early time series literature. More recent literature however [see e.g Perron (1989)] has revealed that the unit root tests have low power in applications to processes with level shifts. In Section 2, we pointed out two major declines in the oil prices, which may have led to spurious results of unit root test. Moreover, the unit root tests are flawed in applications to stationary recurrent processes with non-normal distributions that display local trends [see, Gourieroux and Jasiak (2018)]. This is because the unit root test does not distinguish between the stationary processes with local trends and nonstationary processes with global trends. Hence, the unit root tests tend to accept the null hypothesis in the presence of either type of trend.

In this section, we examine various models to find a suitable stationary representation for the WTI crude oil price process. Let us first examine its distributional properties while assuming its stationarity.

Table 4.12: Summary Statistics

	count	mean	min	max	sd	skewness	kurtosis
WTI	228	62.010	19.39	133.88	26.79109	0.344	2.141

Table 4.13: Summary Statistics - quantiles of empirical density

	1%	5%	10%	25%	Median	75%	90%	95%	99%
WTI	19.72	26.43	28.39	38.56	59.175	84.96	100.54	104.67	125.4

The summary statistics in Tables 4.12 and 4.13 show that the sample mean of WTI is 62.01, and the median is 59.17, respectively. The standard deviation of WTI is 26.79 and the range is 114.49. respectively. Figure 4.7 below shows the histogram of the Oil Price.

[Insert Figure 4.7: Histogram: WTI Crude Oil Price]

The sample distribution reveals a bimodal pattern. We compare this distribution to the Normal distribution in the quantile plot (QQ-plot) displayed in Figure 4.8 below.

[Insert Figure 4.8: Crude Oil - QQ Plot]

Figure 4.8 reveals departures of the WTI crude oil price from Normality due to heavy tails, despite the low sample kurtosis value. Such a contradictory outcome can arise in multimodal distributions. These finding motivates us to fit to the WTI crude oil prices a stationary mixture model, to accommodate both the multimodality, and heavy tails displayed in Figure 4.9 below:

[Insert Figure 4.9: Crude Oil - Hill Estimator]

Below, we explore the fit of the mixed causal-noncausal MAR models that can accommodate various asymmetric local trends. As an alternative mixture specification, we also explore a convolution model with causal and noncausal components.

All estimations reported below concern the process $y_t = oil_t - median(oil)$. We refer to this process as the "demeaned" oil prices.

The behaviour of the demeaned series over time is displayed in Figure 4.10 below.

[Insert 4.10: Demeaned WTI Crude Oil Prices]

We observe that the series tends to revert to the level close to 0 over time. It displays spikes and bubbles which are short lasting local trends with a growth phase followed by a sudden drop, like the one observed around observation 100 in Figure 4.8, for example.

4.4.1 Noncausal Models

The noncausal processes have simple linear dynamics in the reverse time, while displaying non-linearities in the calendar time. They can accommodate spikes and local trends in the trajectory of a process, including the bubbles.

The mixed autoregressive causal-noncausal model is:

$$\Phi(L)\Psi(L^{-1})y_t = \epsilon_t, \tag{4.4.3}$$

where $\Phi(L)$ is the autoregressive polynomial in lag operator L of order r, $\Psi(L^{-1})$ is the autoregressive polynomial in lead operator L^{-1} of order s and errors ϵ_t are i.i.d. Cauchy distributed variables with location 0 and scale coefficient γ. For stationarity, we require both autoregressive polynomials to have roots outside the unit circle.

Let $\theta = [\phi_1, ...\phi_r, \psi_1, ..., \psi_s, \gamma]'$. We use the Approximate Likelihood Method (AML) [Lanne, Saikkonnen(2011)] and maximize the log-likelihood function [11]:

$$L(\theta; y_1, ...y_T) = \sum_{t=r+1}^{T-s-1} [-ln(\pi) - ln\gamma - ln(1 + (\epsilon_t(\theta)^2/(\gamma^2))];$$

[11] The standard error and t-ratio are asymptotically valid for the scale estimator γ only [Andrews et.al (2009)]. The standard errors of the remaining coefficients tend to be overestimated by the AML method.

where ϵ_t denotes the error term from one of the specifications presented below. Let us first, examine pure noncausal processes. These models with Cauchy distributed errors can replicate the bubbles, i.e. the aforementioned local trends followed by sudden bursts. First, we consider a Cauchy noncausal AR(1) (r=1, s=0).

$$y_t = \psi y_{t+1} + \epsilon_t$$

It is estimated with an objective function value of 713.486537:

Parameters	Estimates	st error*	t-ratio*
ψ	0.9894	0.0096	102.8088
γ	2.6200	0.2223	11.7821

We find that the autoregressive coefficient is close to 1. The root of the autoregressive polynomial is 1.01.

Next, we consider the noncausal AR(2) process (r=2, s=0).

$$y_t = \psi_1 y_{t+1} + \psi_2 y_{t+2} + \epsilon_t$$

It is estimated with an objective function value of 711.17401:

Parameters	Estimates	st error*	t-ratio*
ψ_1	1.1217	0.0582	19.2673
ψ_2	-0.1328	0.0576	-2.3048
γ	2.5812	0.2183	11.8225

The roots of the autoregressive polynomial are 7.4335 and 1.013 and one root is still close to the unit circle.

Let us now consider the mixed causal-noncausal processes, which allow for more complex dynamics including the spikes and bubbles with possibly asymmetric patterns of growth and burst. We start with a noncausal MAR(1,1) process that combines the causal and noncausal components.

$$(1 - \phi L)(1 - \psi L^{-1})y_t = \epsilon_t, \tag{4.4.4}$$

It is estimated with an objective function value of 706.773270:

Parameters	Estimates	st error*	t-ratio*
ψ	0.9843	0.0703	2.1383
ϕ	0.1504	0.0120	81.4352
γ	2.5944	0.2297	11.2947

The roots of the MAR(1,1) lie outside the unit circle. The root of the non-causal polynomial is 1.02. The MAR(2,2) model below

$$(1-\phi_1 L-\phi_2 L^2)(1-\psi_1 L^{-1}-\psi_2 L^{-2})y_t=\epsilon_t,$$

is estimated with an objective function value of 696.384026:

Parameters	Estimates	st error*	t-ratio*
ψ_1	0.6492	0.0939	6.9114
ψ_2	0.3092	0.0950	3.2540
ϕ_1	0.5530	0.0977	5.6583
ϕ_2	-0.0580	0.0785	-0.7397
γ	2.6438	0.2192	12.0579

The roots of both polynomials are 5.4035 and 4.1309 and -1.631393 and 3.73171 for both polynomials, respectively. They are outside the unit circle.

The MAR(2,1) model:

$$(1-\psi_1 L^{-1}-\psi_2 L^{-2})(1-\phi L)y_t=\epsilon_t,$$

is estimated with an objective function value of 696.634560:

Parameters	Estimates	st error*	t-ratio*
ψ_1	0.6816	0.0782	8.7109
ψ_2	0.2708	0.0738	3.6666
ϕ	0.4945	0.0614	8.0435
γ	2.6283	0.2252	11.6676

The roots of the autoregressive polynomials are 4.1199 and -1.59770 and are outside the unit circle.

We conclude that the stationary mixed causal-noncausal processes can accommodate the dynamics of the WTI crude oil prices. The best fit is provided by the MAR(1,1) process which has the highest value of the log-likelihood function at the maximum and the roots of autoregressive polynomials outside the unit circle.

The forecasts from the MAR(1,1) process can be computed from a closed-form formula of the predictive density as shown in Section .

4.4.2 Convolution Model

4.2.1 The Model

An alternative approach that accommodates the mixture representation is based on the assumption that process Y_t is a convolution of a Gaussian causal AR(1) and a Cauchy noncausal MAR(0,1):

$$Y_t = X_t + Z_t,$$

where

$$X_t = \rho X_{t-1} + \sigma \epsilon_t,\ |\rho| < 1$$

is a stationary Gaussian AR(1) with $\epsilon_t \sim IIN(0,1)$. The marginal distribution of X_t is $X_t \sim N(0, \frac{\sigma^2}{1-\rho^2})$.

The stationary noncausal component is:

$$Z_t = r Z_{t+1} + \gamma \varepsilon_t,\ |r| < 1$$

where ε_t is i.i.d. Cauchy $\mathcal{C}(0,1)$. The marginal distribution of Z_t is such that $Z_t(1 - |r|)/\gamma \sim C(0,1)$.

The noise processes ϵ_t and ε_t are independent. Therefore, the distribution of (Y_t) is the convoluate of the distributions of (X_t) and (Z_t). The joint distribution of $(Y_t Y_{t-1})$ can be examined from its joint characteristic function.

Let $\varphi(u,v)$ denote the characteristic function of Y_t, Y_{t-1}:

$$
\begin{aligned}
\varphi(u,v) &= E[\exp\ i(uY_t + vY_{t-1})] \\
&= E\{\exp[i(uX_t + vX_{t-1})]\ \exp[i(uZ_t + vZ_{t-1})]\} \\
&= E\{\exp\ i(uX_t + vX_{t-1})\}\ E\{exp\ i(uZ_t + vZ_{t-1})\}
\end{aligned}
$$

We have $uX_t + vX_{t-1} = (u\rho + v)X_{t-1} + u\sigma\epsilon_t$. Thus the first expectation on the right hand side can be written as:

$$
\begin{aligned}
E\{\exp\ i(uX_t + vX_{t-1})\} &= E\{\exp\ i(u\rho + v)X_{t-1}\}E\{exp(i\sigma u\epsilon_t)\} \\
&= \exp[-(u\rho + v)^2\frac{\sigma^2}{1-\rho^2}]\ \exp(-\frac{\sigma^2}{2}u^2)
\end{aligned}
$$

Similarly, we have $uZ_t + vZ_{t-1} = (u+rv)Z_t + v\gamma\varepsilon_{t-1}$ and $E[exp(iv\gamma\varepsilon_{t-1})] = exp[-\gamma|v|]$.

