

Development of an Orbital LIDAR System for Mars Atmospheric Research

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Abstract

A Monte Carlo model was developed to simulate multiple scattering and depolarization of LIDAR signals in water ice clouds and mineral dust in the atmosphere of Mars. This model was used to test the viability of an orbital LIDAR system for the study of the Martian atmosphere. It was found that a system with the same laser as the Earth-based CALIPSO orbital LIDAR system but with a receiver telescope with half the diameter of CALIPSO's is capable of producing reliable measurements of typical Martian cloud and dust conditions.

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1 Introduction

Water ice clouds are a significant feature of the atmosphere of Mars and have been the subject of many studies since the 1970s (Curran et al., 1973; French et al., 1981; Smith, 2008; Whiteway et al., 2009; Wolff et al., 2019). The LIDAR system onboard the Phoenix lander, which landed on Mars in 2008, is the only LIDAR system that has been used to study the atmosphere of Mars (Whiteway et al., 2008). Analysis of the Phoenix LIDAR data revealed that water ice clouds on Mars have similarities to cirrus clouds on Earth (Whiteway et al., 2009). Due to the immobility of the Phoenix lander, the LIDAR system was locked to one position on the surface. This restriction limited the Phoenix LIDAR's observations to the atmosphere above the northern plains of Mars, specifically near 68° N, 233° E (Christensen Research Group, 2023). Consequently, LIDAR systems have not been used to probe the global cloud structure around Mars. Information regarding the global cloud structure around Mars would help us learn more about the water cycle of Mars and help improve Mars global circulation models. The goal of this project is to determine the requirements to make atmospheric measurements from orbit around Mars and investigate possible new capabilities of Mars atmospheric LIDAR systems.

Several satellites have used active and passive techniques to measure properties of the atmosphere and surface of Mars. Notably, the Mars Climate Sounder onboard the Mars Reconnaissance Orbiter passively measures visible and infrared radiation coming from the surface and atmosphere of Mars (NASA, 2024). The Mars Climate Sounder enables the construction of 3D global weather maps of Mars, showing features such as temperature, pressure, humidity, and dust with a vertical resolution of 5 km (NASA, 2024). While passive spectrometers such as MCS provide valuable information regarding the global climate system of Mars, LIDAR offers a unique advantage for studying clouds, dust, and other aerosols in the atmosphere due to its ability to provide high-resolution vertical profiles. For example, the LIDAR system onboard the Phoenix lander had a 50 m vertical resolution in its standard operating mode (Whiteway et al., 2008). LIDAR uses laser pulses to measure the time it takes for light to travel to a target and back, allowing it to detect the precise altitude and distribution of cloud particles and airborne dust. This vertical resolution enables researchers to study the structure and movement of atmospheric layers with greater

detail, even differentiating between types of particles based on their scattering properties. Additionally, LIDAR's active nature means it can work both day and night, making it a versatile tool for continuous monitoring of dynamic atmospheric conditions. A LIDAR system has already been successfully implemented in orbit around Mars, namely the Mars Orbiter Laser Altimeter (MOLA) onboard the Mars Global Surveyor satellite (MGS). MOLA was primarily used to map the surface topography of Mars, and did not map clouds or dust in high detail (Smith et al., 2001). MOLA's success in mapping the surface of Mars is a promising prospect for a satellite LIDAR system designed to study the Martian atmosphere.

There are several satellite LIDAR systems that have been used to study the atmosphere of Earth. The CALIPSO satellite used LIDAR to map the global cloud structure around Earth (Winker et al., 2010). The profiling and polarization capabilities of CALIPSO's LIDAR system provided valuable information regarding the distribution and properties of aerosols and clouds (Winker et al., 2010). Similar to CALIPSO, the newly developed EarthCARE satellite uses a LIDAR system to map the distribution of clouds and aerosols in the atmosphere of Earth (Wehr et al., 2023). The data collected by EarthCARE helps improve climate models and our understanding of radiative transfer within clouds and aerosols (Wehr et al., 2023). The success of such systems on Earth highlight their potential value for studying the atmosphere of Mars.

2 LIDAR and Scattering Background

2.1 Basic LIDAR Principles

LIDAR (light detection and ranging) systems operate by emitting pulses of laser light and detecting backscattered photons that correspond to the pulses. The time, t , between pulse emission and backscatter detection gives the distance between the detector and the scattering event through the relationship $d = \frac{ct}{2}$, where d is the distance, and c is the speed of light; the factor of 2 in the denominator accounts for light travelling from the detector to the scattering particle and then back to the detector from the scattering particle. A simple depiction of the implementation of a LIDAR system for atmospheric studies is shown in Figure 1.

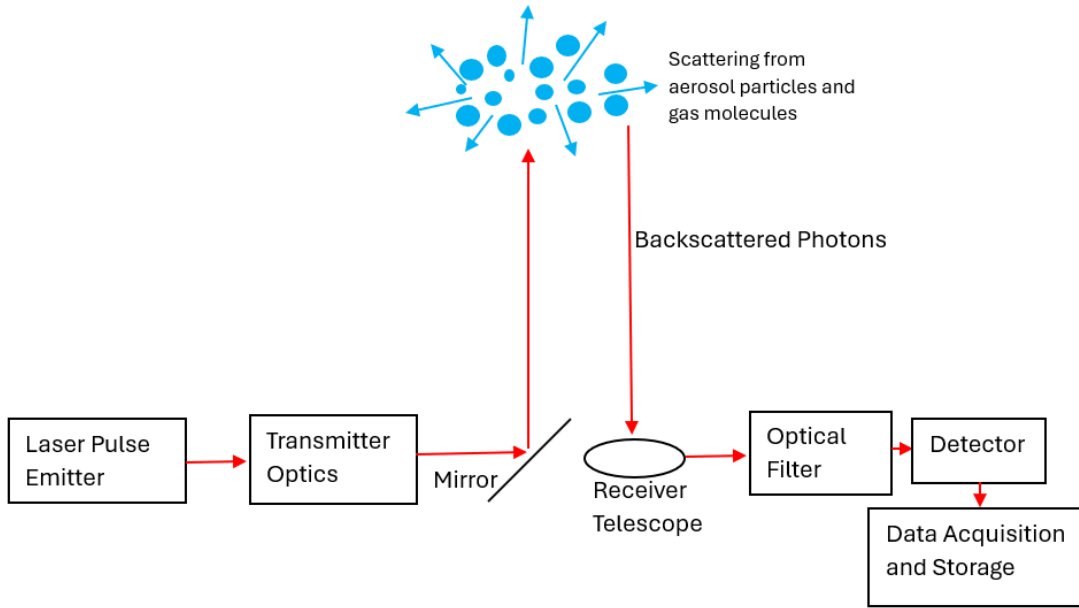


Figure 1: LIDAR system configuration used in this study.

The intensity, $I(f)$, of laser pulses of frequency, f , travelling (in the z -direction) through the atmosphere obey Beer's law, given by

$$dI(f) = -I(f)\sigma(f,z)dz, \quad (1)$$

where $dI(f)$ represents the change in intensity of a plane wave travelling a distance dz through

a medium with extinction coefficient $\sigma(f, z)$ (Zuev, 1976). The extinction coefficient, $\sigma(f, z)$, accounts for the transmission properties of the medium through which the radiation is travelling, and its units are typically given in the form of km^{-1} (Zuev, 1976). Integrating Equation 1 from z_1 to z_2 gives

$$I(f) = I_0(f) \exp \left(- \int_{z_1}^{z_2} \sigma(f, z) dz \right), \quad (2)$$

where $I(f)$ is the intensity after travelling through the medium, and $I_0(f)$ is the intensity at distance z_1 from the source (Zuev, 1976). We are only interested in the backscattered photons, so we need a term that accounts for the amount of light scattered at an angle of π relative to the incident radiation. This term is known as the backscatter coefficient and is given by

$$\beta = \rho \frac{d\alpha(\pi)}{d\Omega}, \quad (3)$$

where ρ is the number density of scattering particles, $\alpha(\pi)$ is the effective backscatter area per scattering particle, and Ω is the solid angle of the receiver as seen from the scattering particle (Collis and Russell, 1976). The backscatter coefficient, β , represents the fractional amount of incident energy that is backscattered per unit solid angle and per unit length. Typical units for β are $\text{m}^{-1}\text{Sr}^{-1}$. With all of this in hand, the number of backscattered photons detected by a LIDAR system from a layer of atmosphere of thickness Δz is given by the single-scattering LIDAR equation:

$$N(z) = Q \frac{A}{z^2} \beta(z) \Delta z N_0 \exp \left(-2 \int_{z_0}^z \sigma(z') dz' \right). \quad (4)$$

In the LIDAR equation,

- Q is a dimensionless quantity that accounts for the efficiency of the LIDAR system.
- $\frac{A}{z^2}$ is the area of the receiver telescope divided by the square of the distance to the scattering event, or the solid angle of the receiver as seen by the scattering event (units of Sr). The $\frac{1}{z^2}$ factor accounts for the decreasing solid angle of the receiver with increasing distance z .
- $\beta(z)$ is the backscatter coefficient as a function of height. It also depends on the number and type of scattering particles.

- Δz is the heights interval over which the received photons are being measured.
- $N_0 \exp \left(-2 \int_{z_1}^{z_2} \sigma(z) dz \right)$ is Beer's Law in terms of power, where N_0 is the number of photons in the emitted laser pulse. The factor of 2 accounts for the roundtrip distance.

Signals detected by a LIDAR system are represented by $N(z)$ in Equation 4. This means that the LIDAR signal can be used to obtain information about the backscatter (β) and extinction (σ) coefficients of the medium through which the laser is propagating.

A typical LIDAR system is composed of a laser pulse emitter, transmitter optics, a receiver telescope, an optical filter, a photodetector, typically a photomultiplier tube (PMT) or avalanche photodiode (APD), and a data acquisition and storage system. As illustrated in Figure 1, portions of each laser pulse are backscattered off of atmospheric constituents and collected by the telescope. The FOV is set by a circular aperture field stop. After passing through the telescope, the light is passed through a collimating lens. The collimated light is then sent through an interference filter with a small pass band centered on the laser wavelength. The filtered light then enters the photodetector. To enable the counting of individual photons for relatively weak signals, PMTs are typically used. PMTs use electron multiplication to generate current that corresponds to detected photons. When counting photons, a discriminator is used to set a voltage cut-off for pulses from the PMT. Only pulses that exceed the cut-off are counted as photons, allowing noise to be filtered out of the signal. For each pulse, an electrical signal is sent to the scaler. The scaler is used to count pulses from the discriminator. The scaler is set to count pulses for a certain period of time, which determines the height resolution of the system. The scaler data is sent to the multi-channel averager (MCA). The dwell clock outputs a square wave with a period corresponding to the time over which the scaler counts pulses (the height resolution). The MCA receives a trigger signal each time a laser pulse is emitted, at which point it starts adding contents from the scaler to memory bins. On each positive edge of the dwell clock square wave signal, the data from the scaler is transferred to a bin in the MCA, where it is averaged. Each bin then corresponds to the average signal over each height interval. The data processing configuration is shown in Figure 2.

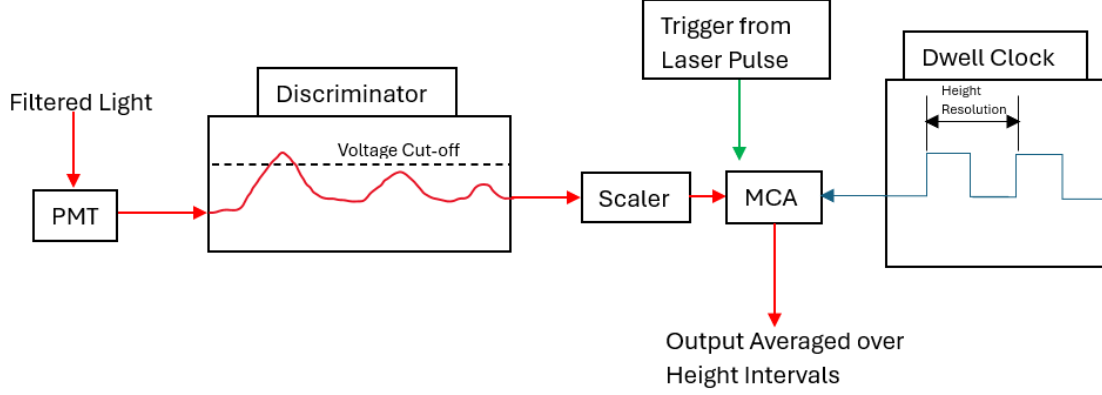


Figure 2: LIDAR data processing configuration.

The laser wavelength for LIDAR systems varies depending on the application. Systems used to study clouds and dust in planetary atmospheres typically use a laser wavelength of 532 nm or 1064 nm. Light at these wavelengths undergoes Mie scattering with most aerosols, and laser pulse emitters are often commercially available at these wavelengths. In this study, we will use a wavelength of 532 nm due to the success of the 532 nm LIDAR system onboard Phoenix as well as the success of the 532 nm LIDAR system onboard the CALIPSO satellite.

When measuring backscattered photons with a LIDAR system, the height from which the photons are detected are grouped into vertical bins. The size of each bin is determined by the time resolution of the data acquisition equipment. For example, if the vertical resolution is 10 m, the time resolution of the data acquisition equipment is $t = (10 \text{ m}) \cdot \frac{2}{c}$, where c is the speed of light. Additionally, there is an integration time over which the photon counts from individual bins are averaged. Since a satellite-based LIDAR system is moving while taking measurements, the integration time, combined with the speed of the satellite and the rotational speed of the orbited planet, determine the resolution of the LIDAR system along the surface (or along the layer of atmosphere being measured).

2.2 Single Scattering

The way in which light scatters off of particles depends on the wavelength of the light and the scattering properties of the particles. Specifically, the scattering angle and polarization state of a

photon at a specific wavelength are given by the matrix

$$\mathbf{F}(\theta) = \begin{bmatrix} F_{11} & F_{12} & 0 & 0 \\ F_{21} & F_{22} & 0 & 0 \\ 0 & 0 & F_{33} & F_{34} \\ 0 & 0 & F_{43} & F_{44} \end{bmatrix}, \quad (5)$$

where θ is the scattering angle. Using the terminology of Hovenier and van der Mee (1983), we will refer to the matrix $\mathbf{F}(\theta)$ as the *scattering matrix*. The scattering matrix is characteristic of the scattering particle at a specific wavelength of incident light. For completely unpolarized light,

$$\mathcal{F}(\theta) = F_{11}(\theta), \quad (6)$$

where $\mathcal{F}(\theta)$ is known as the *phase function*. $\mathcal{F}(\theta)$ gives a probability distribution for the scattering angle of the incident light. For polarized light, the phase function depends on both the scattering angle, θ , and the angle of rotation into the scattering plane, ξ . The phase function for polarized light is given by

$$\mathcal{F}(\theta, \xi) = F_{11}(\theta)I + F_{12}(\theta) (Q \cos(2\xi) + U \sin(2\xi)), \quad (7)$$

where I , Q , and U are elements of the Stokes vector of the incident light (Ramella-Roman et al., 2005). The Stokes vector will be explained in Section 2.4.

