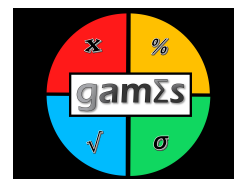
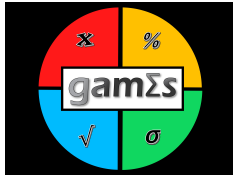


GAMES Practice Problem Solutions – Multivariable Calculus



1. (a) $\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} = -y^2 e^{-t} + 2xy \cos t = -(\sin^2 t)e^{-t} + 2e^{-t} \sin t \cos t$
 (b) $\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} = 2e^y - 2txe^y = 2e^{1-t^2} - 4t^2 x e^{1-t^2}$
2. (a) $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial u} = e^y(2u) + xe^y(2u) = e^{u^2-v^2}(2u) + 2u(u^2 + v^2)e^{u^2-v^2}$
 $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial v} = e^y(2v) - xe^y(2v) = 2ve^{u^2-v^2} - 2v(u^2 + v^2)e^{u^2-v^2}$
 (b) $\frac{\partial z}{\partial u} = \frac{1}{uv} \cos\left(\frac{\ln u}{v}\right), \frac{\partial z}{\partial v} = -\frac{1}{uv^2} \cos\left(\frac{\ln u}{v}\right)$
3. $\frac{dV}{dt} = \frac{dI}{dt}R + I\frac{dR}{dt} = \frac{dI}{dt}R + I\left(\frac{\partial R}{\partial R_1} \frac{dR_1}{dt} + \frac{\partial R}{\partial R_2} \frac{dR_2}{dt}\right) \approx 0.3812$
4. (a) $f_x = 6xy + 5y^3, f_y = 3x^2 + 15xy^2, f_{xy} = 6x + 15y^2, f_{yx} = 6x + 15y^2, f_{xx} = 6y, f_{yy} = 30xy,$
 (b) $f_x = 3(x+y)^2, f_y = 3(x+y)^2, f_{xy} = 6(x+y), f_{yx} = 6(x+y), f_{xx} = 6(x+y), f_{yy} = 6(x+y)$
 (c) $f_x = 2ye^{2xy}, f_y = 2xe^{2xy}, f_{xy} = 2e^{2xy} + 4xye^{2xy}, f_{yx} = 2e^{2xy} + 4xye^{2xy}, f_{xx} = 4y^2e^{2xy},$
 $f_{yy} = 4x^2e^{2xy}$
5. $f(x, y) \approx f(0, 0) + f_x(0, 0)(x - 0) + f_y(0, 0)(y - 0) + \frac{1}{2}f_{xx}(0, 0)(x - 0)^2 + \frac{1}{2}f_{yy}(0, 0)(y - 0)^2 + f_{xy}(0, 0)(x - 0)(y - 0) = 1 + 2x - 2y + x^2 - 2xy + y^2.$
6. (a) $\frac{\partial m}{\partial t} = rPe^{rt}, \frac{\partial m}{\partial P} = e^{rt}$
 $\frac{\partial m}{\partial t}$ represents the change in the amount of money in the bank as one unit of time passes by.
 $\frac{\partial m}{\partial P}$ represents the change in the amount of money in the bank given an increase of \$1 in the principal.
 (b) If the price of salmon, p_S , increases, we expect that the amount sold, s , should drop. Thus we expect $\frac{\partial S}{\partial p_S}$ to be negative.
 (c) If the price of trout, p_T , increases, then we expect there might be some people who will now buy salmon instead. Thus we expect $\frac{\partial S}{\partial p_T}$ to be positive.
 (d) If $\frac{\partial S}{\partial p_S} = -50$, it means (approximately) for a \$1 increase in the price of salmon, s , the store will sell 50 kg less salmon per week.
7. (a) Global maximum is at $(0, 0)$; global minimum on the boundary points $(-1, -1), (-1, 1), (1, -1), (1, 1)$
 (b) Global maximum is at $(1, 0)$ and $(-1, 0)$; global minimum on the boundary points $(0, -1), (0, 1).$
8. (a) No global maximum or minimum
 (b) No global maximum, global minimum at $(0, 0)$
 (c) No global maximum or minimum
 (d) No global minimum, global maximum at $(0, 0)$
 (e) No global maximum or minimum.



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