Also $Z_t = \gamma\varepsilon_t + \gamma r\varepsilon_{t+1} + \gamma r^2\varepsilon_{t+2} + \cdots$.

Moreover,

$$
\begin{aligned}
E[\exp\ (iuZ_t)] &= \exp -(\gamma|u| + \gamma r|u| + \gamma r^2|u| + \cdots) \\
&= \exp\left[-\gamma\frac{|u|}{1-|r|}\right]
\end{aligned}
$$

Therefore, the second expectation can be written as:

$$
\begin{aligned}
E\{\exp\ i(uZ_t + vZ_{t-1})\} &= E\exp\,[i(u+rv)Z_{t-1}]\ E[\exp v\gamma\varepsilon_{t-1}] \\
&= \exp\left[-\gamma\frac{|u+rv|}{1-|r|} - \gamma|v|\right]
\end{aligned}
$$

Proposition:

$$
\begin{aligned}
\varphi(u,v) &= E[\exp\ i(uY_t + vY_{t-1})] \\
&= \exp\left[-(u\rho+v)^2\frac{\sigma^2}{1-\rho^2}]\ - \frac{\sigma^2}{2}u^2 - \gamma\frac{|u+rv|}{1-|r|} - \gamma|v|\right]
\end{aligned}
$$

4.2.2 Estimation

We can estimate the parameter vector $\theta = [\rho, r, \sigma^2, \gamma]'$ from observations $y_1, ..., y_T$ by finding

$$
\hat{\theta} = argmin\ ||\tilde{\varphi}_T(u,v) - \varphi(u,v)||^2,
$$

where $||.||^2$ denotes a norm on the space of functions u, v and

$$
\tilde{\varphi}_T(u,v) = \frac{1}{T}\sum_{t=1}^{T} cos[uY_t + vY_{t-1})].
$$

given the symmetry of the Normal and Cauchy distributions [see, Gourieroux and Zakoian (2017)].

For estimation, we consider an Euclidean norm constructed from a grid of values $(u_j, v_j), j = 1, ..., J$. The grid covers the interval (-0.5,1.5) with increments of 0.001.

We estimate the process and obtain [12] :

Parameters	ρ	σ	r	γ
estimates	0.506	2.880	0.910	1.995
st. error	0.027	0.600	0.083	0.191

In order to verify if the deconvolution model can replicate the bimodal sample density of the data, we simulate a sample of mixture process with the values of coefficients equal to the estimates and the error from Cauchy and Normal distributions with variances set equal to the estimates given above. Next, we compute its histogram and kernel-smoothed density, which is displayed in Figure 4.11 below.

[Insert Figure 4.11: Histogram, simulated y]

The simulated model can replicate the bimodality revealed in the sample distribution of oil prices.

4.5 Nonlinear Forecast

The MAR(1,1) specification can provide the forecasts of WTI series.

It follows from Lanne and Saikkonen (2011), and Lanne, Luoto, and Saikkonen (2012), that process (y_t) has the following unobserved components u_t, v_t defined by:

[12]See Gourieroux and Zakoian (2018) for the asymptotic validity of the standard errors

$$u_t \equiv (1 - \phi L) y_t \leftrightarrow (1 - \psi L^{-1}) u_t = \epsilon_t, \tag{4.5.5}$$

and

$$v_t \equiv (1 - \psi L^{-1}) y_t \leftrightarrow (1 - \phi L) v_t = \epsilon_t, \tag{4.5.6}$$

which can be interpreted as the "causal" and "noncausal" components of process (y_t). Precisely, (u_t) is a pure noncausal and (v_t) is a pure causal autoregressive process of order 1. Moreover, i) u_t is ϵ-noncausal and y-causal and ii) v_t is ϵ-causal and y-noncausal. However, these processes are based on the same noise ϵ_t and are not independent. They can be combined to construct the series y_t.

The above unobserved component representation of process y_t can be used for filtering and forecasting.

4.5.1 Filtering and Simulation

The filtering procedure allows us to compute the unobserved components given the observations on process (y_t), over a period of length T. Let $(y_1, \ldots, y_T)$ denote the observed sequence.

The values of unobserved components u and v and errors ϵ can be computed from a set of observations $(y_1, \ldots, y_T)$ as follows:

(i) From equation 4.4.4 for $t = 2, \dots, T-1$, we obtain the values $\epsilon_2, \dots, \epsilon_{T-1}$ as functions of $(y_1, \dots, y_T)$.

(ii) From equation 4.5.5 : $u_t = (1-\phi L)y_t, t = 2, \dots, T$, we obtain $u_2, \dots, u_T$.

(iii) From equation 4.5.6 : $v_t = (1-\psi L^{-1})y_t, t = 1, \dots, T-1$, we obtain $v_1, \dots, v_{T-1}$.

When an additional observation y_{T+1} becomes available, the set of unobserved components can be updated by computing ε_T, u_{T+1} and v_T.

The above formulas can be used to simulate the trajectories of process (y_t) as follows:

step 1: Simulate a path of i.i.d. errors $\varepsilon_t^s,\ t = 1, ..., T$.

step 2: Use formulas 4.5.5 - 4.5.6 to obtain the simulated paths of the ε-causal and ε-noncausal components :

$$u_t^s = \varepsilon_t^s + \psi u_{t+1}^s,\ t = 1, ..., 2T,$$

$$v_t^s = \varepsilon_t^s + \phi v_{t-1}^s,\ t = -T, ..., T,$$

starting from a far terminal condition (resp. far initial condition) $u_{2T}^s = u_0$, say (resp. $v_{-T}^s = v_0$).

step 3: The simulated trajectory (y_t^s) is obtained from either one of the two partial fraction representations given below:

$$y_t^s = \frac{1}{1-\phi\psi}(u_t^s + \phi v_{t-1}^s) = \frac{1}{1-\phi\psi}(v_t^s + \psi u_{t+1}^s),\ t = 1, ..., T. \tag{4.5.7}$$

4.5.2 Forecasting from the MAR(1,1) Model

The information set $(y_1, \ldots, y_T)$ is equivalent to the information set $(v_1, \ldots, v_2, \varepsilon_2, \ldots, \varepsilon_{T-1}, u_T, \ldots, u_T)$, as shown in Gourieroux and Jasiak (2016). Therefore, the information contained in $(y_1, \ldots, y_{T+H})$ is equivalent to the information in $(v_1, \varepsilon_2, \ldots, \varepsilon_{T+H-1}, u_{T+1}, \ldots, u_{T+H})$, and it is also equivalent to that in $(v_1, \varepsilon_2, ..., \varepsilon_{T-1}, u_T, ..., u_{T+H})$, because $(1-\psi(L^{-1}))u_t = \varepsilon_t, t = T, \ldots, T+H-s$ by formula 4.5.5.

Thus, instead of predicting the future value of y, at horizon H, we can equivalently predict the future value of the ε-noncausal component u, by finding the predictive density $\hat{\Pi}$ at horizon H for a noncausal process of order 1:

For a given error density g and for known values of coefficients ϕ, ψ we get :

$$\begin{aligned} & \hat{\Pi}(u_{T+1}, \ldots, u_{T+H}|\hat{u}_T) \\ = \ & \frac{g(\hat{u}_T - \psi u_{T+1})g(u_{T+1} - \psi u_{T+2})g(u_{T+H-1} - \psi u_{T+H}) \sum_{t=1}^{T} g(u_{T+H} - \psi \hat{u}_t)}{\sum_{t=1}^{T} g(\hat{u}_T - \psi \hat{u}_t)}. \end{aligned} \tag{4.5.8}$$

where $\hat{u}_t,\ t = 1+1, ..., T$ are the filtered values of the ϵ-noncausal component, that are functions of $y_1, ..., y_T$ and of coefficients ϕ, ψ.

The predictive density given above has a closed-form representation when the error density g is known. In particular, when ϵ follows a Cauchy distribution, the one-step ahead predictive density is:

$$\pi(u_{T+1}|u_T) = \frac{1}{\pi}\frac{1}{1+(u_T-\psi u_{T+1})^2}\frac{1+(1-\psi)^2u_T^2}{1+(1-\psi)^2u_{T+1}^2}, \tag{4.5.9}$$

and the predictive joint distribution of two future values is:

$$\begin{aligned}\pi(u_{T+1}, u_{T+2}|u_T) \;\; = \;\; & \frac{1}{\pi^2}\frac{1}{1+(u_T-\psi u_{T+1})^2}\frac{1}{1+(u_{T+1}-\psi u_{T+2})^2} \times \\ & \frac{1+(1-\psi)^2u_T^2}{1+(1-\psi)^2u_{T+2}^2}. \end{aligned} \tag{4.5.10}$$

The above forecasting method developed by Gourieroux and Jasiak (2016) relies on a closed-form formula of the estimated predictive density $\hat{\Pi}$ given above.