When a photon interacts with a scattering particle, there is a chance that it will be absorbed by the particle, rather than scattered. Again, the amount of light which is absorbed is dependent on the characteristics of the scattering particle and the wavelength of incident light. The previously-discussed extinction coefficient, σ , gives the amount of energy removed from a laser pulse per unit length. The amount of energy that is scattered can be represented by σ_{scat} . Of the energy that is removed from the laser pulse, the fraction that is scattered is given by the single scattering albedo:

$$\omega = \frac{\sigma_{scat}}{\sigma}, \quad \omega \leq 1. \quad (8)$$

In a collision between a photon and a scattering particle, ω represents the probability that the

photon will be scattered, while $1 - \omega$ represents the probability that the photon will be absorbed. The scattering matrix, phase function, and single scattering albedo are important quantities that dictate the signal strength and depolarization ratios measured by an atmospheric LIDAR system. These quantities can theoretically be computed for aerosols, like clouds and dust, using Mie scattering theory, developed by Gustav Mie to describe the scattering of light from large, spherical particles. For atmospheric molecules, these quantities can be computed using Rayleigh scattering theory, developed by Lord Rayleigh to describe the scattering of light from small particles, such as gas molecules.

2.3 Multiple Scattering

The LIDAR equation given by Equation 4 assumes single scattering only. This assumption means that only photons that undergo one scattering event that directs them towards the receiver are considered. However, many photons scatter in other directions besides that of the receiver. These photons can scatter several times, especially within dense media, such as clouds and dust. Some of these multiply scattered photons may scatter into the receiver. Since these multiply scattered photons don't obey the simple $d = \frac{ct}{2}$ relationship between distance and time, a significant amount of multiple scattering can distort the signal. As proposed by Kunkel and Weinman (1976), the LIDAR Equation can be corrected for multiple scattering by adding a factor to the extinction coefficient, as follows:

$$N(z) = Q \frac{A}{z^2} \beta(z) \Delta z N_0 \exp \left(-2 \int_{z_0}^z \sigma(z') (1 - F(z')) dz' \right), \quad (9)$$

where $F(z')$ is a factor that accounts for multiple scattering (Kunkel and Weinman, 1976). The amount of multiple scattering that a LIDAR system detects is dependent on several factors. The scattering angle probability distribution of the scattering particles plays a role in the amount of multiple scattering, as well as the density of the scattering medium. Additionally, the field-of-view of the receiver affects the amount of multiple scattering seen by the LIDAR system. With a larger FOV, the receiver can 'see' more multiply scattered photons, leading to more signal distortion. Especially in the case of satellites, the distance of the scattering medium from the LIDAR system also affects the amount of signal distortion due to multiple scattering. This is because the footprint of the FOV gets larger with increased distance from the receiver, meaning that the receiver can 'see'

more multiply scattered photons at larger distances. This effect is more significant in spaceborne LIDAR systems than in ground-based LIDAR systems due to the large separation of satellite-based LIDAR systems from the scattering media being measured (Hu et al., 2001). Figure 3 shows how the average distance from a photon in the beam to the boundary of the FOV increases with increasing distance from the LIDAR system. If a photon is farther from the FOV boundary, it has a higher chance of scattering before leaving the FOV, leading to more multiple scattering within the FOV. We will use a Monte Carlo method to simulate multiple scattering for our satellite-based LIDAR system.

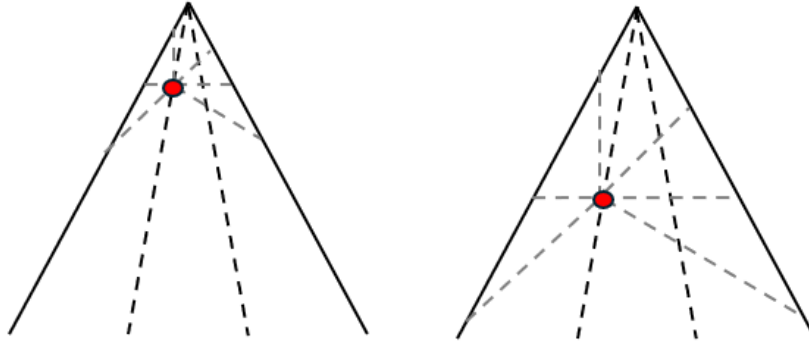


Figure 3: Diagram showing the relationship between multiple scattering and distance from the receiver. The inner lines represent the beam divergence, and the outer lines represent the FOV. The red dots represent photons. The lines radiating from the red dots represent the distance from the photon to the edge of the FOV.

2.4 Depolarization

The mathematical description of the polarization state of a beam of light is described nicely by Hansen and Travis (1974). A short summary of their description will be provided here. The intensity, I , of a beam of light can be written as

$$I = I_{up} + I_p, \quad (10)$$

where I_{up} is the unpolarized part of the beam, and I_p is the polarized part of the beam. The polarized part of the beam, I_p can be written as

$$I_p = \sqrt{I_l^2 + I_c^2}, \quad (11)$$

where I_l is the linearly polarized part of the beam, and I_c is the circularly polarized part of the beam. Continuing to follow the description given by Hansen and Travis (1974), we define the components of the electric field of a beam of light as

$$E_l = a_l e^{i(\omega t - kz - \epsilon_l)}, \quad (12)$$

$$E_r = a_r e^{i(\omega t - kz - \epsilon_r)}, \quad (13)$$

where the subscripts r and l represent mutually perpendicular directions represented by unit vectors $\hat{r}$ and $\hat{l}$. The a_i terms represent the electric field amplitudes in each direction, and the terms ϵ_i represent the phases. The term ω represents the circular frequency of the light beam, t represents time, and k represents the wavenumber. The direction of propagation is positive z , given by the cross product $\hat{r} \times \hat{l}$. A set of parameters called the Stokes parameters are time averages that can be used to completely describe the polarization state of light. Once again following from Hansen and Travis (1974), the Stokes parameters can be written as

$$I = \langle E_l E_l^* + E_r E_r^* \rangle, \quad (14)$$

$$Q = \langle E_l E_l^* - E_r E_r^* \rangle, \quad (15)$$

$$U = \langle E_l E_r^* + E_r E_l^* \rangle, \quad (16)$$

$$V = i \langle E_l E_r^* - E_r E_l^* \rangle, \quad (17)$$

where $\langle \rangle$ indicates a time average. The I and Q Stokes parameters can be written as

$$I = I_l + I_r, \quad (18)$$

$$Q = I_l - I_r, \quad (19)$$

where I_l and I_r are the intensities of light in the $\hat{l}$ and $\hat{r}$ directions, respectively. In practice, the $\hat{l}$ and $\hat{r}$ components of intensity can be measured using a polarizer. When making measurements

with LIDAR systems, a linear depolarization ratio, δ , is commonly defined as

$$\delta = \frac{I - Q}{I + Q} = \frac{I_r}{I_l}. \quad (20)$$

In practice, the linear depolarization ratio is represented by

$$\delta = C \frac{N_r}{N_l}, \quad (21)$$

where N_r and N_l are the perpendicular and parallel components of the measured signal, respectively. C is a constant that accounts for differences in the channels used to measure the parallel and perpendicular components of the signal. Laser light is linearly polarized, with Stokes vector

$$\begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}. \quad (22)$$

The depolarization ratio of linearly polarized laser light is

$$\delta = \frac{I - Q}{I + Q} = \frac{1 - 1}{1 + 1} = 0. \quad (23)$$

When light scatters, it tends to depolarize both from single scattering and multiple scattering. The extent to which the light is depolarized depends on the shape, size and composition of the scattering particles. The depolarization ratio can be useful in differentiating between scattering media, such as clouds and dust. Since multiple scattering depolarizes light, the measured depolarization ratio can be distorted by a large amount of multiple scattering. As mentioned previously, this effect is especially significant in the case of orbital LIDAR systems due to their large FOV footprints. The Monte Carlo model described in this report also tracks the polarization state of scattered photons in order to simulate the depolarization measured by LIDAR systems.

3 Monte Carlo Model

3.1 Monte Carlo Basics

Monte Carlo techniques are probabilistic in nature. Using Monte Carlo techniques, individual events can be simulated, if the probability distributions representing all possible outcomes for such events are known. Monte Carlo simulations use random number sampling to determine outcomes for individual events based on the underlying probability distributions. The simulation of many individual events allows for the simulation of an entire process, with the outcome of the process being entirely dependent on the probability distributions associated with the individual events. The process we follow in this work starts with computing a cumulative probability distribution, $\mathcal{P}(x)$:

$$\mathcal{P}(x) = P(\epsilon \leq x) = \int_{-\infty}^x f(x)dx, \quad (24)$$

representing the probability that a random variable ϵ is less than or equal to x . We sample values of x from this distribution by setting the cumulative probability distribution equal to a random number, ρ :

$$\mathcal{P}(x) = \rho, \quad (25)$$

where ρ is a random number distributed uniformly between 0 and 1. The functional inverse of $\mathcal{P}(x)$ is given by

$$\mathcal{G}(\mathcal{P}(x)) = x. \quad (26)$$

The values of x sampled from $\mathcal{P}(x)$ are given by

$$x_i = \mathcal{G}(\rho_i). \quad (27)$$

This concept is illustrated in Figure 4.

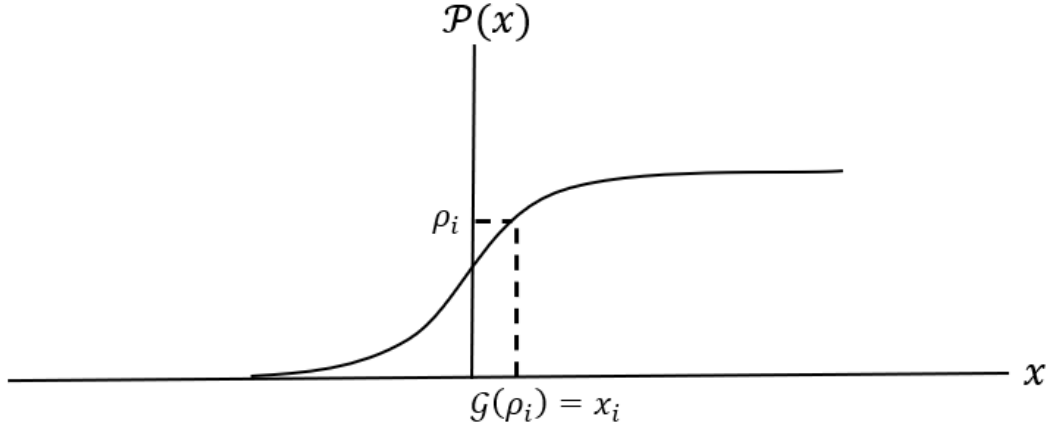


Figure 4: An illustration of sampling a random variable, x_i , from a cumulative probability distribution, $\mathcal{P}(x)$.

Monte Carlo techniques have been used extensively to simulate particle and light transport (Spanier and Gelbard, 1969; Collins and Wells, 1965; Haghighat, 2020; Ramella-Roman et al., 2005). Monte Carlo techniques have been successfully implemented to study multiple scattering and depolarization from atmospheric LIDAR signals (Kunkel and Weinman, 1976; Kattawar and Plass, 1968; Donovan et al., 2015). Previous studies have used Monte Carlo techniques to simulate the effects of multiple scattering on signals from the orbital LIDAR systems CALIPSO (Hu et al., 2001) and EarthCARE (Reverdy et al., 2015). When simulating scattering of LIDAR signals using a Monte Carlo approach, there are several events that must be sampled from pre-computed probability distributions. The distance that a photon travels between collisions is randomly sampled from an exponential distribution (Kunkel and Weinman, 1976), as described in full in Step 3 of Section 3.3. In the case of a mixture of different types of scattering particles, the type of particle with which the photon collides is randomly sampled from the ratio of extinction coefficients, as described in full in Step 4 of Section 3.3. At each collision, the photon is either scattered or absorbed as determined by random sampling from the single scattering albedo, as described in Step 5 of Section 3.3. The scattering angle that a photon takes after colliding with a particle is randomly sampled from a distribution computed from the phase function of the scattering particle, as described in full in Step 8 of Section 3.3. At each collision, the probability of the photon entering the telescope receiver is computed, as described in full in Step 6 of Section 3.3. The sum of these

probabilities is used to represent the LIDAR signal strength within height bins that represent the vertical resolution of the LIDAR system (see Step 6 of Section 3.3). Using this probability summation technique to represent the signal strength is much faster than tracking each photon until it enters the receiver and has been successfully implemented in several previous studies (Kunkel and Weinman, 1976; Hu et al., 2001). As described in Step 7 of Section 3.3, this same probability summation technique is used to simulate the Stokes vector of the received signal in a process which is implemented and described by Hu et al. (2001). Many of these individual probabilistic events are carried out in order to simulate the process of laser pulses scattering in a planetary atmosphere and a LIDAR receiver measuring the corresponding backscatter signal. The random number generator used in our Monte Carlo model is the permuted congruential generator (64-bit, PCG64) provided in the Python library NumPy (NumPy Developers, 2024). The PCG64 random number generator is an implementation of the permutation congruential generator described by O'Neill (2014) and was chosen due to its reproducibility of number sequences while passing many statistical randomness tests (O'Neill, 2014).

3.2 Scattering Properties

The way in which light scatters from particles in the atmosphere depends on the scattering properties of the particles. In the Monte Carlo model, the extinction coefficient, phase function, and full scattering matrix are used to determine the scattering path and polarization state of a photon. An extinction profile (extinction coefficient as a function of height) is needed as input for the model. In the model's current configuration, only vertical variation is allowed in the extinction profile. The photons can move in three dimensions, but the path length can only be computed for extinction profiles that only vary in the z -direction. The extinction profiles used in our model are typically based on LIDAR measurements from the Phoenix mission or from simulations of Martian ice clouds. When multiple types of scattering particles, such as cloud and dust, are mixed together at a given altitude, the total extinction is given by the sum of the extinction coefficients from each particle type. The concept of an extinction profile with variation only in the z -direction

is shown in Figure 5. The extinction profile is converted into an optical depth profile as follows:

$$\tau(z_n) = \sum_{i=0}^n \sigma(z_i)(z_{i+1} - z_i), \quad (28)$$

where n represents the index of the height bin for which τ is computed.

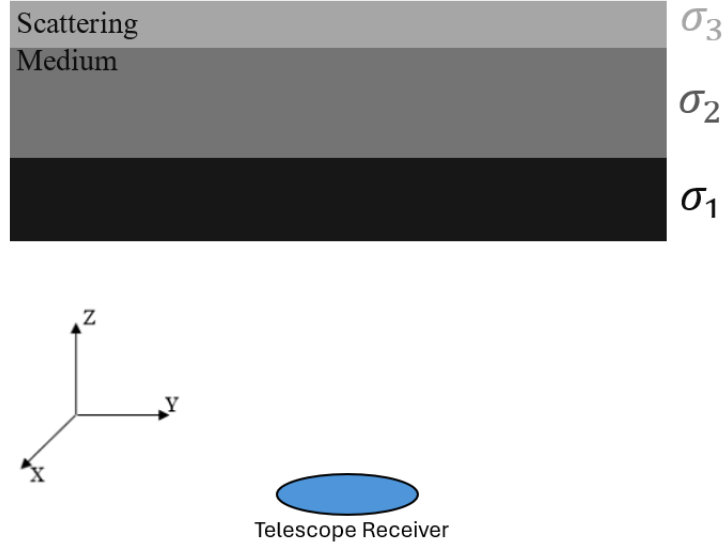


Figure 5: A visualization of a vertical extinction profile used to set up the scattering medium for the Monte Carlo model.

