GAMES Practice Problem Solutions – Multivariable Calculus

1. (a) $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = -y^2e^{-t} + 2xy\cos t = -(\sin^2 t)e^{-t} + 2e^{-t}\sin t\cos t$

   (b) $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = 2e^y - 2txe^y = 2e^{1-t^2} - 4t^2xe^{1-t^2}$

2. (a) $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = e^y(2u) + xe^y(2u) = e^{u^2-v^2}(2u) + 2u(u^2+v^2)e^{u^2-v^2}$

   $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} = e^y(2v) - xe^y(2v) = 2ve^{u^2-v^2} - 2v(u^2+v^2)e^{u^2-v^2}$

   (b) $\frac{\partial z}{\partial u} = \frac{1}{uv}\cos(\frac{\ln u}{v})$, $\frac{\partial z}{\partial v} = -\frac{1}{uv^2}\cos(\frac{\ln u}{v})$

3. $\frac{dV}{dt} = \frac{dI}{dt}R + I\frac{dR}{dt} = \frac{dI}{dt}R + I(\frac{\partial R}{\partial R_1}\frac{dR_1}{dt} + \frac{\partial R}{\partial R_2}\frac{dR_2}{dt}) \approx 0.3812$

4. (a) $f_x = 6xy+5y^3$, $f_y = 3x^2+15xy^2$, $f_{xy} = 6x+15y^2$,, $f_{yx} = 6x+15y^2$, $f_{xx} = 6y$, $f_{yy} = 30xy$,

   (b) $f_x = 3(x+y)^2$, $f_y = 3(x+y)^2$, $f_{xy} = 6(x+y)$, $f_{yx} = 6(x+y)$, $f_{xx} = 6(x+y)$, $f_{yy} = 6(x+y)$

   (c) $f_x = 2ye^{2xy}$, $f_y = 2xe^{2xy}$, $f_{xy} = 2e^{2xy} + 4xye^{2xy}$, $f_{yx} = 2e^{2xy} + 4xye^{2xy}$, $f_{xx} = 4y^2e^{2xy}$, $f_{yy} = 4x^2e^{2xy}$

5. $f(x,y) \approx f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(x-0) + \frac{1}{2}f_{xx}(0,0)(x-0)^2 + \frac{1}{2}f_{yy}(0,0)(y-0)^2 + f_{xy}(0,0)(x-0)(y-0) = 1 + 2x - 2y + x^2 - 2xy + y^2$.

6. (a) $\frac{\partial m}{\partial t} = rPe^{rt}$, $\frac{\partial m}{\partial P} = e^{rt}$

   $\frac{\partial m}{\partial t}$ represents the change in the amount of money in the bank as one unit of time passes by.

   $\frac{\partial m}{\partial P}$ represents the change in the amount of money in the bank given an increase of \$1 in the principal.

   (b) If the price of salmon, p_S, increases, we expect that the amount sold, s, should drop. Thus we expect $\frac{\partial S}{\partial p_S}$ to be negative.

   (c) If the price of trout, p_T, increases, then we expect there might be some people who will now buy salmon instead. Thus we expect $\frac{\partial S}{\partial p_T}$ to be positive.

   (d) If $\frac{\partial S}{\partial p_S} = -50$, it means (approximately) for a \$1 increase in the price of salmon, s, the store will sell 50 kg less salmon per week.

7. (a) Global maximum is at $(0,0)$; global minimum on the boundary points $(-1,-1)$, $(-1,1)$, $(1,-1)$, $(1,1)$

   (b) Global maximum is at $(1,0)$ and $(-1,0)$; global minimum on the boundary points $(0,-1)$, $(0,1)$.

8. (a) No global maximum or minimum

   (b) No global maximum, global minimum at $(0,0)$

   (c) No global maximum or minimum

   (d) No global minimum, global maximum at $(0,0)$

   (e) No global maximum or minimum.

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