An alternative forecasting method proposed by Lanne, Luoto, and Saikkonen (2012) relies on the simulations of long paths of future $\varepsilon_{T+1}, ..., \varepsilon_{T+M}$, from which the future vectors $u_{T+1}, ..., u_{T+H}$ are recovered. That method approximates numerically the predictive density from a large number of simulations and is more computationally demanding. The approximation to the predictive density is based on a truncation $u_T \approx \sum_{j=1}^{M} \beta_j \varepsilon_{T+j}$, which entails a truncation bias. That bias can be arbitrarily reduced by sufficiently increasing the truncation parameter M.

4.5.3 MAR Forecasts of WTI Crude Oil Prices

Let us now examine the forecasting performance of the MAR(1,1) model in application to the WTI crude oil price data over the last 8 months of the sampling period, i.e. for $T = 221$ to 228.

The one step ahead out-of-sample forecasts of WTI crude oil prices are computed from the MAR(1,1) model, by applying the "GJ" method of Gourieroux and Jasiak (2016), and the "LLS" method of Lanne, Luoto, and Saikkonen (2012). Next, the forecasts are compared to the true values of median adjusted oil prices and the "no-change" forecast, denoted by "NC" and equal to the last observed value of the process.

The GJ forecasts of the last 8 values of the process are given in Table 4.14 below in column 3. The true values of the process are given in column 1, Column 2 reports the filtered values of the noncausal component u. Column 4 contains the prediction error of that forecast. Columns 5 and 6 present the lower and upper prediction interval at 50% obtained from the lower and upper quartile of the predictive density. The level of 50% is chosen to eliminate the effect of long tails. Column 7 shows the forecast from the MAR(1,1) process based on the LLS method with a t-student approximation of the error distribution. Column 8 provides the prediction error of that forecast. Column 9 provides the "no-change" forecast NC. Column 10 reports the forecast error of the NC forecast.

Table 4.14: One Step Ahead Out-Of-Sample Forecasts of Demeaned WTI Crude Oil

true y_{T+1}	u_{T+1}	$\hat{y}_{T+1}$	er.($\hat{y}_{T+1}$)	PI($\hat{y}_{T+1}$) L	PI($\hat{y}_{T+1}$) U	$\tilde{y}_{T+1}$	er.($\tilde{y}_{T+1}$)	NC	er(NC)
y(221) = 10.805	9.743	7.463	3.341	-13.136	16.863	6.919	3.885	7.075	3.73
y(222) = 8.695	7.0742	11.235	-2.540	-9.464	20.535	11.308	-2.613	10.805	-2.11
y(223) = 11.805	10.500	8.358	3.446	-11.541	18.458	8.654	3.150	8.695	3.11
y(224) = 8.885	7.114	12.119	-3.234	-8.480	21.519	12.044	-3.159	11.805	-2.92
y(225) = 11.055	9.722	8.946	2.109	-11.354	18.645	9.171	1.883	8.885	2.17
y(226) = 11.575	9.916	11.281	0.293	-9.218	20.781	10.944	0.630	11.055	0.52
y(227) = -2.215	-3.951	11.493	-13.708	-8.706	21.293	11.077	-13.292	-1 11.575	-13.79
y(228) = -9.655	-9.332	-3.180	6.474	-22.280	7.719	-3.304	6.350	-2.215	-7.44

The forecasts from the MAR(1,1) based on both methods provide very close results in the application to the WTI crude oil prices. The median forecast error based on Lanne et al. is 1.2565 as compared to 1.201 for Gourieroux and Jasiak. The median forecast error of the no-change method is -0.795. The prediction interval, obtained from the predictive density of Gourieroux and Jasiak method contains the true values of the process for each y_{T+1}. The method of Lanne et al. available on-line does not provide a prediction interval.

The GJ method provides forecasts with mean forecast error of -0.477 and mean squared error (MSE) of 34.287, which are slightly above the mean forecast error of -0.395 and MSE of 32.845 for the LSS. The "no-change" method has the worse performance with mean forecast error of -2.0912 and MSE of 35.882. Hence, the forecasts from the mixed causal-noncausal model outperform the "no-change" in one step-ahead forecasts.

4.6 Conclusion

This chapter introduced a new convolution model for monthly WTI crude oil prices, which accommodates local trends in its dynamics and a multimodal sample density evidenced in Section 4.2. The results show that the convolution approach presents a promising approach for modelling the monthly WTI crude oil prices. We also estimated the comovements between the Ontario consumer price and energy price indexes and the WTI crude oil prices. The comovements between these series allow for forecasting the Ontario price indexes using either the VEC model, or linear functions of the forecasts of oil prices provided from the mixed autoregressive causal-noncausal model.

The forecasts based on the noncausal MAR(1,1) model of the WTI crude oil prices were computed from two methods of forecasting for autoregressive causal-noncausal processes. Both the methods using MAR model to forecast the WTI crude oil price outperformed the "no change" forecast. The results suggest the proposed model estimated under both method outperforms existing models and provide close approximations of out of sample values, as the model can accommodate the various asymmetric local trends observed in the WTI data distribution.

Under a more turbulent economic environment, for example with the COVID 19 pandemic causing a steep change in oil price, a separate cointegration analysis should be performed on the relationship between the WTI and Ontario price indices to verify any changes and the validity of the VEC models. The convolution model and the forecasts based on the noncausal MAR(1,1) model should be estimated with updated data. The forecast under these models should still produce a close approximation, as the model can accommodate various asymmetric local trends.