As discussed previously, the scattering angle is dependent on the phase function of the scattering particle, and the polarization state is dependent on the full scattering matrix. For isotropic homogeneous spheres, the scattering matrix can be represented by four independent parameters (Hansen and Travis, 1974):

$$\mathbf{F}(\theta) = \begin{bmatrix} F_{11} & F_{21} & 0 & 0 \\ F_{21} & F_{11} & 0 & 0 \\ 0 & 0 & F_{33} & -F_{43} \\ 0 & 0 & F_{43} & F_{33} \end{bmatrix}. \quad (29)$$

For spherical particles, the scattering matrix can be computed using Mie theory. The scattering matrix depends on both the composition of the scattering particle and the size of the scattering particle. It is often not practical to consider only one size of scattering particle, in which case the

particle concentration distribution as a function of radius, $n(r)$ (typical units of $\mu\text{m}^{-1}\text{L}^{-1}$), can be used to compute the scattering matrix over a range of sizes. Following from Deirmendjian (1969), for a polydispersion of spherical particles of common composition, each element of the scattering matrix (normalized over full solid angle) is given by

$$\frac{F_j(\theta)}{4\pi} = \frac{1}{k^3\sigma_{\text{scat}}} \int_0^\infty n(x)i_j(\theta)dx, \quad (30)$$

where $x = kr$ is a dimensionless size parameter that relates the particle radius, r , to the wavelength of incident light, λ , and $k = \frac{2\pi}{\lambda}$. σ_{scat} is the scattering coefficient. $n(x)$ is the scattering particle concentration as a function of the dimensionless size parameter, x . $i(\theta)$ is known as the intensity parameter and is described nicely by Deirmendjian (1969). The intensity parameter can be computed using Mie theory; in our model we use the Python module `miepython` (Prah, 2024) to compute intensity parameters for spherical particles. The scattering efficiency, $Q_{\text{scat}}(x)$, is defined as the scattering cross section normalized by the geometrical cross section (Deirmendjian, 1969). Following from Deirmendjian (1969), σ_{scat} is related to $Q_{\text{scat}}(x)$ by

$$\sigma_{\text{scat}} = \pi k^{-3} \int_0^\infty x^2 n(x) Q_{\text{scat}}(x) dx. \quad (31)$$

$Q_{\text{scat}}(x)$ can be computed using Mie theory. Once again, our model uses the Python module `miepython` (Prah, 2024) to compute scattering efficiencies for spherical particles. Equations 30 and 31 can be easily converted to sums for computation using discrete datasets.

In many cases, assuming that the scattering particles are spherical is not very accurate. This is certainly so for water ice crystals in clouds and aerosolized dust particles. For non-spherical scattering particles, the scattering matrix can be represented by six independent parameters (Hansen and Travis, 1974):

$$\mathbf{F}(\theta) = \begin{bmatrix} F_{11} & F_{21} & 0 & 0 \\ F_{21} & F_{22} & 0 & 0 \\ 0 & 0 & F_{33} & -F_{43} \\ 0 & 0 & F_{43} & F_{44} \end{bmatrix}. \quad (32)$$

The scattering matrix given by Equation 32 can only be used for a group of randomly oriented particles in which every particle has at least one plane of symmetry **OR** for a group of randomly oriented asymmetric particles in which half of the particles are mirror images of the other half (Hansen and Travis, 1974). Computing scattering matrices for non-spherical particles is more complicated than the spherical case. Consequently, we use data libraries of scattering matrix parameters to construct scattering matrices for water ice crystals and aerosolized dust. For water ice crystals, we use the data library developed by Yang et al. (2013) for randomly oriented atmospheric ice crystals. For dust, we use the data library developed by Saito et al. (2021) for irregular hexahedral aerosol particles. Again, it is often useful to consider a range of particle shapes and sizes, which can be accomplished by averaging the elements of the scattering matrix. For non-spherical particles of common composition, the average scattering matrix elements are given by

$$\langle F_j(\theta) \rangle = \sum_{i=0}^N W_i \cdot F_i(\theta), \quad (33)$$

where W_i is a weight parameter given by

$$W_i = \frac{n_i \cdot A_i \cdot Q_{ext,i} \cdot \omega_i}{\sum_{k=0}^N n_k \cdot A_k \cdot Q_{ext,k} \cdot \omega_k}, \quad (34)$$

where N represents the total number of combinations of particle size and particle shape, and i indicates an individual combination of particle size and shape. n_i represents the concentration $[\mu\text{m}^{-1}\text{L}^{-1}]$ of a particle of a certain size and shape. A_i represents the projected area of the particle. $Q_{ext,i}$ represents the extinction efficiency of the particle. The extinction efficiency is simply the extinction cross section normalized by the geometrical cross section (Deirmendjian, 1969). ω_i represents the single scattering albedo of the particle. Similarly, the average single scattering albedo for an ensemble of particles of common composition is given by

$$\langle \omega \rangle = \sum_{i=0}^N W_i \cdot \omega_i, \quad (35)$$

where W_i is a weight parameter given by

$$W_i = \frac{n_i \cdot A_i \cdot Q_{ext,i}}{\sum_{k=0}^N n_k A_k Q_{ext,k}}. \quad (36)$$

For Martian water ice clouds, we will base the particle size and shape distributions on measurements of Earth-based cirrus clouds (Gallagher et al., 2005). This assumption is made due to similar temperature and humidity conditions in the cloud formation regions of Earth-based cirrus clouds and Martian water ice clouds (Dickinson et al., 2011). This assumption is further supported by size and shape analysis of Martian water ice cloud crystals based on fall speed data measured by the Phoenix LIDAR (Dickinson et al., 2011). The cirrus cloud ice crystal shapes measured by Gallagher et al. (2005) are broken down into five categories: irregular, spheroidal, column, bullet rosette, and plate. From the database provided by Yang et al. (2013), we will use six categories to represent the ice crystals measured by Gallagher et al. (2005): 5-plate aggregate, 8-column aggregate, solid bullet rosette, droxtal, solid column, and plate. Based on descriptions and data provided by Gallagher et al. (2005), we classify the irregular ice crystals as a combination of 5-plate aggregates, 8-column aggregates, and solid bullet rosettes, and we classify the spheroidal ice crystals as droxtals. According to the observations presented by Gallagher et al. (2005), the size distribution of irregular ice crystals peaks at small particle size, while the size distributions of columns, bullet rosettes, and plates peak at larger sizes. Accordingly, we break down the particle shape distribution into two groups: max dimension (D) less than or equal to $50 \mu\text{m}$ and max dimension (D) greater than $50 \mu\text{m}$. The fraction of the total number density distributed by ice crystal shape can be found in Table 1. The shape distribution presented in Table 1 is based on data and descriptions given by Gallagher et al. (2005). The size distribution used for each particle shape is the same and is based on the average size distribution measured by Gallagher et al. (2005). The maximum dimension of the ice crystals is in the range $4 - 1000 \mu\text{m}$. The scattering matrix elements for each individual particle size/shape combination is taken from the database provided by Yang et al. (2013). The crystals are assumed to be part of a randomly oriented ensemble. The average single-scattering albedo for the water ice crystals is $\langle \omega \rangle \approx 0.99999$. Plots of the average scattering matrix elements for the ice crystals can be found in Figure 6.

$D \leq 50 \mu m$

Measured Shape	Fraction of Total	Database Shape	Fraction of Total
Irregular	60%	5-plate Aggregate	20%
		8-column Aggregate	20%
		Solid Bullet Rosette	20%
Spheroidal	24%	Droxtal	24%
Column	10%	Solid Column	10%
Bullet Rosette	4%	Solid Bullet Rosette	4%
Plate	2%	Plate	2%

$D > 50 \mu m$

Measured Shape	Fraction of Total	Database Shape	Fraction of Total
Irregular	6%	5-plate Aggregate	2%
		8-column Aggregate	2%
		Solid Bullet Rosette	2%
Spheroidal	24%	Droxtal	24%
Column	40%	Solid Column	40%
Bullet Rosette	20%	Solid Bullet Rosette	20%
Plate	10%	Plate	10%

Table 1: Number of ice crystals of specific shapes as a percentage of the total number density. The measured shapes include those measured by Gallagher et al. (2005), and the database shapes include those modelled by Yang et al. (2013).

For Martian dust, we follow from Komguem et al. (2013), Dickinson et al. (2011), and Wolff et al. (2009). For the size distribution, we employ the gamma distribution in the form presented by Hansen and Travis (1974):

$$n(r) = \frac{N(ab)^{\frac{2b-1}{b}}}{\Gamma\left[\frac{1-2b}{b}\right]} r^{\frac{1-3b}{b}} e^{-\frac{r}{ab}}, \quad (37)$$

where r is particle size, N is number density, a is effective radius, b is effective variance, and $\Gamma[x]$ is the Gamma function. Typical units of $n(r)$ are $[\mu m^{-1} L^{-1}]$ or $[\mu m^{-1} cm^{-3}]$. Based on the work of Komguem et al. (2013), Dickinson et al. (2011), and Wolff et al. (2009), we use $a = 1.4 \mu m$, $b = 0.2$, and $N = 4.4 cm^{-3}$. The wavelength of the incident light is $0.532 \mu m$. The refractive index of the dust at this wavelength is $m = 1.5 - 0.011i$ (Komguem et al., 2013). The max dimension of the dust particles is in the range $0.02 - 11 \mu m$. The database described, compiled, and maintained by Saito et al. (2021) is used to compute the scattering matrix elements for an ensemble of hexahedral particles. The dust particles are assumed to be randomly oriented. The average single-scattering albedo for the dust particles is $\langle \omega \rangle \approx 0.88522$. The average scattering matrix elements for the Martian dust can be found in Figure 7.

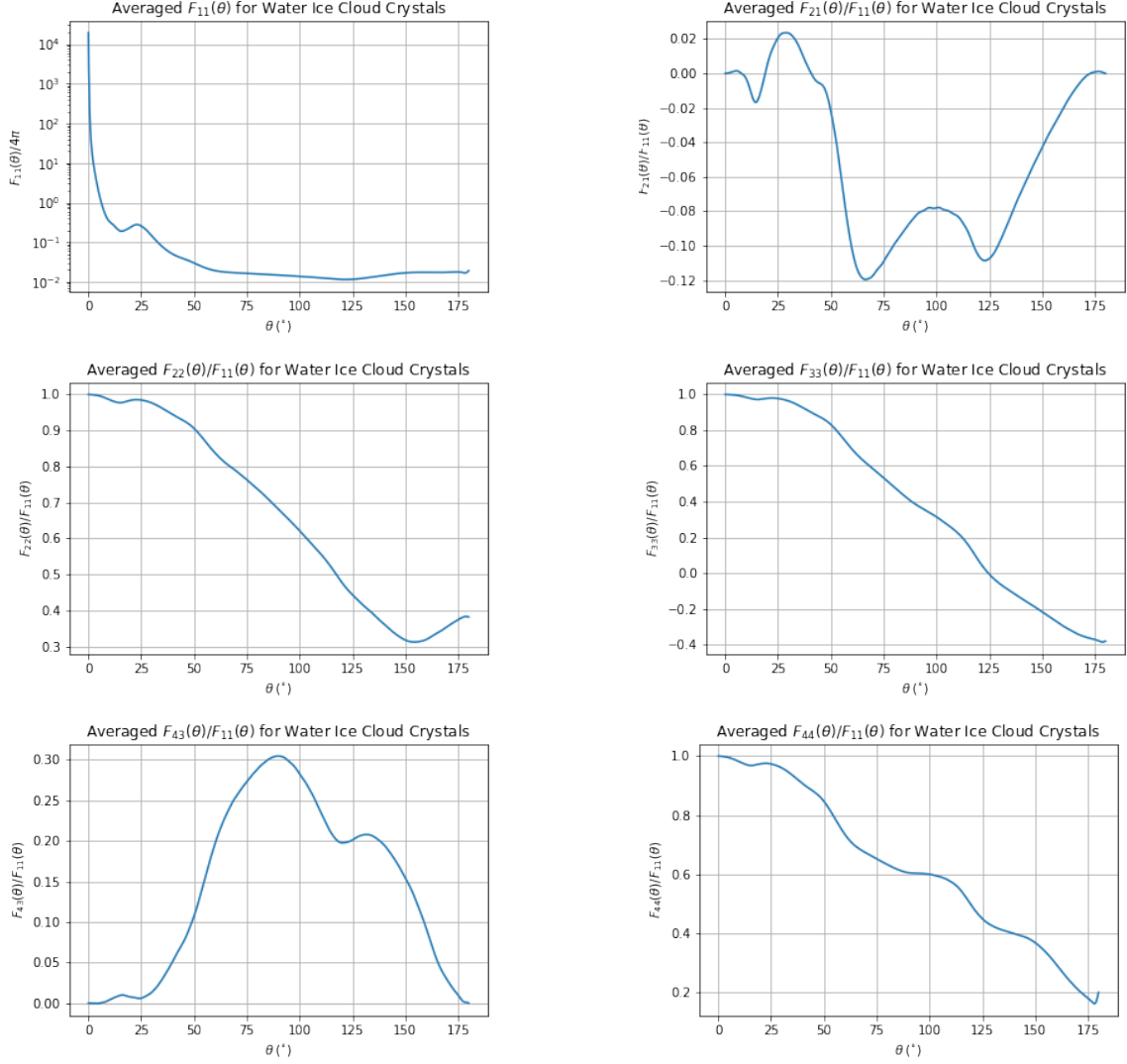


Figure 6: Averaged scattering matrix elements for water ice cloud crystals. The wavelength of the incident light is $0.53 \mu\text{m}$. The maximum ice crystal dimension ranges from $4 - 1000 \mu\text{m}$. The averaging includes 5-plate aggregates, 8-column aggregates, solid bullet rosettes, droxtals, solid columns, and plates. The size distribution of each particle mode is based on measurements of Earth-based cirrus clouds (Gallagher et al., 2005). The individual matrix elements used in the averaging are sourced from the database described, created, and maintained by Yang et al. (2013).