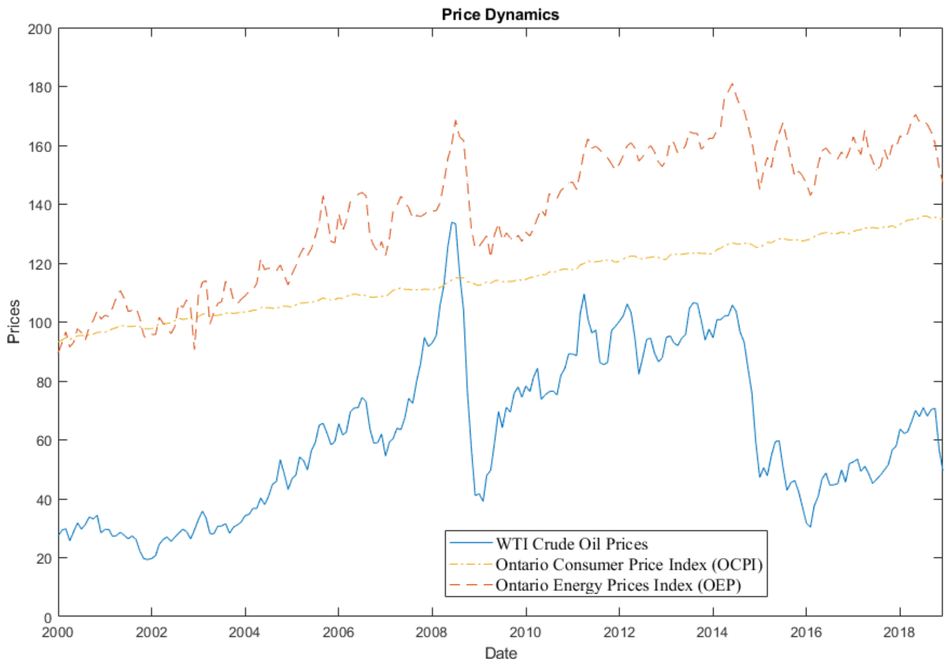

Figure 4.1: Dynamics of OCPI, OEP and WTI Crude oil prices

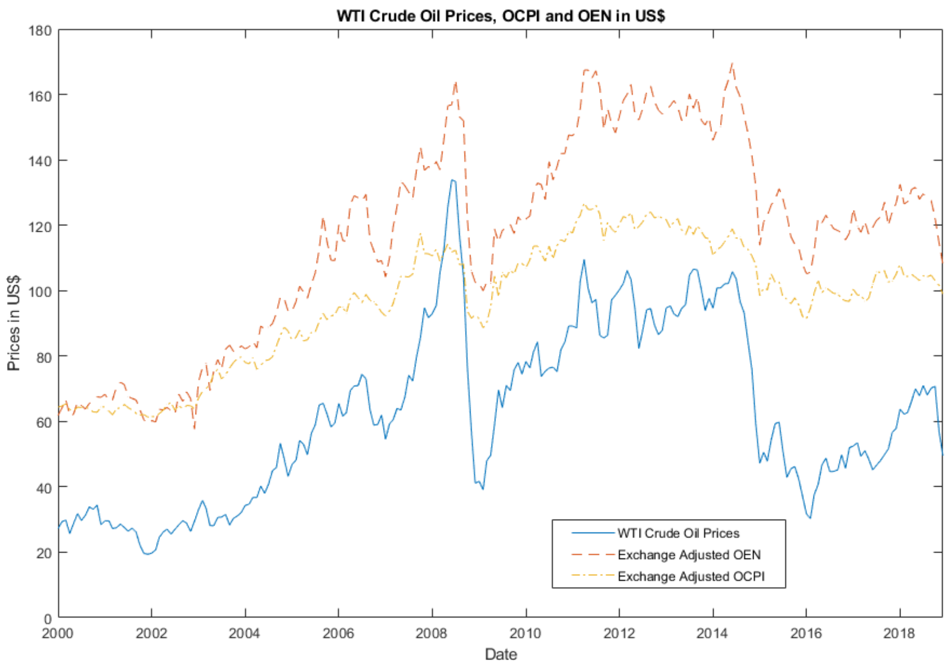

Figure 4.2: Dynamics of transformed OCPI, OEP and WTI crude oil price in US$

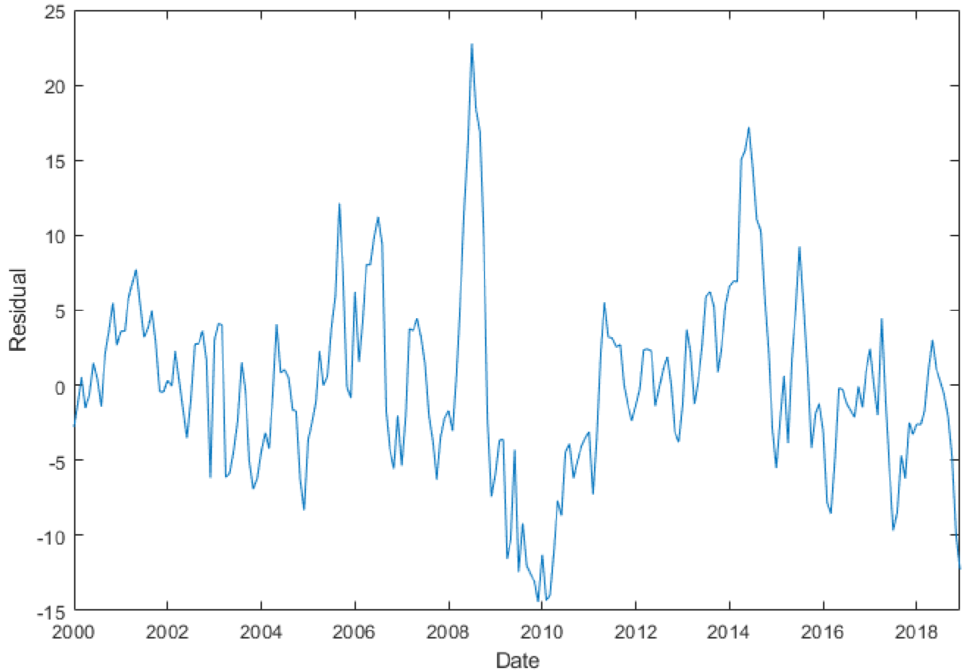

Figure 4.3: Departures from long-term relation of OEP and OCPI- Residuals Model (4.3.1)

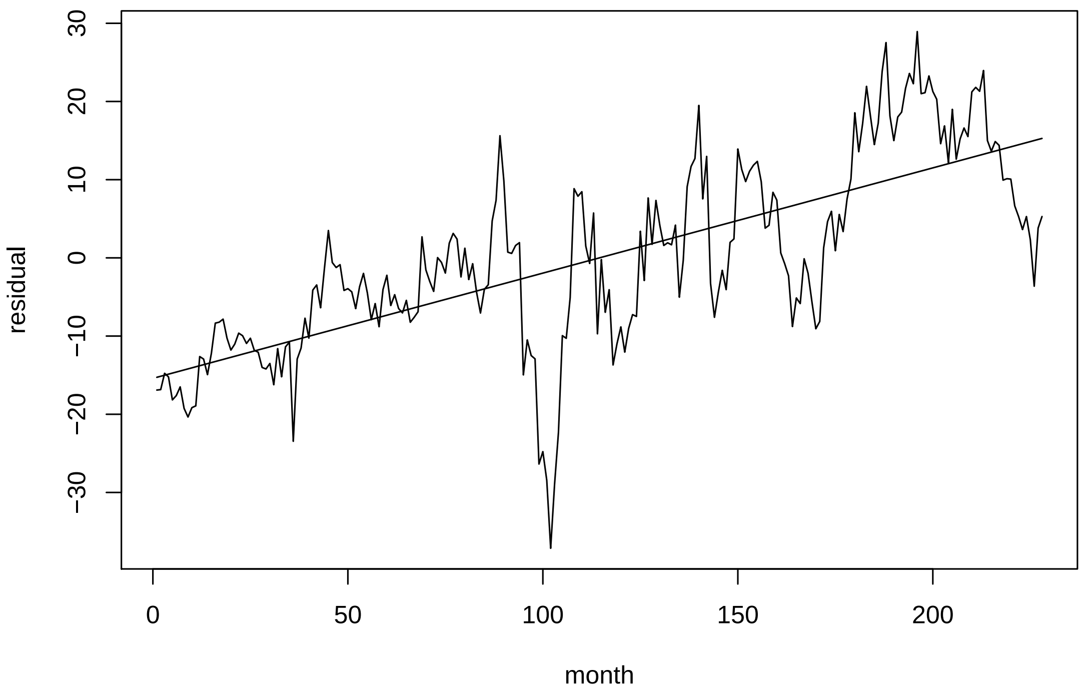

Figure 4.4: Rgression of OEP on oil prices - Residuals

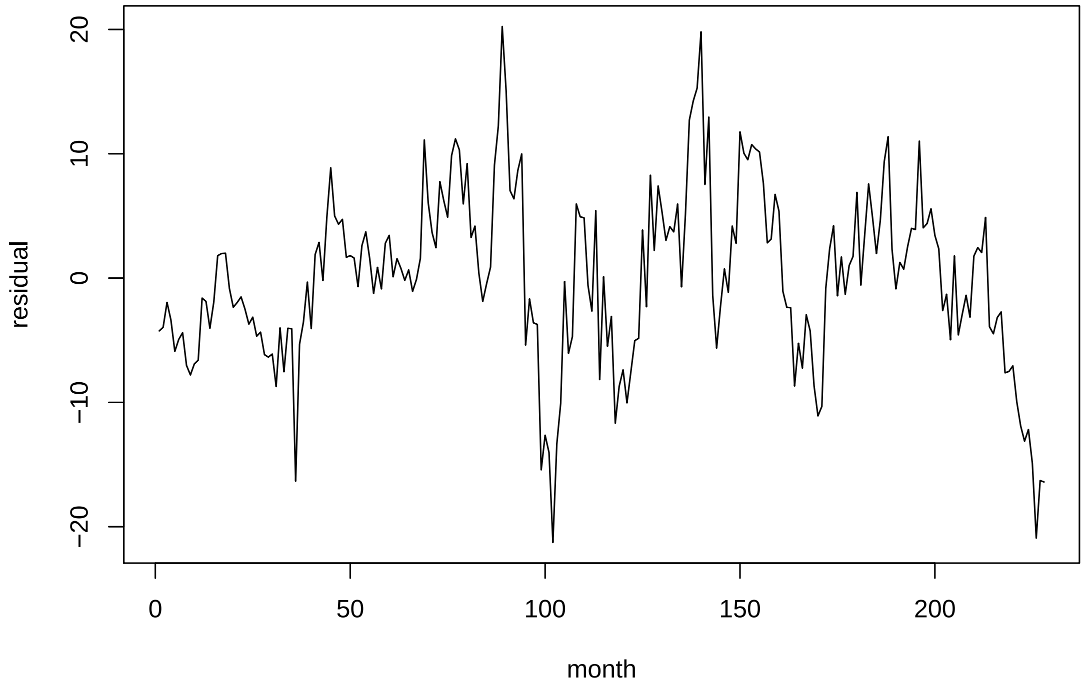

Figure 4.5: Departures from long-term relation of oil prices and OEP - Residuals (4.3.2)

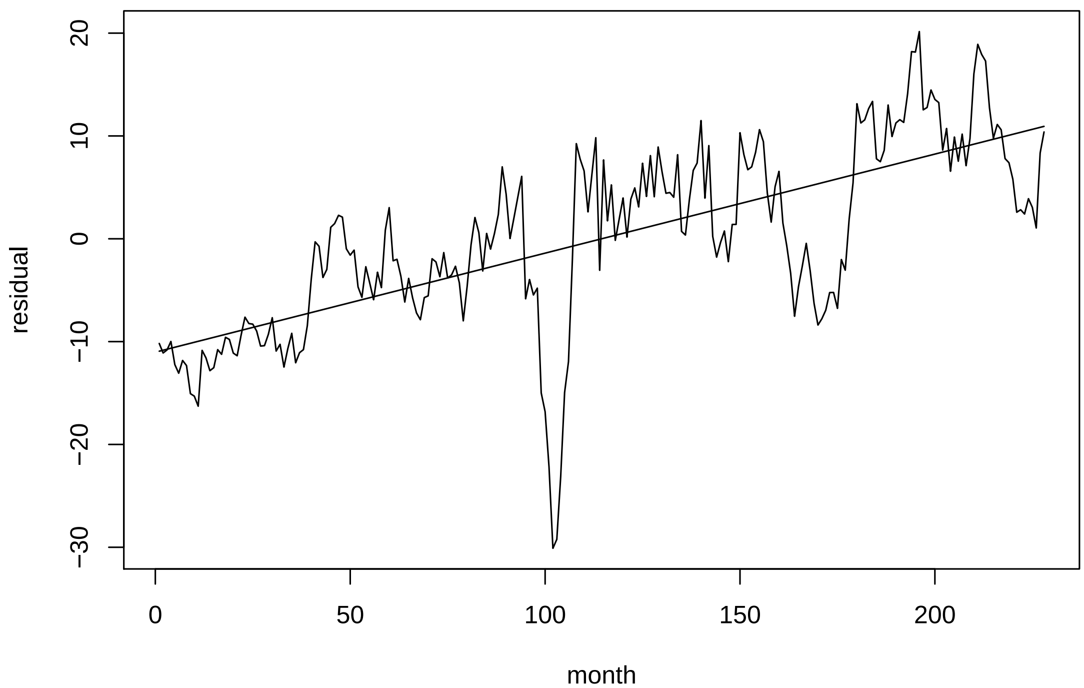

Figure 4.6: Regression on OCPI on oil prices - Residuals

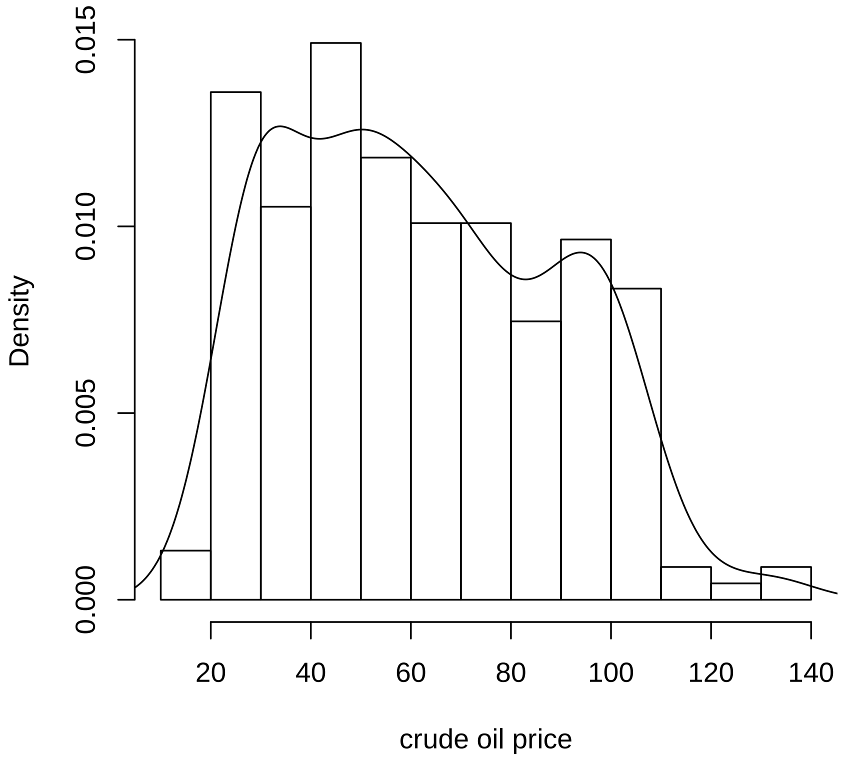

Figure 4.7: WTI Crude Oil Prices: Histogram

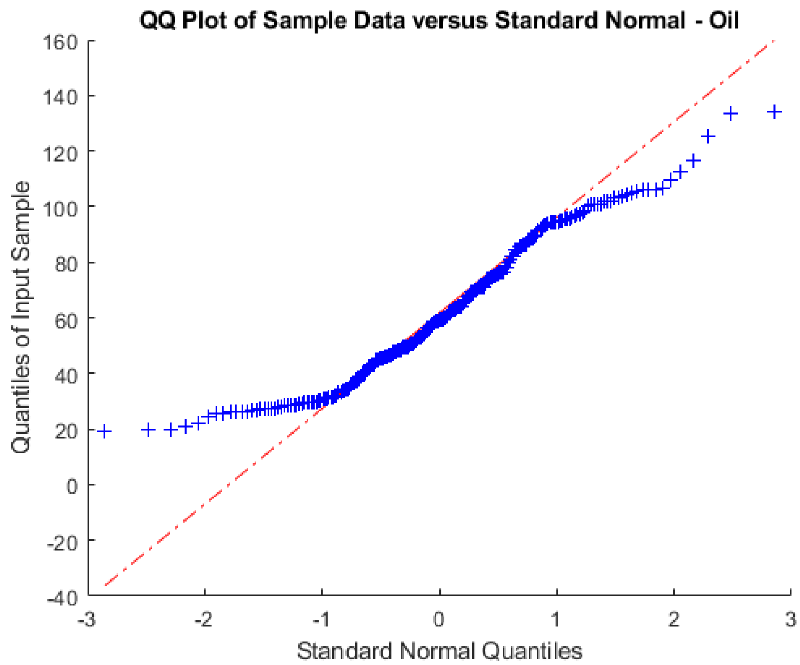

Figure 4.8: Crude Oil - QQ Plot

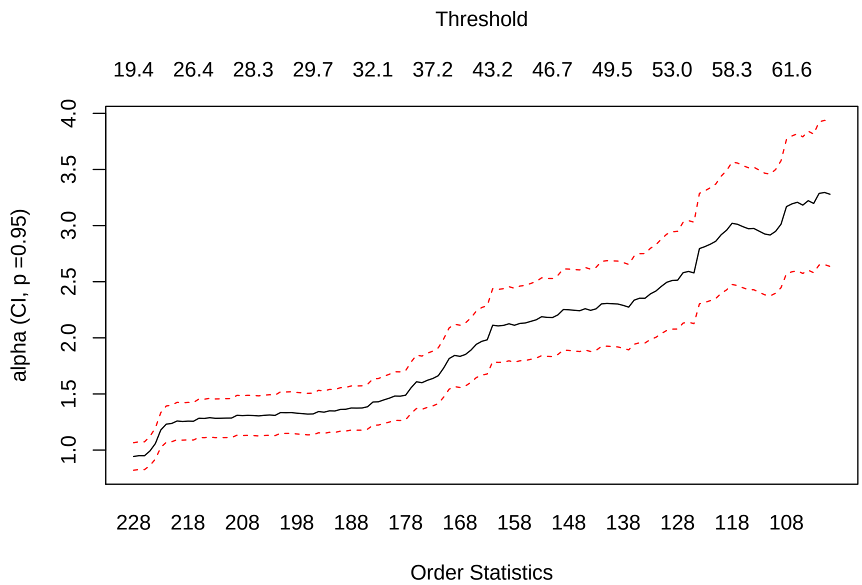

Figure 4.9: Crude Oil - QQ Plot

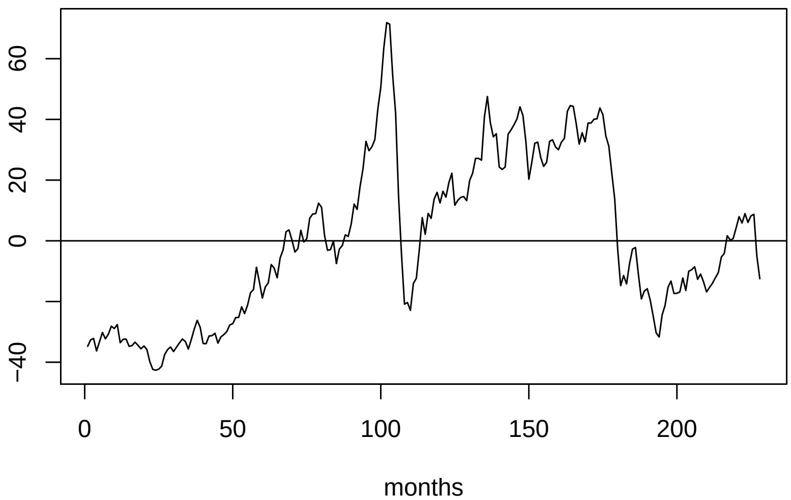

Figure 4.10: Demeaned WTI Crude Oil Prices

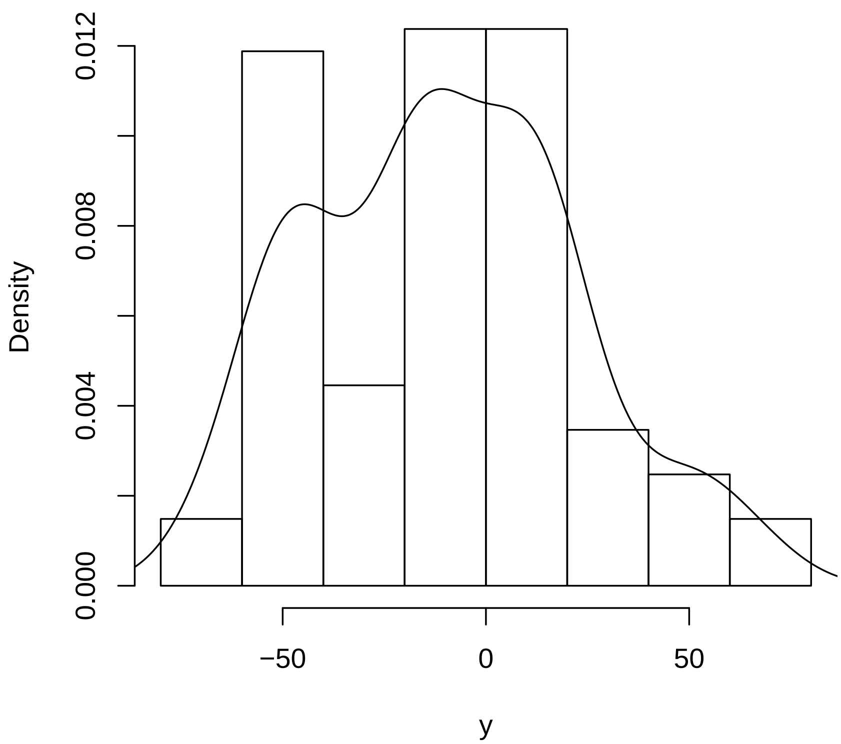

Figure 4.11: Density simulated y

Conclusion

Given the volatility in commodity prices and the prevalent use of financial derivative to hedge price volatility, this thesis focuses on two themes: (i) how to align hedging decision between mangers and shareholder using executive compensation to maximize firm value, and (ii) how the commodity/index prices can be forecasted.