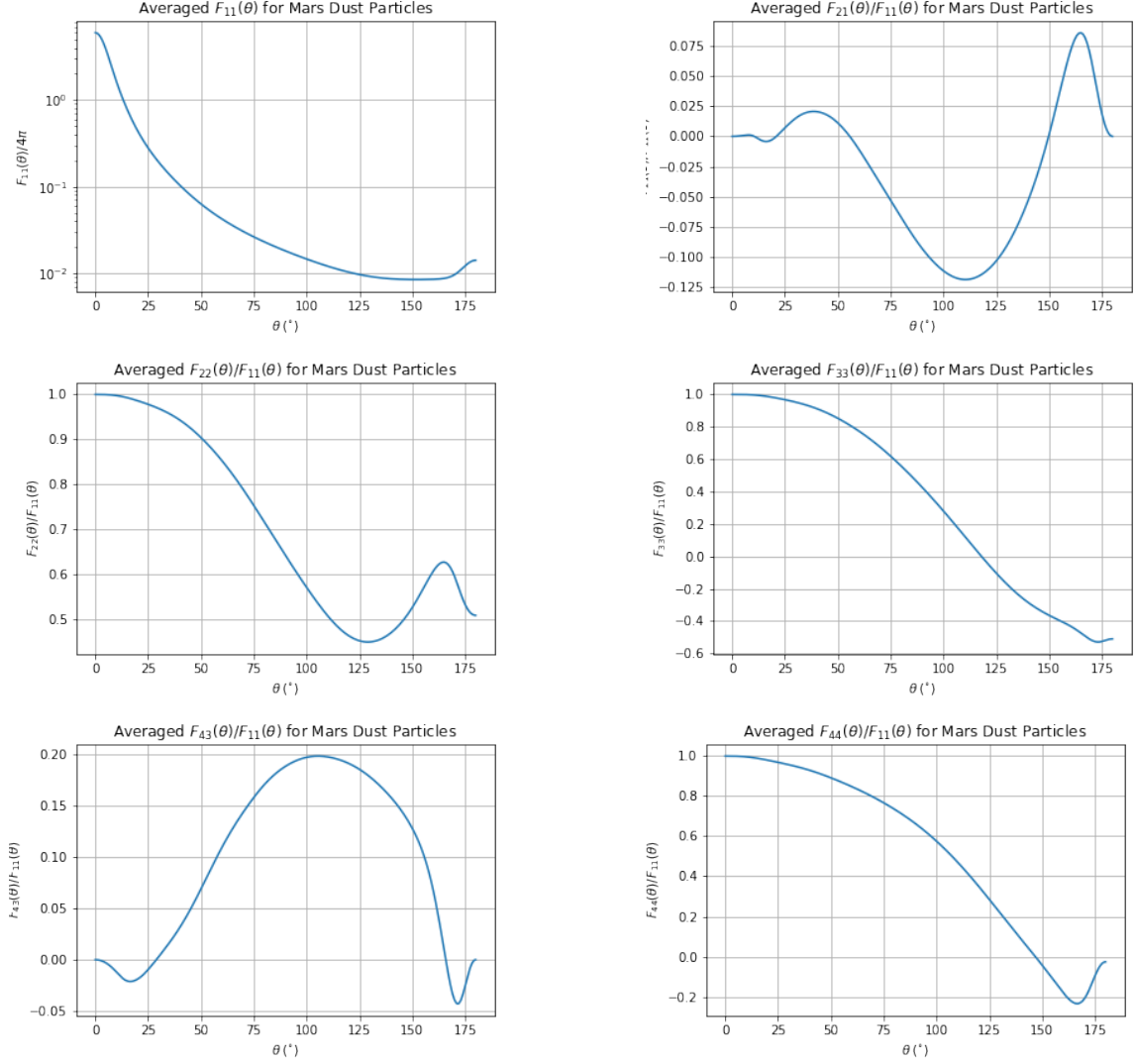


Figure 7: Averaged scattering matrix elements for Martian dust particles. The wavelength of the incident light is $0.532 \mu\text{m}$. The maximum ice crystal dimension ranges from $0.02 - 11 \mu\text{m}$. The size distribution is based on the work of Komguem et al. (2013), Dickinson et al. (2011), and Wolff et al. (2009). The individual matrix elements used in the averaging are sourced from the database described, created, and maintained by Saito et al. (2021).

We will consider multiple cases of Martian water ice clouds and mineral dust. We will consider three cases with no dust: a low altitude water ice cloud, based on clouds observed by the Phoenix lander (Dickinson et al., 2011), a high altitude water ice cloud based on modeling (Frank Daerden, Personal Communication) and observations from the NOMAD instrument on ExoMars Trace Gas Orbiter (Daerden et al., 2022), and a double cloud layer composed of both the low altitude and high

altitude clouds. We will consider 4 cases with mineral dust: a dust-only case based on Martian dust observed by the Phoenix lander (Komguem et al., 2013) and the three cloud cases discussed previously combined with dust. The extinction profiles for each case can be found in Figures 8 and 9. The cloud extinction coefficient value is set to 0.2 km^{-1} based on measurements made by Phoenix (Dickinson et al., 2011). The dust extinction profile is also based on measurements made by Phoenix and is taken from Komguem et al. (2013).

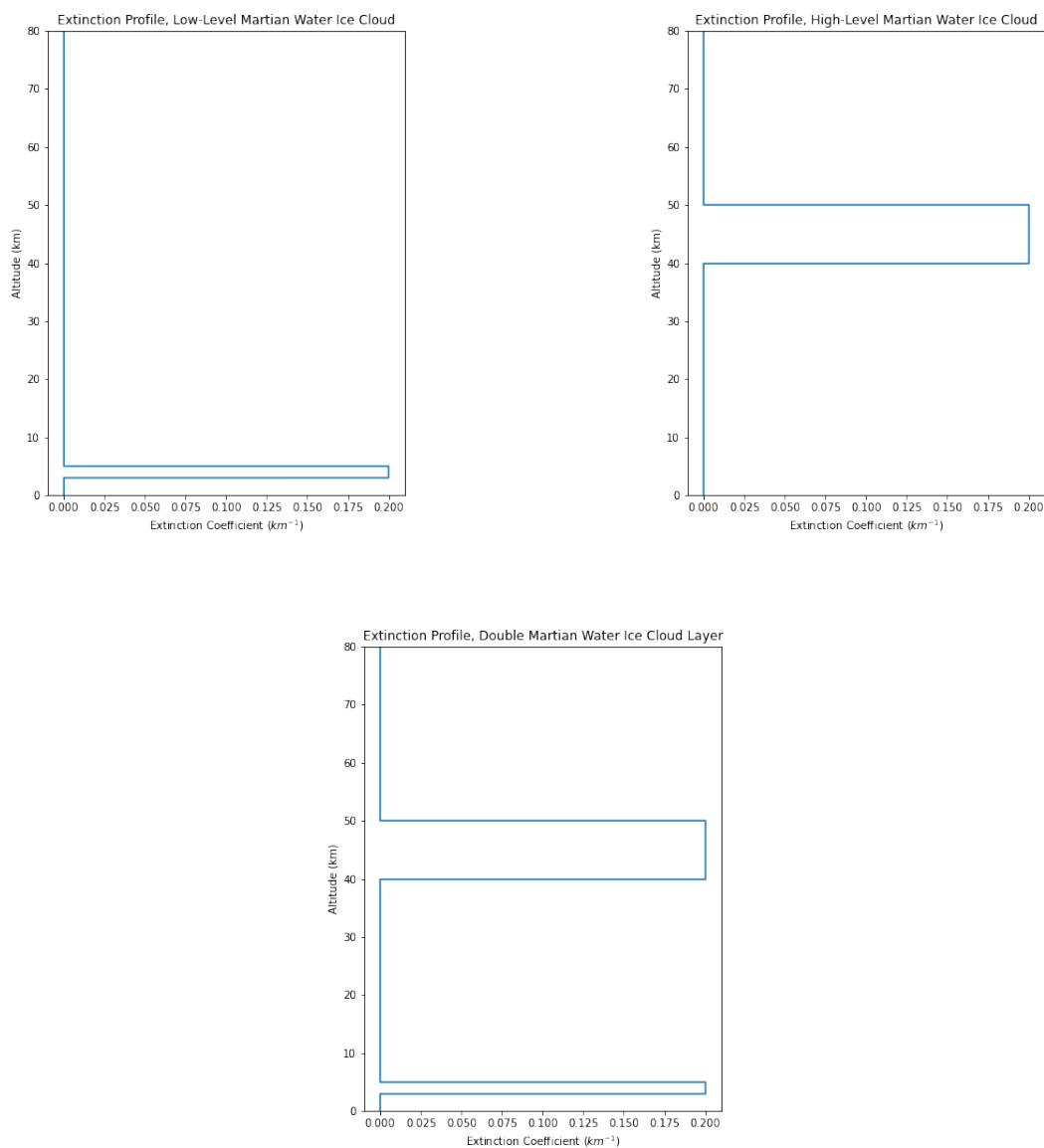


Figure 8: Extinction profiles used in the simulations (cloud only). *Top Left*: Low altitude cloud. *Top Right*: High altitude cloud. *Bottom*: Double cloud layer.

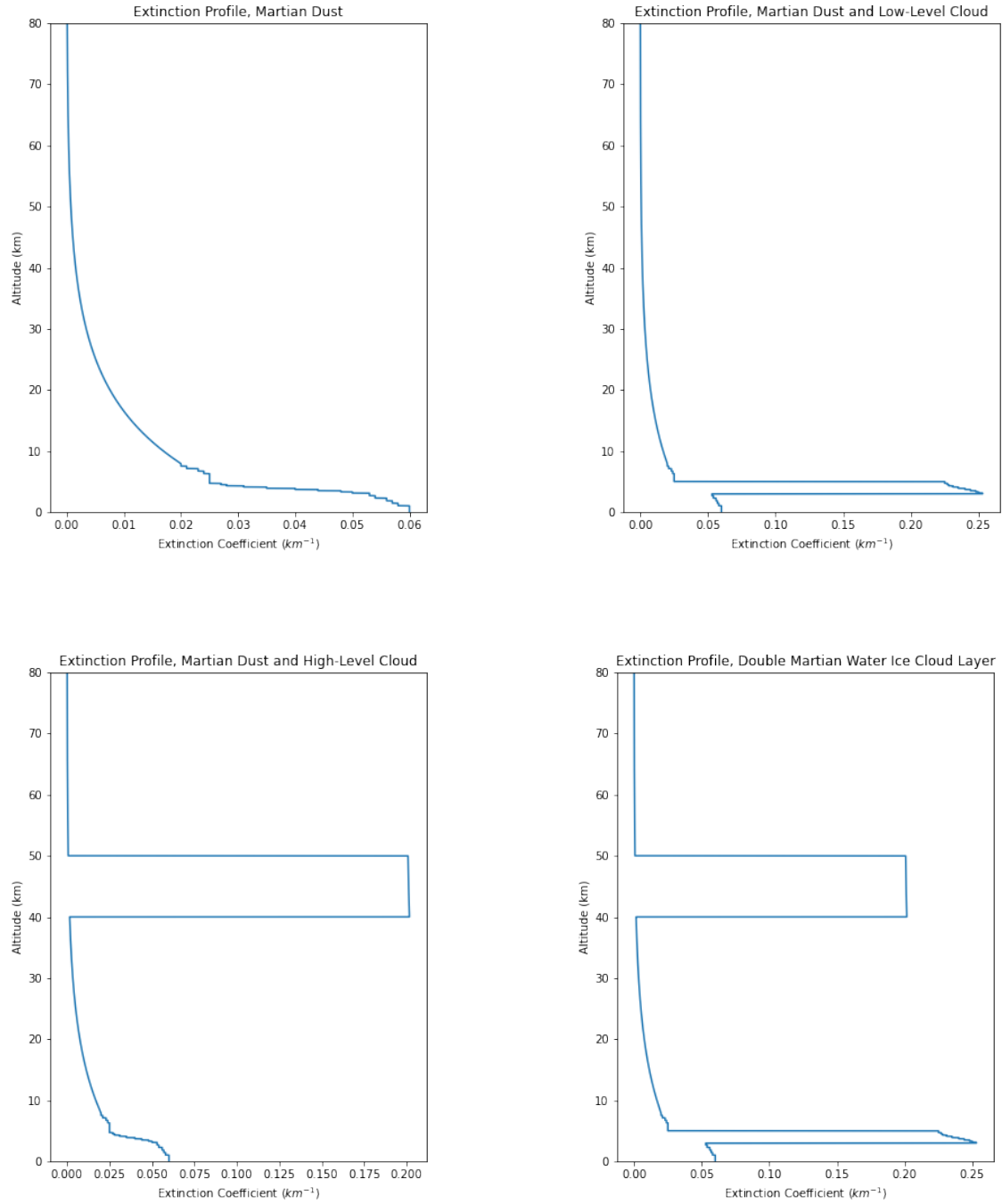


Figure 9: Extinction profiles used in the simulations (including dust). *Top Left*: Dust only. *Top Right*: Dust and low altitude cloud. *Bottom Left*: Dust and high altitude cloud. *Bottom Right*: Dust and double cloud layer.

Together, the extinction and optical depth profiles, the scattering matrix, and the phase function,

allow us to simulate the free path length, scattering direction, and Stokes vector of LIDAR pulses as they scatter within a planetary atmosphere.

3.3 Monte Carlo Simulation Process

To simulate a received LIDAR signal, we use a Monte Carlo approach. The Monte Carlo model tracks the magnitude and polarization state of received LIDAR signals from multiple orders of scattering. The Monte Carlo process used for simulating multiple scattering is generally based on the work of Kunkel and Weinman (1976). Computations of the path length are based on the work of Collins and Wells (1965). Definitions of the direction cosines used for defining photon trajectories are based on the definitions provided by Spanier and Gelbard (1969), Carter and Cashwell (1975), and Peplow (1999). The process used for simulating depolarization is based on the work of Hu et al. (2001), and the relationships used to define the Stokes vector transformations are largely based on the descriptions provided by Hovenier and van der Mee (1983).

We start by defining a telescope receiver of a desired radius and field-of-view. We also set a certain number of photon groups to be simulated, with a desired number of photons in each group. Additionally, we set a beam divergence for the laser and define a scattering medium at a desired distance from the telescope receiver. A visualization of the simulation environment is shown in Figure 10.

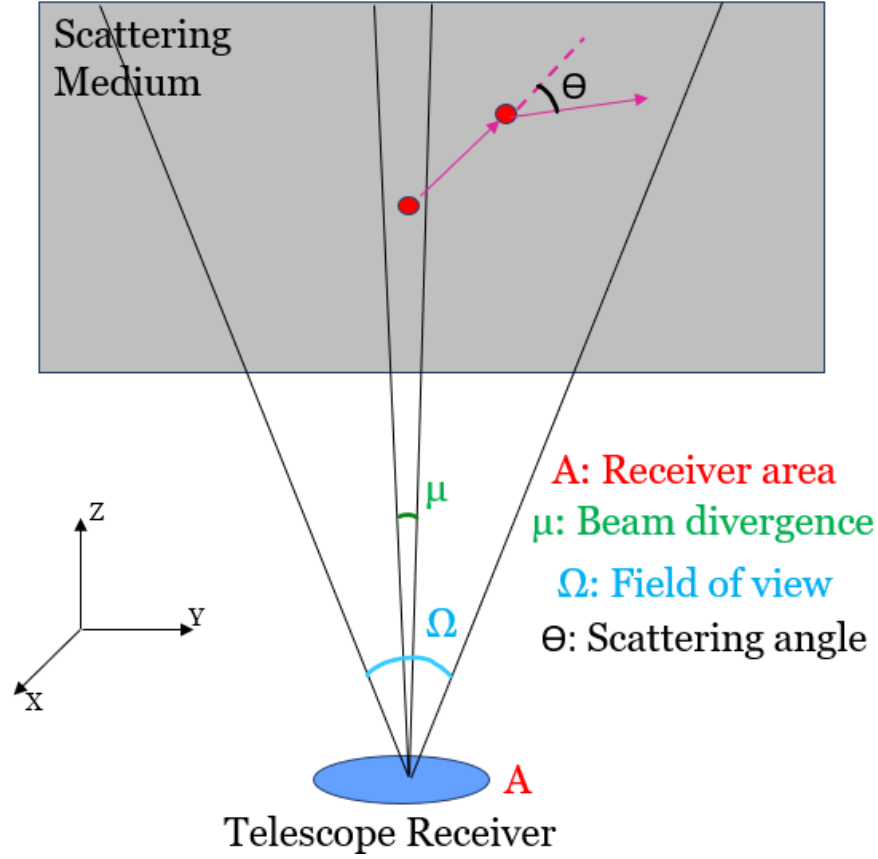


Figure 10: Diagram of the environment used for Monte Carlo simulations.

The steps taken to execute the Monte Carlo model are as follows:

1. A group of photons starts at a heights z_0 from the telescope receiver. The positive z -direction is pointed away from the telescope receiver and towards the scattering medium. The initial distance of each photon from the z -axis is given by

$$R_0 = z_0 \tan \mu \sqrt{\rho}, \quad (38)$$

where μ is the beam divergence, and ρ is a random number sampled uniformly between 0 and 1. The initial azimuthal position of each photon is given by

$$\psi_{az,0} = 2\pi\rho, \quad (39)$$

where ρ is a different random number sampled uniformly between 0 and 1. The initial x and y positions of each photon are given by

$$x_0 = R_0 \cos \psi_{az,0}, \quad (40)$$

$$y_0 = R_0 \sin \psi_{az,0}. \quad (41)$$

The initial geometry of the photons is illustrated in Figure 11.