Chapters Two and Three addresses the agency problem presented in past literature whereby risk-averse mangers tends to over-hedge (Holmstorm and Ricart i. Costa, 1986; Smith and Stulz, 1985). I developed a model to mitigate the agency problem by aligning hedging with the use of executive compensation. My first contribution is determining the factors that affect hedging decision and optimal executive compensation. Second, using these factors I show how the agency problem can be mitigated in practice. In the third chapter, I empirically test my model findings, the additional contribution is using manually collected firm hedging data from the firms' 10K reports to show empirically equity-based executive compensation is negatively related to hedging which is consistent with prior research Chen, Jin, and Wen (2011), and Tufano (1996).

Future research will be focused on further examining the relationship between executive compensation and hedging. For example, developing a model to study the effects of different types of derivatives while considering the role of the risk-averse managers, or studying the effects in a multi-period model. Another possible study can examine the not only the hedging of price risk, but also quantity risk.

Chapter Four focuses on how commodity/index prices can be forecasted. Being able to forecast the commodity prices provides insight into the economic environment, which can then be used to determine the optimal compensation scheme, as one of the factors affecting optimal executive compensation is volatility of commodity prices. The chapter's first contribution is modelling and forecasting the WTI oil prices by estimating the long-run relationships between the Ontario price indexes, and the WTI oil prices. The forecasts of the Ontario price indexes and WTI crude oil prices are obtained by the cointegration analysis. Alternatively, the forecasts of Ontario price indexes are provided by the functions of the WTI crude oil price forecasts combined with the forecast of the stationary series of departures from the long run equilibrium. Second, this chapter provide a new method to model the oil price process (i.e. a convolution of stationary causal and noncausal processes). The forecast of the WTI crude oil prices can be obtained by forecasting the convoluted series or conversely, from a simple mixed causal -noncausal model of WTI crude oil prices. The latter approach allows for the comparison of the two methods of forecasting for noncausal processes developed by Lanne, Luoto and Saikkonen (2012) and by Gourieroux and Jasiak (2016).

Related future research will be focused on the use of cointegration analysis for different commodities and series and the use of the convolution approach. Also, we can study the forecasts obtained under the convolution approach of the predicted oil prices.

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Appendix

Appendix 1: Chapter 2

To get $E_t(\pi_t)$ and $Var_t(\pi_t)$, first get $E(S_t)$ and $Var_t S$

$$E(S_t) = \frac{S_o}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} [(e^{\mu t - \frac{\sigma_s^2 t}{2} + \sigma_s \chi})(e^{\frac{-\chi^2}{2t}})]\partial\chi$$

$$E(S_t) = \frac{S_o}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} [e^{-\frac{(\chi - \sigma_s t)^2}{2t} + \mu t}]$$

$$E(S_t) = S_o e^{\mu t} [\frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} e^{-\frac{(\chi - \sigma_s t)^2}{2t}}]$$

$$E(S_t) = S_o e^{\mu t}$$

Now to get $VarS_t$, $VarS_t = E(S^2) - (ES)^2$

$$E(S_t^2) = \frac{S_o^2}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} [(e^{2\mu t - \sigma_s^2 t + 2\sigma_s \chi})(e^{\frac{-\chi^2}{2t}})]\partial\chi$$

$$E(S_t^2) = \frac{S_o^2}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} [e^{-\frac{(\chi - 2\sigma_s t)^2}{2t} + \sigma_s^2 t + 2\mu t}]$$

$$E(S_t^2) = S_o^2 e^{\sigma_s^2 t + 2\mu t} [\frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} e^{-\frac{(\chi - \sigma_s t)^2}{2t}}]$$

$$E(S_t^2) = S_o^2 e^{\sigma_s^2 t + 2\mu t}$$

$$VarS_t = S_o^2 e^{\sigma_s^2 t + 2\mu t} - S_o^2 e^{2\mu t}$$

$$VarS_t = S_o^2 e^{2\mu t}(e^{\sigma_s^2 t} - 1)$$

Also,

$$E(q(a)) = f(a)$$

$$E(q(a)^2) = (f(a) + \varepsilon)(f(a) + \varepsilon) \; E(q(a)^2) = (f(a)^2 + 2f(a)E(\varepsilon) + E(\varepsilon^2)$$

$$= f(a)^2 + \sigma_\varepsilon^2)$$

$$E(S_t^2 q(a)) = E[S_t^2]E[q(a)] - cov(S_t^2, q(a)) = f(a) S_o e^{\sigma_s^2 t + 2\mu t}$$

$$E(S_t q(a)^2) = E(S_t)E(q(a)^2) - cov(S, q(a)^2) = f(a)^2 S_o e^{\mu t} + S_o e^{\mu t} \sigma_\varepsilon^2$$

$$E(S_t^2 q(a)^2) = E[S_t^2]E[q(a)^2] - cov(S_t^2, q(a)^2) = S_o e^{\sigma_s^2 t + 2\mu t}[f(a)^2 + \sigma_\varepsilon^2)]$$

$$= f(a)^2 S_o^2 e^{\sigma_s^2 t+2\mu t} + S_o^2 e^{\sigma_s^2 t+2\mu t}\sigma_\varepsilon^2$$

The covariance between S_t and q(a) and the co variance of their 2nd moments are zero.

$$E_t(\pi_t) = s + b[h(F_o - E(S_t)) + E(q(a))S_t - c_1 E(f(a)+\varepsilon) - c_2] - g(a)$$

$$Var_t(\pi_t) = Var[s + bhF_o + b[-hE(S_t) + q(a)S_t - c_1(q(a))] - bc_2 - g(a)]$$

$$Var_t(\pi_t) = Var[b[-hE(S_t) + q(a)S_t - c_1(q(a))]$$

$$X = b[-hE(S_t)) + q(a)S_t - c_1(q(a))$$

$$Var_t(\pi_t) = Var_t X = E(X^2) - (EX)^2$$

$$(EX)^2 = E[b[-hE(S_t) + q(a)S_t - c_1(q(a))]^2 = [-bhS_o e^{\mu t} + bf(a)S_o e^{\mu t} - bc_1 f(a)]^2$$

$$(EX)^2 = (bhS_o e^{\mu t})^2 - 2b^2 hS_o^2 e^{2\mu t} f(a) + 2b^2 hS_o e^{\mu t} c_1 f(a) + (bf(a)S_o e^{\mu t})^2$$

$$-2b^2 f(a)^2 c_1 S_o e^{\mu t} + (bc_1 f(a))^2$$

$$X^2 = [-bhS + bq(a)S - bc_1 q(a)][-bhS + bq(a)S - bc_1 q(a)]$$

$$X^2 = (bhS)^2 - 2b^2 hq(a)S^2 + 2b^2 hc_1 q(a)S + (bq(a)S)^2 - 2b^2 c_1 q(a)^2 S + (bc_1 q(a))^2]$$

$$E(X^2) = (bh)^2(S_o^2 e^{\sigma_s^2 t+2\mu t}) - 2b^2 h(S_o^2 e^{\sigma_s^2 t+2\mu t})f(a) + 2b^2 hc_1(S_o e^{\mu t})f(a)$$

$$+b^2 S_o^2(f(a)^2 e^{\sigma_s^2 t+2\mu t} + \sigma_\varepsilon^2 e^{\sigma^2 t+2\mu t}) - 2b^2 c_1 S_o(f(a)^2 e^{\mu t} + \sigma_\varepsilon^2 e^{\mu t}) + b^2 c_1^2(f(a)^2 + \sigma_\varepsilon^2)$$

$$Var(X) = S_o^2 b^2(e^{\sigma_s^2 t} - 1)e^{2\mu t}(h - f(a))^2 + b^2\sigma_\varepsilon^2[S_o^2 e^{\sigma_s^2 t+2\mu t} - 2S_o c_1 e^{\mu t} + c_1^2]$$

Appendix 2: Chapter 4

Additional Figures

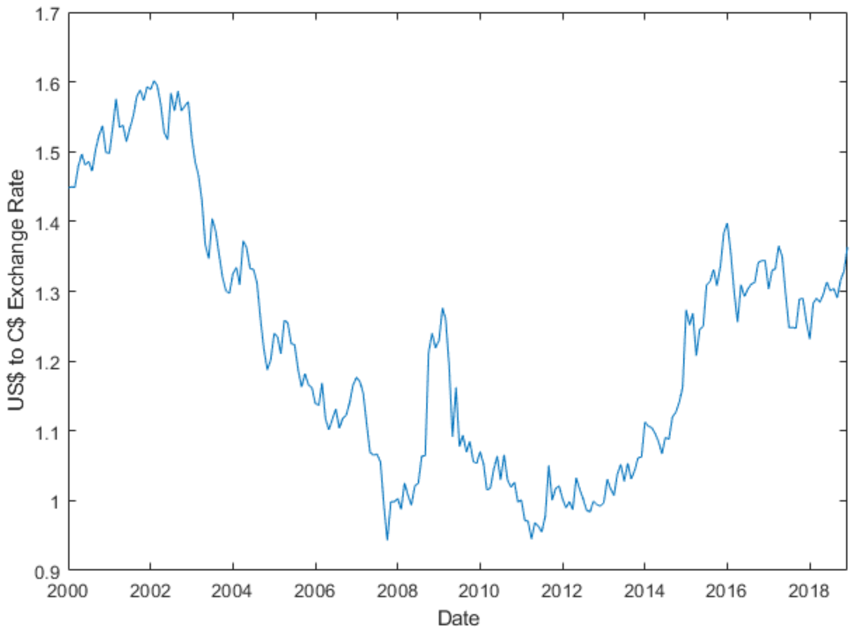

Figure 4.12: CAD/USD Exchange Rates

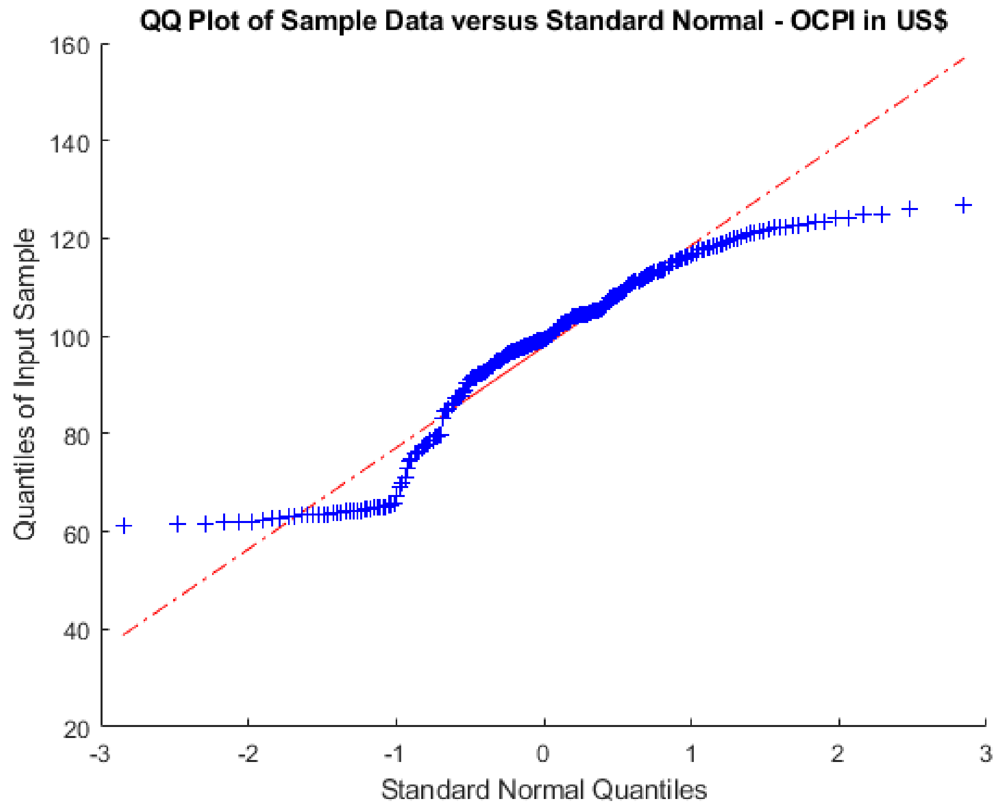

Figure 4.13: OCPI in US$ - QQ Plot

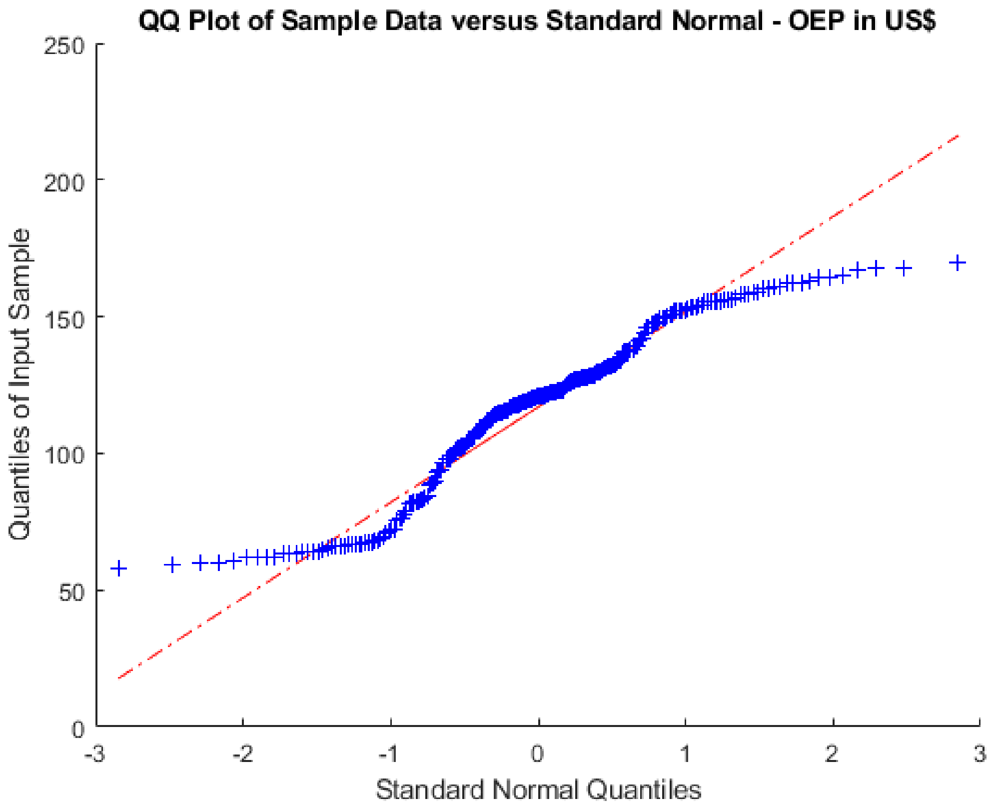

Figure 4.14: OCPI in US$ - QQ Plot

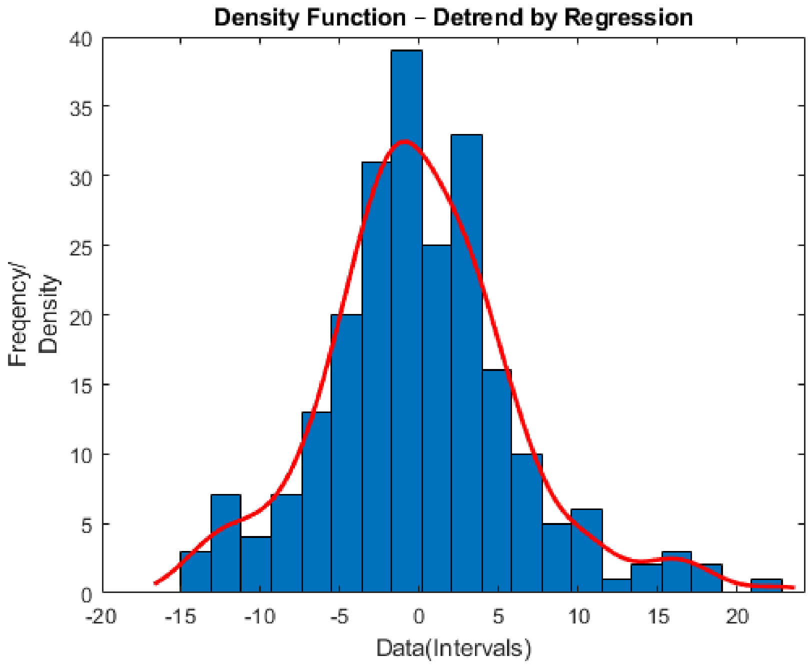

Figure 4.15: Residual Model (4.3.1) - Density

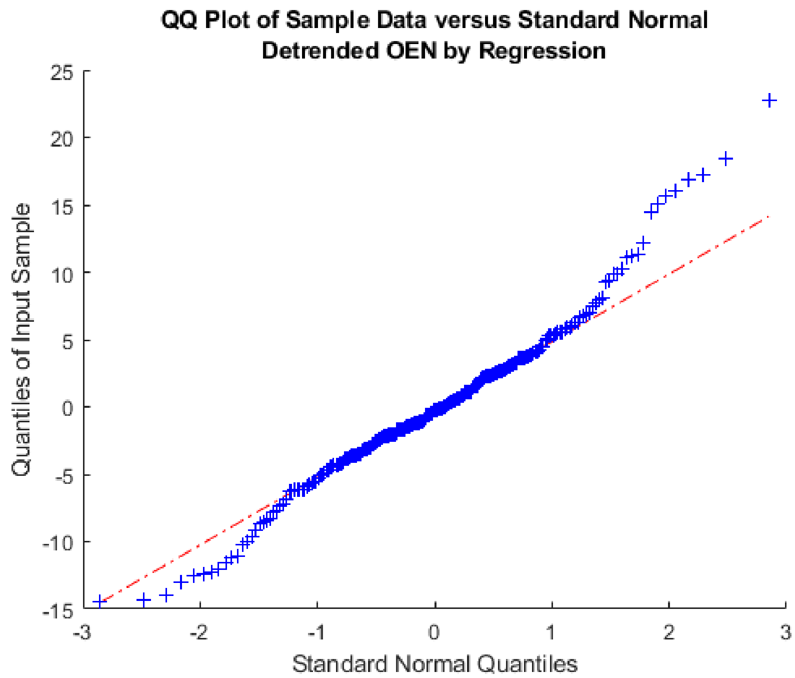

Figure 4.16: Residual Model (4.3.1) - QQ Plot

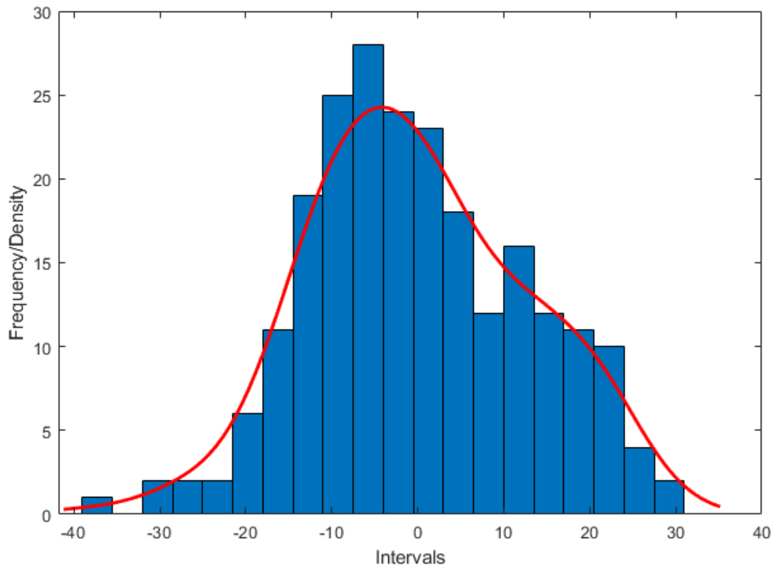

Figure 4.17: Residual Model (4.3.2) - Density

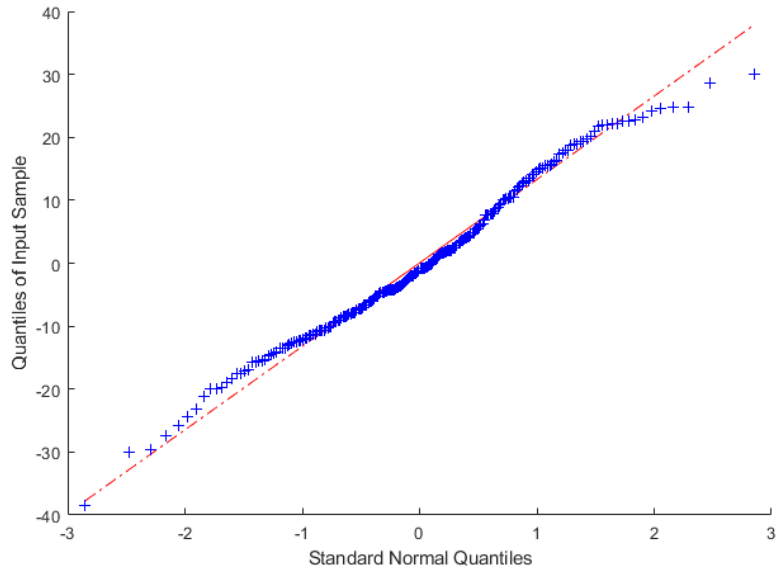

Figure 4.18: Residual Model (4.3.2) - QQ Plot

Appendix 3: Chapter 4 - Additional Tables

Table 4.15: VEC Model Parameter Estimates

Model Parameter Estimates						
Equation	Parameter	Estimate	Standard Error	t Value	Pr > |t|	Variable
D_oep	CONST1	-5.71160	0.99573	-5.74	<.0001	1
	XL0_1_1	0.18984	0.02346	8.09	<.0001	oil(t)
	AR1_1_1	-0.47599	0.05328	-8.93	<.0001	oep(t-1)
	AR1_1_2	0.51319	0.05744	8.93	<.0001	ocpi(t-1)
	AR2_1_1	0.22190	0.09463	2.34	0.0199	D_oep(t-1)
	AR2_1_2	-0.40504	0.18945	-2.14	0.0336	D_ocpi(t-1)
D_ocpi	CONST2	-1.24219	0.55613	-2.23	0.0265	1
	XL0_2_1	0.05177	0.01310	3.95	0.0001	oil(t)
	AR1_2_1	-0.14589	0.02976	-4.90	<.0001	oep(t-1)
	AR1_2_2	0.15730	0.03208	4.90	<.0001	ocpi(t-1)
	AR2_2_1	0.03879	0.05285	0.73	0.4637	D_oep(t-1)
	AR2_2_2	-0.18278	0.10581	-1.73	0.0855	D_ocpi(t-1)

Table 4.16: VEC Model Parameter Estimates

Model Parameter Estimates						
Equation	Parameter	Estimate	Standard Error	t Value	Pr > |t|	Variable
D_oep	CONST1	-9.35870	2.57980	-3.63	0.0004	1
	LTREND1	-0.01176	0.00316	-3.72	0.0003	t, EC
	AR1_1_1	-0.16123	0.04336	-3.72	0.0003	oep(t-1)
	AR1_1_2	0.33340	0.08966	3.72	0.0003	ocpi(t-1)
	AR1_1_3	-0.04081	0.01098	-3.72	0.0003	oil(t-1)
	AR2_1_1	-0.17660	0.12492	-1.41	0.1589	D_oep(t-1)
	AR2_1_2	-0.07533	0.21450	-0.35	0.7258	D_ocpi(t-1)
	AR2_1_3	0.49034	0.07862	6.24	<.0001	D_oil(t-1)
D_ocpi	CONST2	-2.40831	1.37370	-1.75	0.0810	1
	LTREND2	-0.00317	0.00168	-1.88	0.0613	t, EC
	AR1_2_1	-0.04343	0.02309	-1.88	0.0613	oep(t-1)
	AR1_2_2	0.08981	0.04774	1.88	0.0613	ocpi(t-1)
	AR1_2_3	-0.01099	0.00584	-1.88	0.0613	oil(t-1)
	AR2_2_1	-0.14879	0.06652	-2.24	0.0263	D_oep(t-1)
	AR2_2_2	-0.02260	0.11422	-0.20	0.8433	D_ocpi(t-1)
	AR2_2_3	0.21075	0.04186	5.03	<.0001	D_oil(t-1)
D_oil	CONST3	-13.20765	2.48837	-5.31	<.0001	1
	LTREND3	-0.01628	0.00305	-5.34	<.0001	t, EC
	AR1_3_1	-0.22316	0.04182	-5.34	<.0001	oep(t-1)