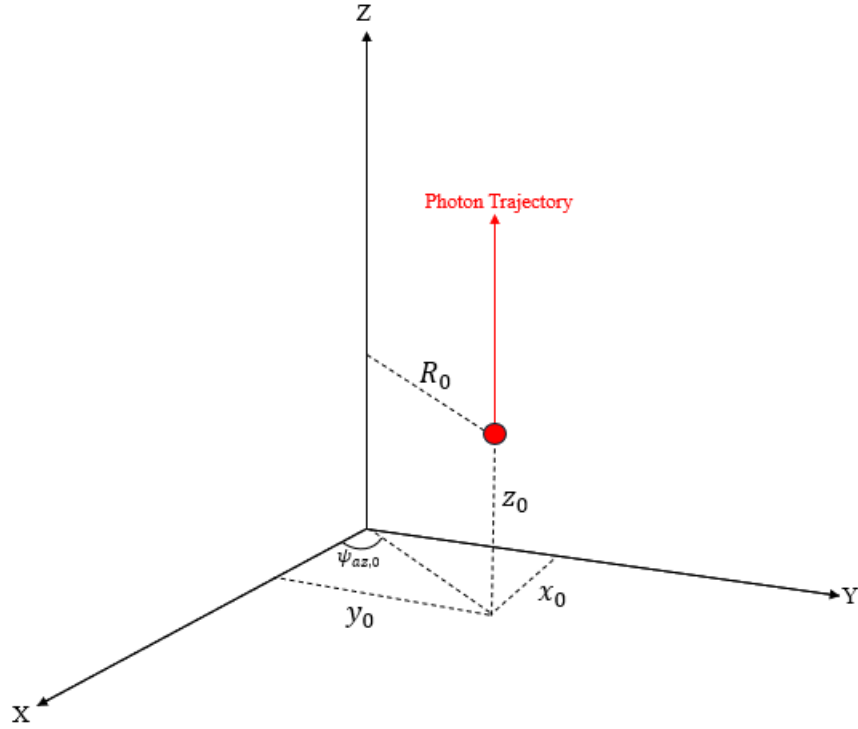


Figure 11: Initial geometry of photons in the Monte Carlo model.

2. Three direction cosines are used to keep track of the trajectory of each photon. The direction cosines are

$$A = \cos \phi_x, \quad (42)$$

$$B = \cos \phi_y, \quad (43)$$

$$C = \cos \phi_z. \quad (44)$$

The geometry of the direction cosines is illustrated in Figure 12.

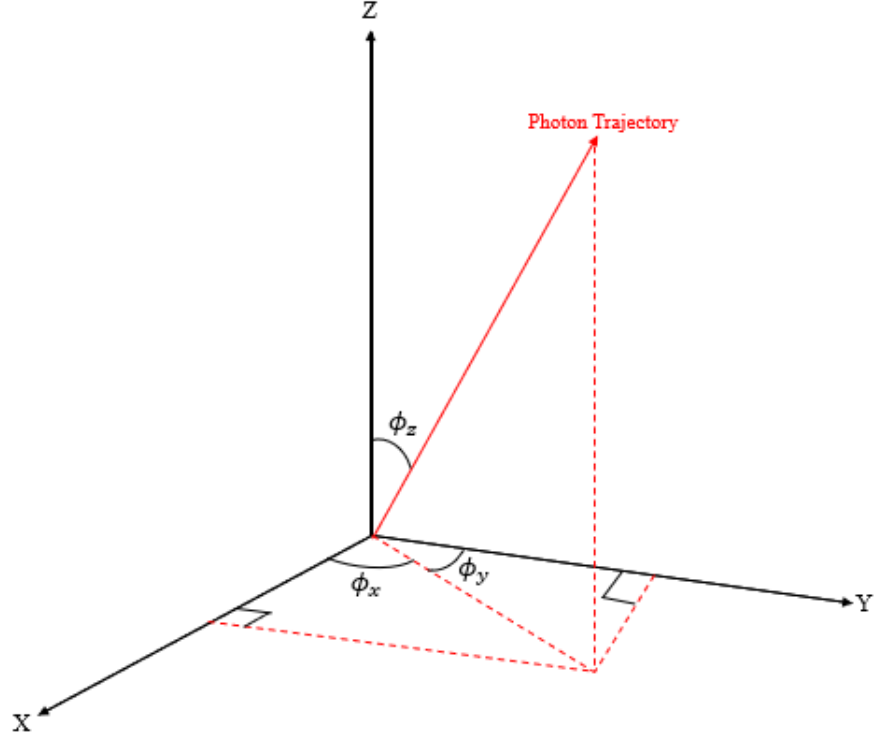


Figure 12: Geometry of the direction cosines used in the Monte Carlo model.

The direction cosines are initialized as $A_0 = 0$, $B_0 = 0$, $C_0 = 1$, so that all of the photons are travelling in the positive z -direction. The initial Stokes vector of each photon is in a linearly polarized state:

$$\begin{bmatrix} I_0 \\ Q_0 \\ U_0 \\ V_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}. \quad (45)$$

3. The vertical optical depth at which a scattering event occurs is given by

$$\tau(z) = -\ln(1 - \rho)C + \tau(z_0), \quad (46)$$

where $\tau(z_0)$ is the initial vertical optical depth, and ρ is another random number sampled uniformly between 0 and 1. The free path length is then given by

$$L = \frac{z - z_0}{C}. \quad (47)$$

An alternative approach must be used when $C = 0$. Note that these relationships are only valid if there is no horizontal variation in extinction coefficient. The geometry of the path length is illustrated in Figure 13.

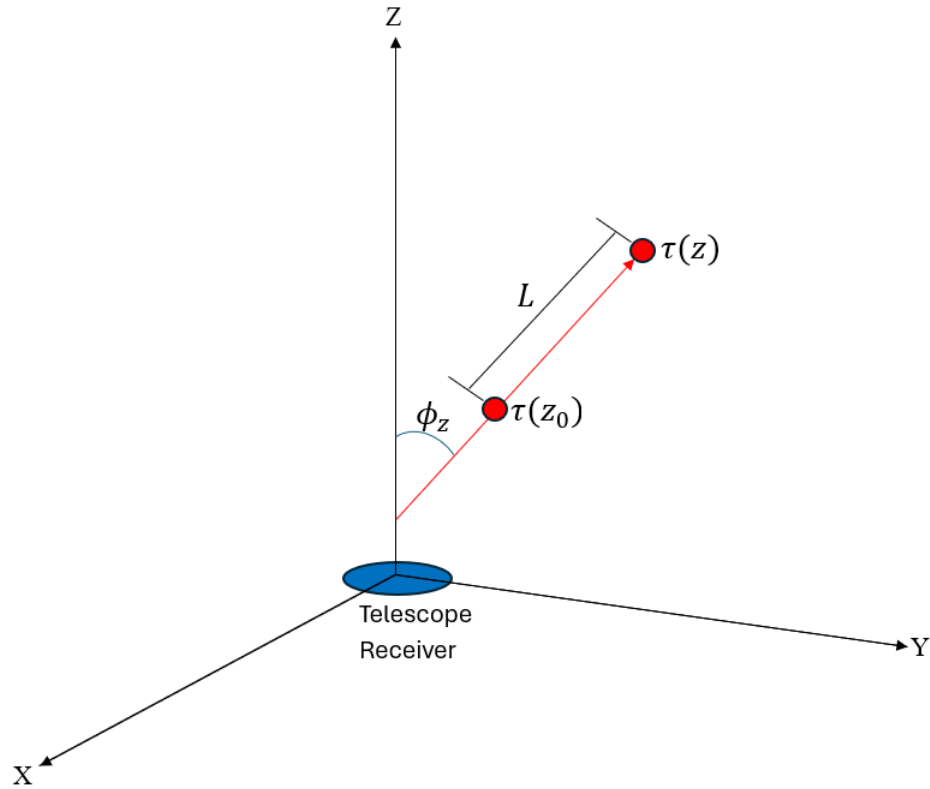


Figure 13: Geometry of the path length used in the Monte Carlo model.

In some cases, a significant number of simulated photons will have free path lengths that send them outside of the scattering medium, or so far outside of the LIDAR field-of-view that they become statistically irrelevant. In order to force more statistically relevant scattering events, the free path length of each photon can be restricted. The goal of this is to reduce the variance of the results without taking a large amount of computation time. To implement

this restriction, the vertical optical depth at which a scattering event occurs is given by

$$\tau(z) = -\ln \left(1 - \rho \left(1 - e^{-\tau(z_{max})} \right) \right) C + \tau(z_0), \quad (48)$$

where $\tau(z_{max})$ is the maximum optical depth that each photon can travel before reaching the boundary within which scattering is forced (Collins and Wells, 1965). In our simulations, this boundary is typically the FOV. Forcing scattering events to occur introduces statistical bias into the results. To account for this, we modify the statistical weight of each photon according to

$$W = W_0 \left(1 - e^{-\tau(z_{max})} \right), \quad (49)$$

where W_0 is the statistical weight of each photon prior to the path length determination (Collins and Wells, 1965). The geometry of the restricted path length is illustrated in Figure 14.

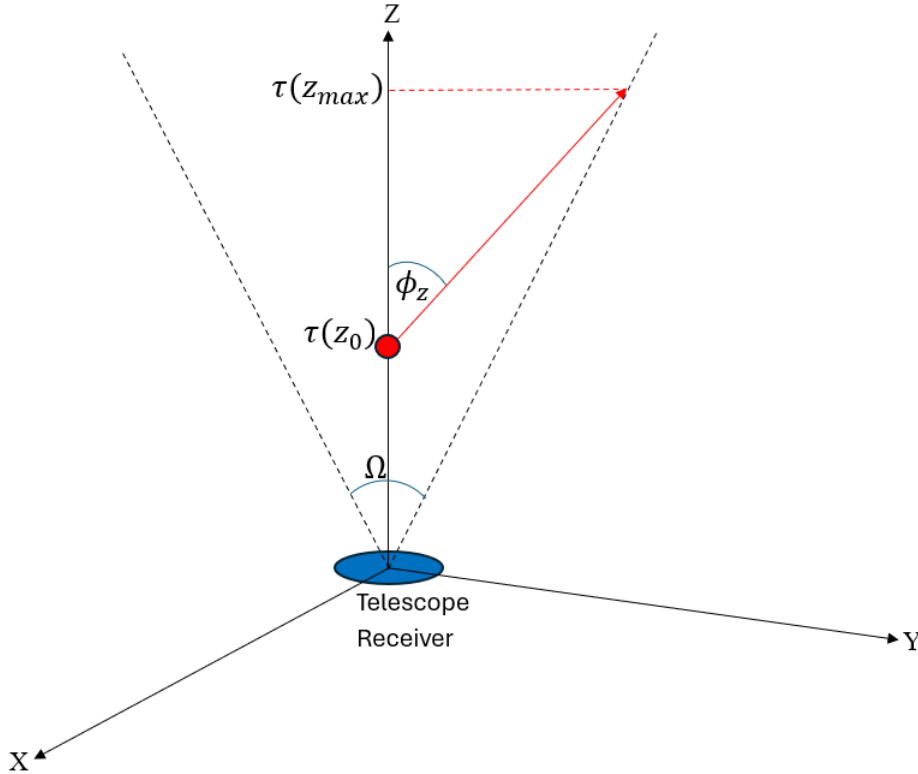


Figure 14: Geometry of the restricted path length used for variance reduction in the Monte Carlo model.

For individual photons, the sum of all free path lengths is tracked and is interpreted as the perceived height of the photons. The position of the scattering event is then given by

$$x = x_0 + AL, \quad (50)$$

$$y = y_0 + BL, \quad (51)$$

$$z = z_0 + CL. \quad (52)$$

With the new scattering locations determined, any photons that have left the scattering medium are removed from the model. The distance of the photons from the z-axis is tracked as

$$R = \sqrt{x^2 + y^2}. \quad (53)$$

The total path length that each photon travels is tracked as

$$L_{tot} = \sum_{i=1}^n L_i, \quad (54)$$

where n is the total number of scattering events for an individual photon.

4. If there is more than one type of scattering particle at a given height, the extinction coefficients of each particle are compared. Given a mixture of only two types of scattering particles, such as dust and cloud, the following ratio is computed using the extinction coefficients at the height of each photon:

$$\mathcal{R} = \frac{\sigma_1(z_1)}{\sigma_1(z_1) + \sigma_2(z_2)}, \quad (55)$$

where $\sigma_1(z_1)$ and $\sigma_2(z_2)$ are the extinction coefficients at heights z_1 and z_2 . The type of scattering particle with which each photon collides is then determined through random number sampling as follows:

$$\mathcal{R} \leq \rho \longrightarrow \text{Particle 1}, \quad (56)$$

$$\mathcal{R} > \rho \longrightarrow \text{Particle 2}, \quad (57)$$

where ρ is a new random number sampled uniformly between 0 and 1. Obviously, this same

process can easily be applied to a mixture of more than two scattering particles.

5. The single scattering albedo of each type of scattering particle is used to account for absorption. Another random number is sampled to determine if a photon is scattered or absorbed:

$$\rho > \omega \longrightarrow \text{absorption}, \quad (58)$$

$$\rho \leq \omega \longrightarrow \text{scatter}, \quad (59)$$

where ω is the single scattering albedo of the scattering particle, and ρ is a new random number sampled uniformly between 0 and 1. Any photons that are absorbed are removed from the simulation.

6. At each scattering event, the received signal is simulated by computing the probability that each photon will scatter directly into the receiver telescope. The geometry of this situation is depicted in Figure 15. The first step in this process is to calculate the return trajectories of each photon. The return scattering angle is given by

$$\theta_r = \pi + \psi_{zen} - \phi_z, \quad (60)$$

where $\psi_{zen} = \arctan\left(\frac{R}{z}\right)$, and $\phi_z = \arccos C$. The return direction cosines are given by

$$A_r = -\frac{x}{L_r}, \quad (61)$$

$$B_r = -\frac{y}{L_r}, \quad (62)$$

$$C_r = -\frac{z}{L_r}, \quad (63)$$

where $L_r = \sqrt{R^2 + z^2}$ is the return path length of the photon. The perceived height of each scattering event is given by

$$H = \frac{L_{tot} + L_r}{2}. \quad (64)$$

The next step in this process is to check if the photons are within the field-of-view. The radius

of the FOV footprint corresponding to the vertical position of each photon is computed as

$$R_{FOV} = z \cdot \tan \Omega_{1/2}, \quad (65)$$

where $\Omega_{1/2}$ is the field-of-view half-angle. If $R > R_{FOV}$, the probability that each photon will scatter into the receiver telescope is set to 0. Following from Kunkel and Weinman (1976), for photons within the field-of-view ($R \leq R_{FOV}$), the probability of directly scattering into the receiver is computed as follows:

$$\mathcal{P} = \mathcal{F}(\theta_r, \xi_{inc}) \Phi e^{-\tau_r}. \quad (66)$$

$0 \leq \theta_r \leq \pi$ is the scattering angle in the direction of the telescope receiver, $0 \leq \xi_{inc} \leq 2\pi$ is the angle of rotation into the scattering plane for the incident direction vector, $\mathcal{F}(\theta_r, \xi_{inc})$ is the phase function corresponding to the return direction (Equation 7), Φ is the solid angle of the receiver, and τ_r is the total optical depth that the photon would traverse along a direct path to the receiver. In short, this probability is a combination of the probability that the photon will scatter in the direction of the receiver and the probability that the photon will not scatter again before exiting the scattering medium. τ_r is computed in a similar manner to the path length in Step 3. The computed values of $\mathcal{P}$ are summed over all scattering events and all photons within *physical* and *perceived* height bins. The *physical* height bins are based on the actual scattering height (z), while the *perceived* height bins are based on the total path length of the photons (H , Equation 64). The *perceived* height bins represent how a measured signal from a real LIDAR system would be interpreted.

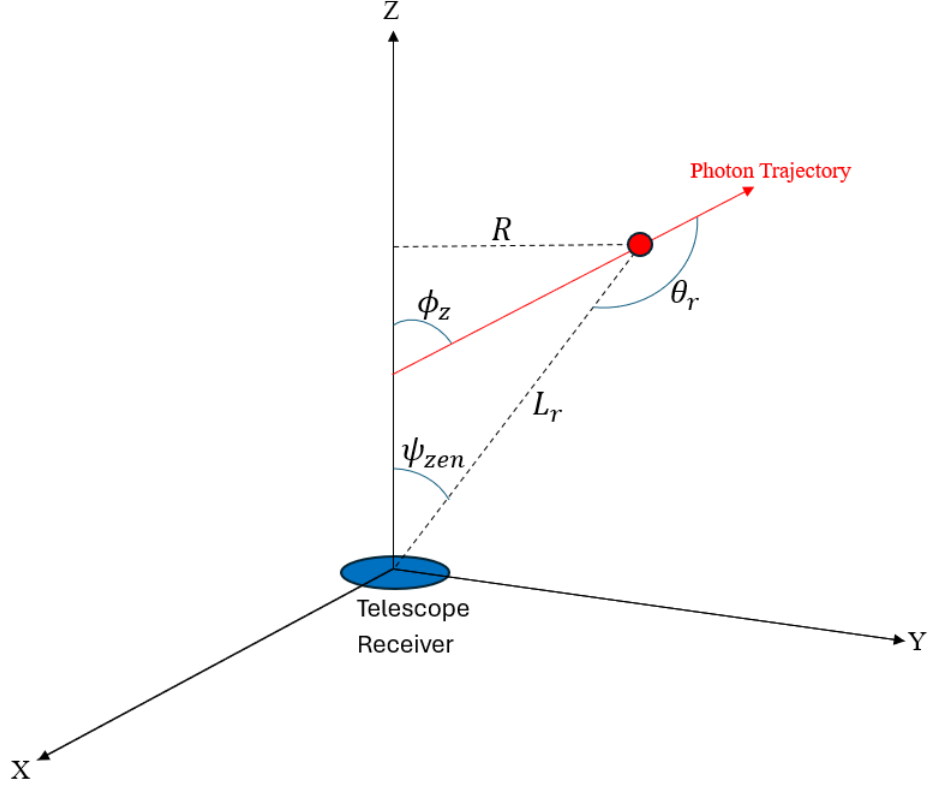


Figure 15: Geometry for photons returning to the receiver in the Monte Carlo model.

7. The polarization state of the photons is simulated in a similar manner. Following from Hu et al. (2001), the probabilistic representation of the received photons is

$$\begin{bmatrix} \mathcal{P}_I \\ \mathcal{P}_Q \\ \mathcal{P}_U \\ \mathcal{P}_V \end{bmatrix} = \begin{bmatrix} I_r \\ Q_r \\ U_r \\ V_r \end{bmatrix} \Omega e^{-\tau_r}, \quad (67)$$

where $\mathcal{P}_i$ represent the probabilistic Stokes parameters, and (I_r, Q_r, U_r, V_r) represent the Stokes parameters that would result from the photon scattering in the direction of the receiver. The sum of these probabilistic Stokes vectors over every photon and every scattering event within the bins described in Step 6 yields a simulated polarization state for the received signal. Computing the Stokes parameters is more complicated than computing the total signal strength. The Stokes parameters require knowledge of the entire scattering ma-

trix of a particle, not simply the F_{11} component alone. The return Stokes vector is given by

$$\begin{bmatrix} I_r \\ Q_r \\ U_r \\ V_r \end{bmatrix} = \mathbf{L}(\pi - \xi_r) \mathbf{F}(\theta_r) \mathbf{L}(-\xi_{inc}) \begin{bmatrix} I_{inc} \\ Q_{inc} \\ U_{inc} \\ V_{inc} \end{bmatrix}, \quad (68)$$

with the the r subscript continuing to represent angles which return the photon to the receiver (Hu et al., 2001). I_{inc} , Q_{inc} , U_{inc} , V_{inc} are the components of the Stokes vector before scattering. $\mathbf{F}(\theta)$ represents the scattering matrix, and $\mathbf{L}(\xi)$ is a rotation matrix. All of the elements of the $\mathbf{F}(\theta)$ matrix are interpolated from pre-computed datasets. The angles ξ_{inc} and ξ_{scat} are related to the direction cosines and scattering angle by

$$\cos \xi_{inc} = \frac{\cos \phi_{z,scat} - \cos \phi_{z,inc} \cdot \cos \theta}{\sin \phi_{z,inc} \sin \theta}, \quad (69)$$

$$\cos \xi_{scat} = \frac{\cos \phi_{z,inc} - \cos \phi_{z,scat} \cdot \cos \theta}{\sin \phi_{z,inc} \sin \theta}, \quad (70)$$

where $C_{inc} = \cos \phi_{z,inc}$ is the incident z-direction cosine, $C_{scat} = \cos \phi_{z,scat}$ is the z-direction cosine after scattering, and θ is the scattering angle (Hovenier and van der Mee, 1983; Hu et al., 2001). For scattering events that direct the photon towards the receiver, C_{scat} is represented by C_r . The rotation matrix $\mathbf{L}(\xi)$ (Hu et al., 2001) is given by

$$\mathbf{L}(\xi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\xi) & \sin(2\xi) & 0 \\ 0 & -\sin(2\xi) & \cos(2\xi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (71)$$

Following from Hu et al. (2001) and Hovenier and van der Mee (1983), when the denominator of Equations 69 and 70 is equal to 0, the following can be used:

If $|C_{inc}| = 1$,

$$\cos \xi_{inc} = -C_{inc} \cos (\varphi_{scat} - \varphi_{inc}), \quad (72)$$

$$\cos \xi_{scat} = C_{inc}. \quad (73)$$

If $|C_{scat}| = 1$,

$$\cos \xi_{inc} = C_{scat}, \quad (74)$$

$$\cos \xi_{scat} = -C_{scat} \cos (\varphi_{scat} - \varphi_{inc}). \quad (75)$$

If $\sin \theta = 0$,

$$\cos \xi_{inc} = 1, \quad (76)$$

$$\cos \xi_{scat} = 1. \quad (77)$$

The values of $0 \leq \varphi_{inc}, \varphi_{scat} \leq 2\pi$ are determined from the following expressions:

$$\mathcal{H}_{inc} = \begin{cases} \frac{C_{inc}}{|C_{inc}|}, & \text{if } |C_{inc}| = 1 \\ \frac{B_{inc}}{\sqrt{1-C_{inc}^2}}, & \text{otherwise} \end{cases}, \quad (78)$$

$$\mathcal{H}_{scat} = \begin{cases} \frac{C_{scat}}{|C_{scat}|}, & \text{if } |C_{scat}| = 1 \\ \frac{B_{scat}}{\sqrt{1-C_{scat}^2}}, & \text{otherwise} \end{cases}, \quad (79)$$

$$\varphi_{inc} = \begin{cases} 2\pi - \arccos (\mathcal{H}_{inc}), & \text{if } A_{inc} < 0 \\ \arccos (\mathcal{H}_{inc}), & \text{otherwise} \end{cases}, \quad (80)$$

$$\varphi_{scat} = \begin{cases} 2\pi - \arccos (\mathcal{H}_{scat}), & \text{if } A_{scat} < 0 \\ \arccos (\mathcal{H}_{scat}), & \text{otherwise} \end{cases}. \quad (81)$$

The cosine terms used in the rotation matrix $L(\xi)$ (Equation 71) are given by

$$\cos(2\xi) = 2(\cos \xi)^2 - 1, \quad (82)$$

$$\sin(2\xi) = \begin{cases} -2\sqrt{(1 - (\cos \xi)^2)} \cos \xi, & \text{if } \pi < \varphi_{scat} - \varphi < 2\pi \\ 2\sqrt{(1 - (\cos \xi)^2)} \cos \xi, & \text{otherwise} \end{cases}. \quad (83)$$

Note that these angles are related to the scattering angle, θ , by Hovenier and van der Mee (1983)

$$\cos \theta = C_{inc} \cdot C_{scat} + \sqrt{1 - C_{inc}^2} \sqrt{1 - C_{scat}^2} \cos(\varphi_{inc} - \varphi_{scat}). \quad (84)$$

The scattering geometry is shown in Figure 16.

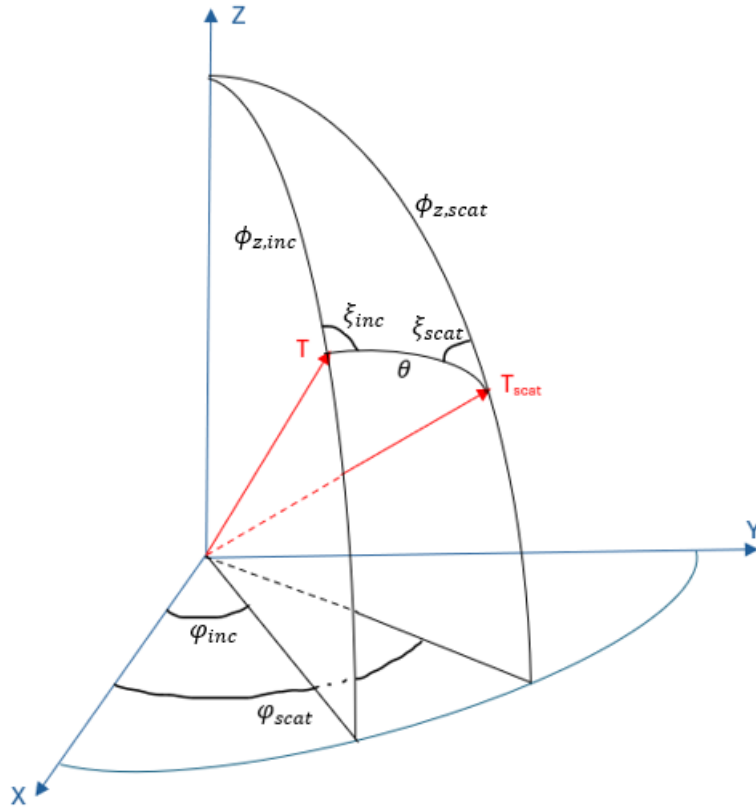


Figure 16: Geometry for photon scattering. T is the trajectory before scattering, and T_{scat} is the trajectory after scattering. The magnitude of the trajectory vectors is 1 (a unit sphere). This diagram is based on Figure 3 from Hovenier and van der Mee (1983).

8. At each scattering event, the post-scattering direction of each photon must be simulated. The scattering angle, θ , and the angle of rotation into the scattering plane, ξ_{inc} , both depend on the Stokes vector of the incident light and the scattering matrix of the scattering particle, according to Equation 7. To start, we sample a random number, ρ , from a uniform distribution between 0 and 1. We then use the following pre-computed cumulative probability distribution to sample the scattering angle, θ , through interpolation:

$$P(\theta) = \frac{1}{2} \int_0^\theta F_{11}(\theta') \sin(\theta') d\theta', \quad (85)$$

where $P(\theta)$ is the probability of choosing an angle between 0 and θ , and F_{11} is the (1,1) component of the scattering matrix. The angle of rotation into the scattering plane, ξ_{inc} , is sampled uniformly between 0 and 2π according to

$$\xi_{inc} = 2\pi\rho, \quad (86)$$

where ρ is a new random number sampled uniformly between 0 and 1. As shown in Equation 7, the angle ξ_{inc} is not truly uniform between 0 and 2π . To account for this, the statistical weight of each photon is modified as follows:

$$W = W_0 \left(\frac{1}{R_{max}} \right) \left(\frac{\mathcal{F}(\theta, \xi_{inc})}{F_{11}(\theta)} \right), \quad (87)$$

where R_{max} is the maximum possible value of $\mathcal{F}(\theta, \xi_{inc}) / F_{11}(\theta)$, given by

$$R_{max} = \text{Max} \left[\frac{F_{11}(\theta) + \sqrt{(Q_{norm}^2 + U_{norm}^2) F_{12}(\theta)^2}}{F_{11}(\theta)} \right], \quad (88)$$

where $Q_{norm} = Q/I$, and $U_{norm} = U/I$ (He et al., 2023). Dividing by R_{max} ensures that the new weight factor is normalized. Following from Equation 7, the phase function used in Equation 87 is given by He et al. (2023):

$$\mathcal{F}(\theta_{rand}, \xi_{inc,rand}) = F_{11}(\theta_{rand}) + F_{12}(\theta_{rand}) (Q_{norm} \cos(2\xi_{inc,rand}) + U_{norm} \sin(2\xi_{inc,rand})). \quad (89)$$

$F_{11}(\theta)$ and $F_{12}(\theta)$ are precomputed datasets. Using this weighting method allows for efficient computation times when computing the scattering angles.

9. With the angles of scattering determined, the new direction cosines need to be computed. If we allow κ to be defined as the cosine of the scattering angle:

$$\kappa = \cos \theta, \quad (90)$$

then the new direction cosines are given by Spanier and Gelbard (1969), Carter and Cashwell (1975), and Peplow (1999):

$$A_{scat} = A_{inc}\kappa + \frac{\sqrt{1-\kappa^2}}{\sqrt{1-C_{inc}^2}} (A_{inc}C_{inc} \cos \xi_{inc} - B \sin \xi_{inc}), \quad (91)$$

$$B_{scat} = B_{inc}\kappa + \frac{\sqrt{1-\kappa^2}}{\sqrt{1-C_{inc}^2}} (B_{inc}C_{inc} \cos \xi_{inc} + A_{inc} \sin \xi_{inc}), \quad (92)$$

$$C_{scat} = C_{inc}\kappa - \sqrt{1-\kappa^2} \sqrt{1-C_{inc}^2} \cos \xi_{inc}. \quad (93)$$

When $|C_{inc}| = 1$, the following can be used:

$$A_{scat} = \sqrt{1-\kappa^2} \cos \xi_{inc}, \quad (94)$$

$$B_{scat} = \sqrt{1-\kappa^2} \sin \xi_{inc}, \quad (95)$$

$$C_{scat} = \frac{C_{inc}}{|C_{inc}|} \kappa. \quad (96)$$

10. As in Step 7, the polarization state of each photon after scattering is given by Hu et al. (2001):

$$\begin{bmatrix} I_{scat} \\ Q_{scat} \\ U_{scat} \\ V_{scat} \end{bmatrix} = \mathbf{L}(\pi - \xi_{scat}) \mathbf{F}(\theta) \mathbf{L}(-\xi_{inc}) \begin{bmatrix} I_{inc} \\ Q_{inc} \\ U_{inc} \\ V_{inc} \end{bmatrix}. \quad (97)$$

The same process outlined in Step 7 is used to compute the new polarization state.

11. Steps 3-9 are repeated until every photon exits the scattering medium or a user-defined maximum number of scattering events is reached. Steps 1-2 are repeated for each new group of photons simulated. The total signal and polarization state of the received photons are summed over all photon groups.