	AR1_3_2	0.46145	0.08648	5.34	<.0001	ocpi(t-1)
	AR1_3_3	-0.05649	0.01059	-5.34	<.0001	oil(t-1)
	AR2_3_1	-0.04939	0.12049	-0.41	0.6823	D_oep(t-1)
	AR2_3_2	0.35424	0.20690	1.71	0.0883	D_ocpi(t-1)
	AR2_3_3	0.32509	0.07583	4.29	<.0001	D_oil(t-1)

The sample standard deviations of the OEP and OCPI residuals are 4.659 and 2.600, respectively. The Kolmogorov-Smirnov test does not reject the null hypothesis of normality of these residuals at 5% with the p-values of 0.083 and 0.086, respectively.

ARMA models of departures from long-run equilibrium

Table 4.17: departures from long-run equilibrium (Model 4.3.1)

	AR1	AR2	MA1	MA2	ARMA1,1	ARMA 1,2	ARMA 2,1
Dtregress							
Constant	-0.575	-1.007	-0.0467	-0.0580	-1.068	-0.899	-0.885
	(2.595)	(3.331)	(0.885)	(1.048)	(3.499)	(3.231)	(3.160)
ARMA							
Lag 1	0.857***	0.636***			0.923***	0.906***	0.397
	(0.0405)	(0.0774)			(0.0329)	(0.0381)	(0.264)
Lag 2		0.259***					0.462*
		(0.0770)					(0.224)
e1			0.601***	0.719***	-0.257**	-0.256**	0.258
			(0.0680)	(0.0592)	(0.0881)	(0.0879)	(0.295)
e2				0.622***		0.0972	
				(0.0648)		(0.0796)	
sigma							
Constant	4.600***	4.441***	6.632***	5.367***	4.465***	4.444***	4.432***
	(0.255)	(0.251)	(0.423)	(0.333)	(0.250)	(0.253)	(0.251)
Observations	156	156	156	156	156	156	156
AIC	926.1	917.3	1039.4	976.2	919.0	919.5	918.7
BIC	935.3	929.5	1048.6	988.4	931.2	934.8	934.0

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The results show that the AR2 model has the best fit.

Table 4.18: OEP regress on WTI - (Model 4.3.2)

	AR1	AR2	MA1	MA2	ARMA1,1	ARMA 1,2	ARMA 2,1
Dtregress							
Constant	-0.143	-0.0574	-0.000797	0.00523	-0.0684	-0.0466	-0.0565
	(2.251)	(1.771)	(0.666)	(0.831)	(1.920)	(1.757)	(1.748)
ARMA							
Lag 1	0.878***	1.086***			0.826***	0.781***	1.138***
	(0.0378)	(0.0746)			(0.0466)	(0.0648)	(0.332)
Lag 2		-0.239***					-0.284
		(0.0673)					(0.293)
e1			0.758***	0.873***	0.240**	0.311***	-0.0548
			(0.0556)	(0.0555)	(0.0769)	(0.0836)	(0.348)
e2				0.692***		0.119	
				(0.0513)		(0.0917)	
sigma							
Constant	3.434***	3.336***	4.661***	3.832***	3.346***	3.332***	3.335***
	(0.172)	(0.164)	(0.224)	(0.180)	(0.165)	(0.166)	(0.165)
Observations	156	156	156	156	156	156	156
AIC	835.2	828.2	929.8	871.5	829.1	829.8	830.2
BIC	844.3	840.4	938.9	883.7	841.3	845.1	845.4

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The results shows that the AR2 and ARMA(1,1) models provide the best fit.