3.4 Validation

It is useful to compare our model with other models. A workshop group called MUSCLE (Multiple SCattering Lidar Experiments) carried out a coordinated effort to compare atmospheric LIDAR models from various research groups. A detailed comparison of the models was carried out by Bissonnette et al. (1995). A liquid water cloud using the C1 size distribution (Deirmendjian, 1969) is used as a comparison case. The laser wavelength is 1064 nm. The base height of the cloud is 1 km above the receiver. The extinction coefficient is 17.25 km^{-1} . The cloud is 300 m thick vertically. The particle refractive index of the liquid water droplets is $m = 1.325$. The laser beam divergence is 0.1 mrad (full angle). Figures 17 and 18 show the ratio of total multiple scattering to single scattering as a function of range in cloud (perceived) for receiver FOVs of 10 mrad and 1 mrad, respectively. The results of our model are shown along with the results of several other models developed by members of the MUSCLE group (Bissonnette et al., 1995). It is important to note that it was assumed that the angle of rotation into the scattering plane was assumed to be uniformly distributed between 0 and 2π when calculating our results in Figures 17 and 18. This assumption was made in order to match the assumptions made by the other research groups. The Florence group's model (Bruscaglioni et al., 1995) is a semi-analytic Monte Carlo model. The NASA Langley group's model (Winker and Poole, 1995) is a Monte Carlo model with no variance reduction. The Munich group's model (Starkov et al., 1995) is a stochastic model that employs analytical calculations of each order of multiple scattering. The Israel group's model (Cohen et al., 1978) is a modified Monte Carlo model. The Swiss group's model (Flesia and Schwendimann, 1993) is an analytical extension of Mie theory. The DREV (Defence Research Establishment Valcartier) group's model (Bissonnette, 1995) is an approximation to the radiative-transfer equation applicable to narrow incident beams. The Minsk group's model (Zege et al., 1995) is a semi-analytic radiative-

transfer model. Figures 17 and 18 show that our model agrees well with several of the MUSCLE group models. The results of our model are closest to those of the Florence group's Monte Carlo model, the Munich group's stochastic model, the Minsk group's semi-analytic radiative-transfer model, and the NASA Langley group's Monte Carlo model. The agreement between our model and these models is a good indicator that the levels of multiple scattering are being tracked correctly in our model.

It is difficult to judge which of the models is closest to reality due to a lack of comparison measurements of actual clouds. Many factors contribute to the differences between the models. Even the various Monte Carlo models yield different results, largely due to differences in variance reduction techniques. Variance reduction is used in many Monte Carlo models to reduce variance by increasing the amount of samples. For example, our model employs variance reduction by forcing photons to scatter before exiting the FOV. This allows for an increased number of scattering events within the FOV without increasing computation time. Variance reduction techniques require a statistical weight to be applied to each photon to account for the introduced statistical bias. Naturally, using different variance reduction methods leads to different results. The non-Monte Carlo models require different assumptions regarding the scattering process. These assumptions lead to differences in results. The radiative transfer models (Minsk and DREV) solve the radiative transfer equation using different algorithms. Consequently, the models have different levels of accuracy for different orders of scattering and for different scattering angles. As explained by Bissonnette et al. (1995), it is important to note that the variance of experimental data from Bissonnette (1995) falls within the variance of the results of the models shown in Figures 17 and 18. Bissonnette et al. (1995) gives a detailed comparison of the scattering models.

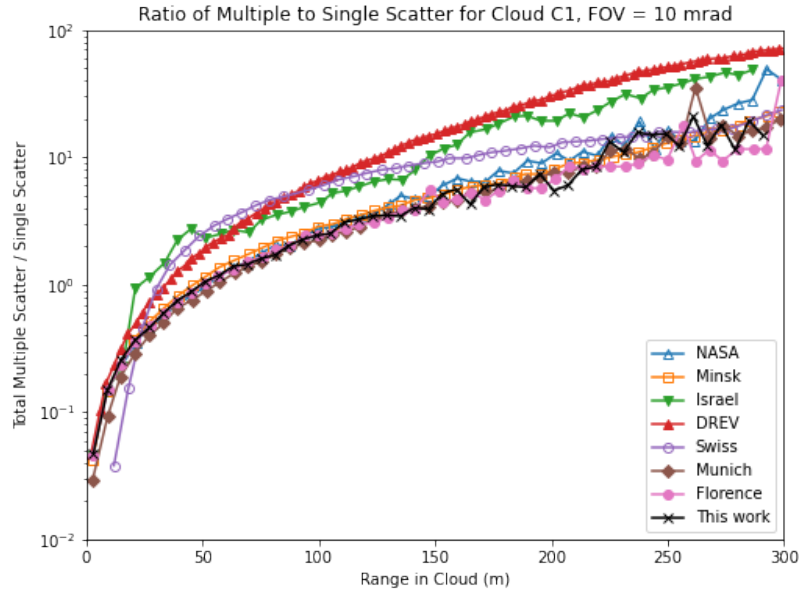


Figure 17: The ratio of multiple scattering to single scattering plotted against range in cloud (perceived). Results of several different models from members of the MUSCLE group are shown. The cloud used is a C1 cloud (Deirmendjian, 1969). The receiver FOV is 10 mrad (full angle). The laser wavelength is 1064 nm. Cloud base height is 1 km above the receiver with an extinction coefficient of 17.25 km^{-1} . The data in this plot can be found in Bissonnette et al. (1995).

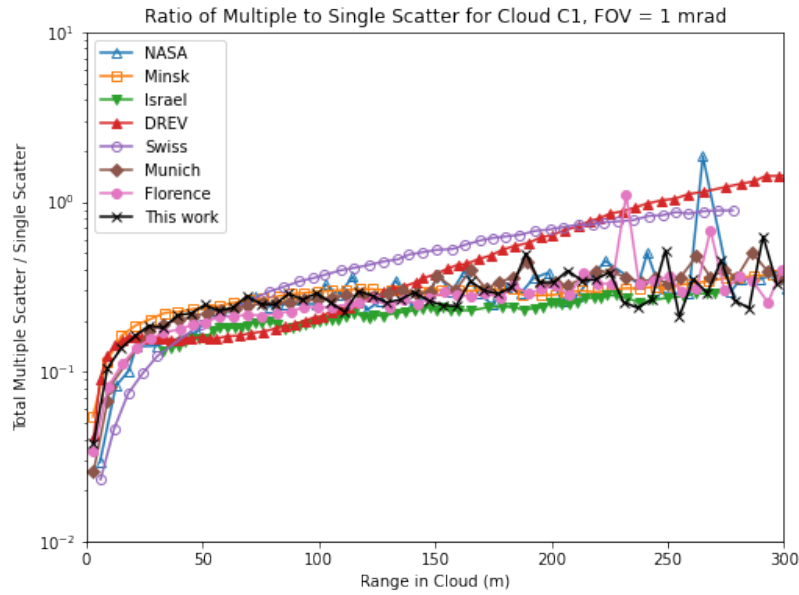


Figure 18: Same as Figure 17 but with a receiver FOV of 1 mrad (full angle).

It is also important to validate the Stokes vector results from our model. We will compare the depolarization ratio computed with our model to the depolarization ratio computed with several other models using the liquid water C1 cloud (Deirmendjian, 1969). The laser wavelength used in the validation case is $0.7 \mu\text{m}$. The distance between the LIDAR receiver and the cloud base is 2 km. The field-of-view is $\sim 1.75 \text{ mrad}$ (full angle). The extinction coefficient is 0.025 m^{-1} . The results of our model are compared to the results of a vectorized physical model and a Monte Carlo model, both developed by Sato et al. (2019). The results of several other models (Alkasem et al., 2017) are also used for comparison. The depolarization ratio computed using each model is shown in Figure 19. As shown in Figure 19, the linear depolarization predicted by our model agrees well with other models. This is a good validation case for our model, as similar results are found using different modeling methods, including a vectorized physical model (Sato et al., 2019), a semi-analytical approach with small-angle approximation (Alkasem et al., 2017) (included in ‘other models’), and several other Monte Carlo models.

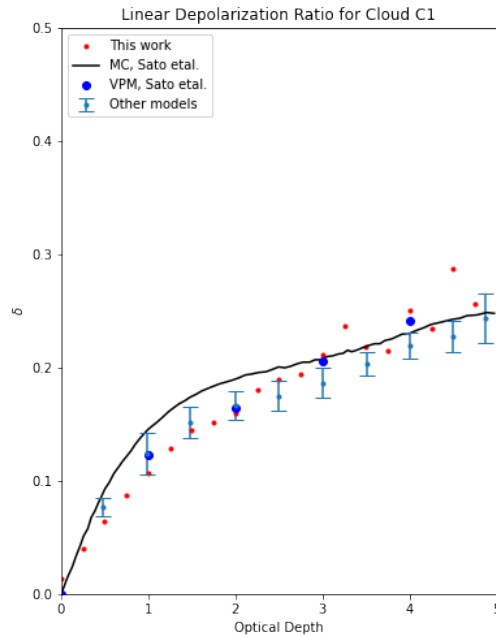


Figure 19: The linear depolarization for the C1 cloud (Deirmendjian, 1969) at a laser wavelength of $0.7 \mu\text{m}$. Vectorized physical model (VPM) and Monte Carlo (MC) results from Sato et al. (2019) are shown alongside a combination of results from other models (Alkasem et al., 2017).

4 Simulated Signal Analysis Methods

4.1 Generation of Simulated LIDAR Signals

The Monte Carlo model can be used to simulate the relative amounts of multiple and single scattering measured by a LIDAR system. However, to analyze the performance of a LIDAR system, we need to model a complete signal. To understand how this is accomplished, we revisit the single scattering LIDAR equation (Equation 4):

$$N(z) = Q \frac{A}{z^2} \beta(z) \Delta z N_0 \exp \left(-2 \int_{z_0}^z \sigma(z') dz' \right).$$

If we have values for the constants N_0 , Q , and A , then we only need an extinction coefficient profile, $\sigma(z)$, and a backscatter coefficient profile, $\beta(z)$, to estimate the levels of single scattering measured by a LIDAR system. It is common to relate the extinction coefficient to the backscatter coefficient through a constant known as the LIDAR ratio (Fernald, 1984):

$$S = \frac{\sigma(z)}{\beta(z)}. \quad (98)$$

Assuming that S is constant with height is equivalent to assuming that the type of scattering particle and size distribution of the scattering particles is constant with height and that the number density is the only variable changing the extinction coefficient (Fernald, 1984). With this simplifying assumption, the LIDAR equation becomes:

$$N(z) = Q \frac{A}{z^2} \frac{\sigma(z)}{S} \Delta z N_0 \exp \left(-2 \int_{z_0}^z \sigma(z') dz' \right). \quad (99)$$

Given a mixture of multiple types of scattering particles, we can represent the extinction coefficient as the sum of the of the individual extinction coefficients:

$$\sigma(z) = \sum_{i=0}^n \sigma_i(z), \quad (100)$$

where i represents the type of scattering particle, with n being the total number of types. For two different types of scattering particle, such as cloud and dust, we can write the LIDAR equation as

(Fernald et al., 1972):

$$N(z) = N_0 Q \frac{A}{z^2} \frac{\sigma_1(z) + \sigma_2(z)}{S} T_1^2(z) T_2^2(z) \Delta z \text{ [photons]}, \quad (101)$$

where

$$T_{1,2}^2 = \exp \left(-2 \int_{z_0}^z \sigma_{1,2}(z') dz' \right). \quad (102)$$

The photon count rate is given by

$$N_{rate}(z) = N_0 Q \frac{A}{z^2} \frac{\sigma_1(z) + \sigma_2(z)}{S} T_1^2(z) T_2^2(z) \frac{c}{2} \text{ [photons/s]}, \quad (103)$$

where c is the speed of light, and the factor of $\frac{1}{2}$ accounts for the two-way transmission of the signal. Simulated profiles of $N(z)$ and $N_{rate}(z)$ can be computed using the same $\sigma(z)$ profiles that are input to the Monte Carlo model. If the vertical resolution of the LIDAR system is different than the vertical spacing of the extinction profile, the photon count is summed within the desired height bins, and the count rate is averaged within the desired height bins.

The total signal, including both single and multiple scattering can then be modeled as

$$N_{total}(z) = N_{single}(z) + N_{single}(z) R_{MS}(z), \quad (104)$$

where $N_{single}(z)$ is the signal profile computed using the single scattering LIDAR equation, and $R_{MS}(z)$ is the multiple to single scatter ratio computed using the Monte Carlo model.

To simulate a LIDAR system that is equipped to measure depolarization, we need to simulate the signal in both the parallel and perpendicular channels. Following from Equation 20, the ratio of parallel signal to total signal is given by

$$R_I = \frac{1}{2} \frac{I + Q}{I}, \quad (105)$$

and the ratio of perpendicular signal to total signal is given by

$$R_r = \frac{1}{2} \frac{I - Q}{I}. \quad (106)$$

Height profiles of these ratios can be computed using the simulated Stokes vectors from the Monte Carlo model. The signal in each channel is then given by

$$N_l(z) = N_{total}(z) \cdot R_l(z), \quad N_r(z) = N_{total} \cdot R_r(z). \quad (107)$$

In order to realistically model a LIDAR signal, we need to include solar background. Despite the use of a spectral filter that blocks out most of the solar background light, solar background light can still pass through at wavelengths within the filter's pass band. In this study, we will use solar background levels measured by the Phoenix LIDAR to estimate solar background levels for a Mars orbital LIDAR. The typical daytime solar background light intensity used for validating the Phoenix LIDAR measurements was $20 \text{ Wm}^{-2} \mu\text{m}^{-1} \text{Sr}^{-1}$. In this study, this value is used as a reliable estimate of the daytime solar background intensity, keeping in mind that this value would vary due to several factors, including dust loading, cloud cover, and time of day. The nighttime solar background case uses a value of $0 \text{ Wm}^{-2} \mu\text{m}^{-1} \text{Sr}^{-1}$. The solar background can simply be added directly to the simulated signal. An equal amount of solar background is added in each height bin because the amount of solar radiation is approximately constant over the timescale of a LIDAR measurement. The background light is split equally between the parallel and perpendicular polarization channels. The final step in simulating a LIDAR signal is to add noise, which is discussed in the following section.

4.2 Signal Noise

The statistical uncertainty corresponding to a number of photons, N , counted in time Δt is given by

$$\Delta N = \sqrt{N}, \quad \Delta N_{rate} = \frac{\sqrt{N}}{\Delta t}, \quad (108)$$

for the photon count and photon count rate, respectively. To simulate signal noise in a LIDAR signal, we use white noise sampling. Consider a standard normal distribution with a mean of 0

and a variance of 1:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}. \quad (109)$$

We randomly sample values of x from this distribution using the `normal()` function from the `random` module of the Python library `numpy` (NumPy Developers, 2024). A different x value is sampled for each vertical bin. The simulated LIDAR signal with noise in each vertical bin is given by

$$N(z_i) = \overline{N(z_i)} + x_i \sqrt{\overline{N(z_i)}}, \quad (110)$$

where $\overline{N(z_i)}$ is the average photon count in each bin. When analyzing the viability of a simulated signal, we need to know if the signal strength is greater than the statistical uncertainty. If the signal strength is not greater than the statistical uncertainty, then LIDAR returns coming from clouds and dust cannot be reliably distinguished from noise. A good way to compare the signal to the statistical uncertainty is by using a signal-to-noise ratio (SNR) profile. Generating a SNR profile of a simulated signal allows us to compare the strength of the signal to that of the noise at every point in the vertical profile. For atmospheric LIDAR measurements to be useful, the SNR needs to be at least greater than 1 (preferably greater than ~ 5) at every point in the vertical profile.

4.3 Retrieval of Extinction Coefficient

As discussed previously, multiple scattering can make it difficult to accurately retrieve parameters, such as the extinction coefficient and backscatter coefficient, from LIDAR signals. Thus, it is important to test if we can accurately retrieve these parameters from our simulated signal. To retrieve the extinction and backscatter coefficients from our simulated signal, we will use the Klett-Fernald method, which is a procedure frequently used in the analysis of real, measured signals. Following from Fernald et al. (1972) and recalling Equation 101, we can write the LIDAR equation for a mixture of two types of scattering particles as

$$N(z) = N_0 Q \frac{A}{z^2} (\beta_1(z) + \beta_2(z)) T_1^2(z) T_2^2(z). \quad (111)$$

Note that the factor of Δz has been omitted, meaning that Equation 111 represents the signal coming from a point at height z rather than over the range Δz . As discussed previously, the

LIDAR ratio $S = \frac{\sigma(z)}{\beta(z)}$ is taken as constant with respect to z . As described by Fernald et al. (1972) and Fernald (1984), the solution to Equation 111 is

$$\beta_1(z) + \beta_2(z) = \frac{X(z) \exp \left(-2 (S_1 - S_2) \int_{z_c}^z \beta_2(z') dz' \right)}{\frac{X(z_c)}{\beta_1(z_c) + \beta_2(z_c)} - 2S_1 \int_{z_c}^z X(z') \exp \left(-2 (S_1 - S_2) \int_{z_c}^{z'} \beta_2(z'') dz'' \right) dz'}, \quad (112)$$

where the constants P_0 , Q , and A have been solved for through calibration at height z_c (Fernald, 1984). $X(z) = N(z)z^2$ is the range-corrected signal. $\beta_2(z)$ typically represents molecular scattering molecules that compose the gaseous atmosphere. In this case, other information, such as radiosonde profiles, can be used to construct a $\beta_2(z)$ profile. In cases for which $\beta_2(z)$ can be pre-computed, Fernald (1984) presents an iterative scheme that can be used to compute $\beta_1(z)$:

$$\beta_1(i+1) + \beta_2(i+1) = \frac{X(i+1) \exp(-A(i, i+1))}{\frac{X(i)}{\beta_1(i) + \beta_2(i)} - S_1 (X(i) + X(i+1) \exp(-A(i, i+1))) \Delta z}, \quad (113)$$

and

$$\beta_1(i-1) + \beta_2(i-1) = \frac{X(i-1) \exp(+A(i-1, i))}{\frac{X(i)}{\beta_1(i) + \beta_2(i)} + S_1 (X(i) + X(i-1) \exp(+A(i-1, i))) \Delta z}, \quad (114)$$

where

$$A(i, i+1) = (S_1 - S_2) (\beta_2(i) + \beta_2(i+1)) \Delta z. \quad (115)$$

Similarly, the extinction coefficient is given by

$$\sigma_1(i+1) + \frac{S_1}{S_2} \sigma_2(i+1) = \frac{X(i+1) \exp(-A(i, i+1))}{\frac{X(i)}{\sigma_1(i) + \frac{S_1}{S_2} \sigma_2(i)} - (X(i) + X(i+1) \exp(-A(i, i+1))) \Delta z}, \quad (116)$$

and

$$\sigma_1(i-1) + \frac{S_1}{S_2} \sigma_2(i-1) = \frac{X(i-1) \exp(+A(i-1, i))}{\frac{X(i)}{\sigma_1(i) + \frac{S_1}{S_2} \sigma_2(i)} + (X(i) + X(i-1) \exp(+A(i-1, i))) \Delta z}. \quad (117)$$

An initial value of β_1 must be given at a calibration range in order to initialize the iterative process. Equations 116 and 117 can be used to separate the extinction coefficients of clouds and dust. When

modeling LIDAR signals from the Martian atmosphere, we typically ignore molecular scattering due to the very low density of the atmosphere. In this case, the expressions are simplified for application to highly turbid ($\sigma_1 \gg \sigma_2$) atmospheres (Fernald, 1984):

$$\beta_1(i-1) = \frac{X(i-1)}{\frac{X(i)}{\beta_1(i)} + S_1 (X(i) + X(i-1)) \Delta z}, \quad (118)$$

and

$$\sigma_1(i-1) = \frac{X(i-1)}{\frac{X(i)}{\sigma_1(i)} + (X(i) + X(i-1)) \Delta z}. \quad (119)$$

The simulated signal can be used with this iterative scheme to retrieve an extinction profile. This extinction profile can be compared to the one used to model the signal. If multiple scattering is significantly distorting the signal, we expect the derived extinction profile to vary noticeably from the source profile.

4.4 Validation

To ensure that our extinction coefficient algorithm is working correctly, we will use LIDAR signals measured by the Phoenix LIDAR to derive extinction profiles and compare to those derived by Komguem et al. (2013) and Dickinson et al. (2011). Figure 20 shows two signals measured by the Phoenix LIDAR and the corresponding extinction profiles derived using the process described in Section 4.3. It is clear that our extinction profiles agree well with those derived by Komguem et al. (2013) and Dickinson et al. (2011). These results provide us with confidence that our extinction coefficient algorithm is working properly and can be reliably used to retrieve the atmospheric extinction coefficient from backscatter signal profiles.

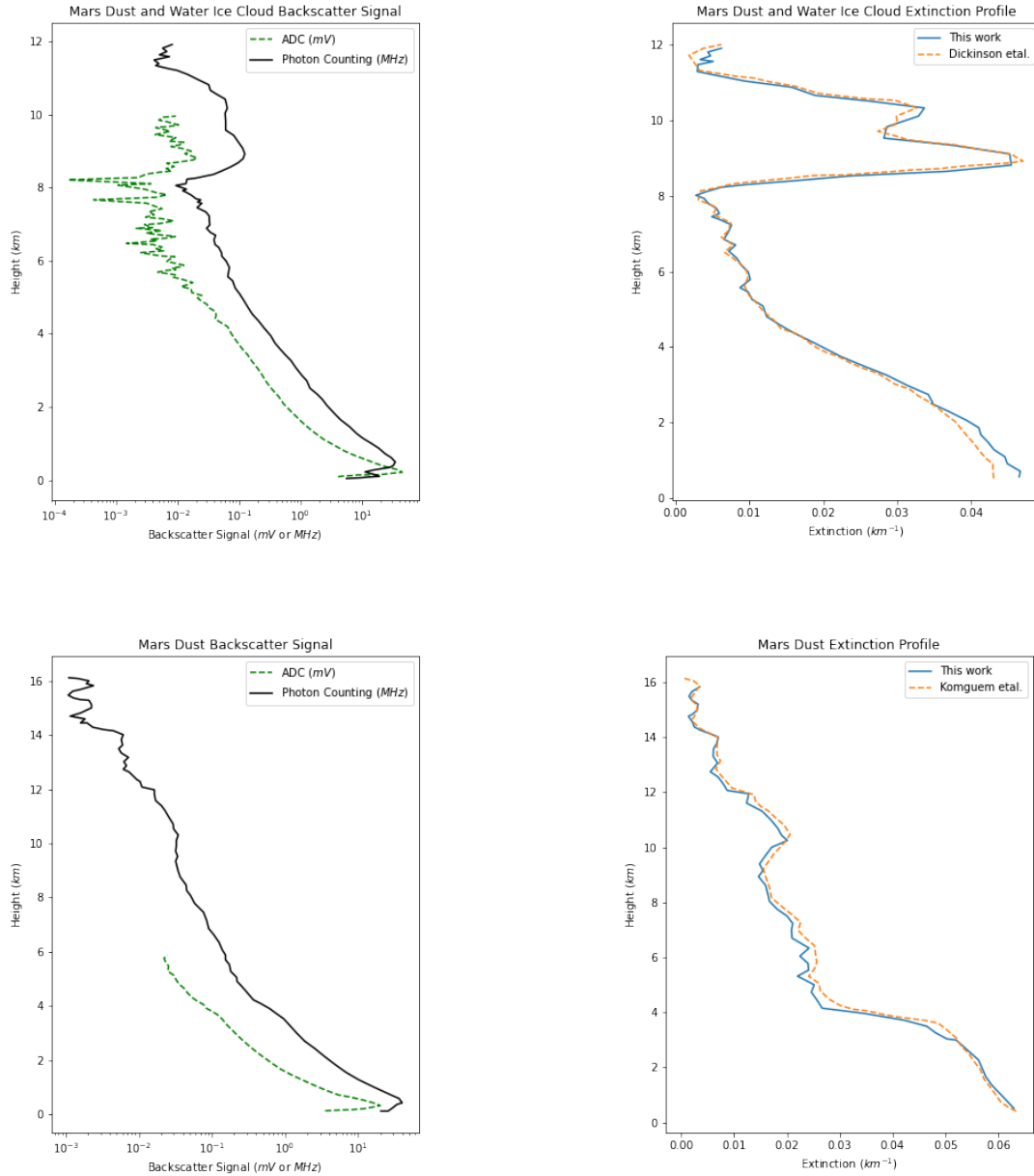


Figure 20: *Top:* Martian water ice cloud and dust measured by the Phoenix LIDAR (*Left*). Extinction profile retrieved from backscatter signal (*Right*). This profile is presented and analysed by Dickinson et al. (2011). The extinction profile derived by Dickinson et al. (2011) is included for comparison to our profile. *Bottom:* Martian dust measured by Phoenix LIDAR (*Left*). Extinction profile retrieved from backscatter signal (*Right*). This profile is presented and analyzed by Komguem et al. (2013). The extinction profile derived by Komguem et al. (2013) is included for comparison to our profile.

5 Results and Analysis

5.1 System Specifications

For the test cases in this report, we will use the system specifications of the CALIPSO LIDAR (Hunt et al., 2009), which can be found in Figure 2. The vertical resolution and pulse averaging were customized for our testing purposes. A 400 km circular orbit was chosen for the satellite. This orbit is used by many Mars observation satellites. Additionally, it should be noted that any reference in this work to ‘day’ measurements correspond to a solar background of $20 \text{ Wm}^{-2}\mu\text{m}^{-1}\text{Sr}^{-1}$, and any reference in this work to ‘night’ measurements correspond to a solar background of $0 \text{ Wm}^{-2}\mu\text{m}^{-1}\text{Sr}^{-1}$.

Laser Wavelength	532 nm
Pulse Energy	110 mJ
Pulse Rate	20.16 Hz
Beam Divergence	110 μrad
Telescope Diameter	1 m
FOV	130 μrad
Filter Bandwidth	35 μm
Detector Efficiency	0.11
Vertical Resolution	10 m
Pulse Averaging	~134 pulses, ~6.665 s
Orbital Altitude	400 km (circular)

Table 2: CALIPSO LIDAR specifications. Many of these values are presented by Hunt et al. (2009).

5.2 Signal Strength

To begin, we will analyze the simulated signals using the CALIPSO system specifications. Figure 21 shows simulated LIDAR signals for the cloud-only test cases. The data in these plots is direct output from the Monte Carlo model, so the units of signal strength are arbitrary. Multiple scattering causes ‘tails’ to be observed below each cloud base. If these ‘tails’ are of significant magnitude compared to the background noise, they can make it difficult to determine the base height of the cloud. Within the clouds, multiple scattering affects the slope of the signal. This can lead to inaccuracies when computing the extinction coefficient (see Section 5.4). When a low altitude cloud is present, it appears that there is signal coming from beneath the ground (negative altitude). This is caused by the total path length of multiply scattered photons being larger than the orbital altitude of the LIDAR satellite.

Figure 22 shows simulated LIDAR signals for the test cases that include dust. Multiple scattering from dust particles appears to not significantly affect the slope of the signal. However, backscatter signals from layers of dust below clouds can be distorted due to multiple scattering within the clouds.

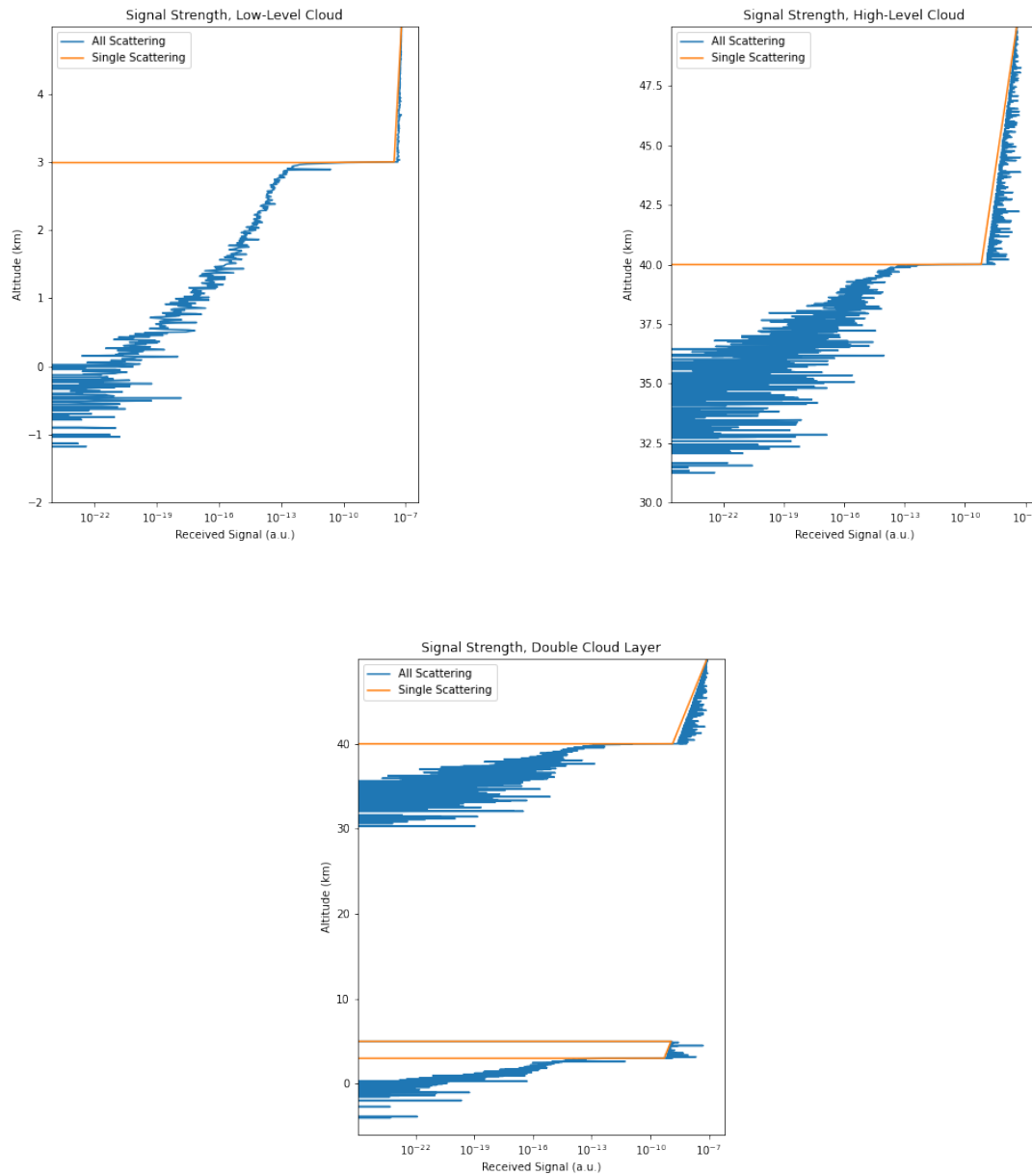


Figure 21: Simulated LIDAR signal for Martian water ice clouds. LIDAR specifications match those of CALIPSO.

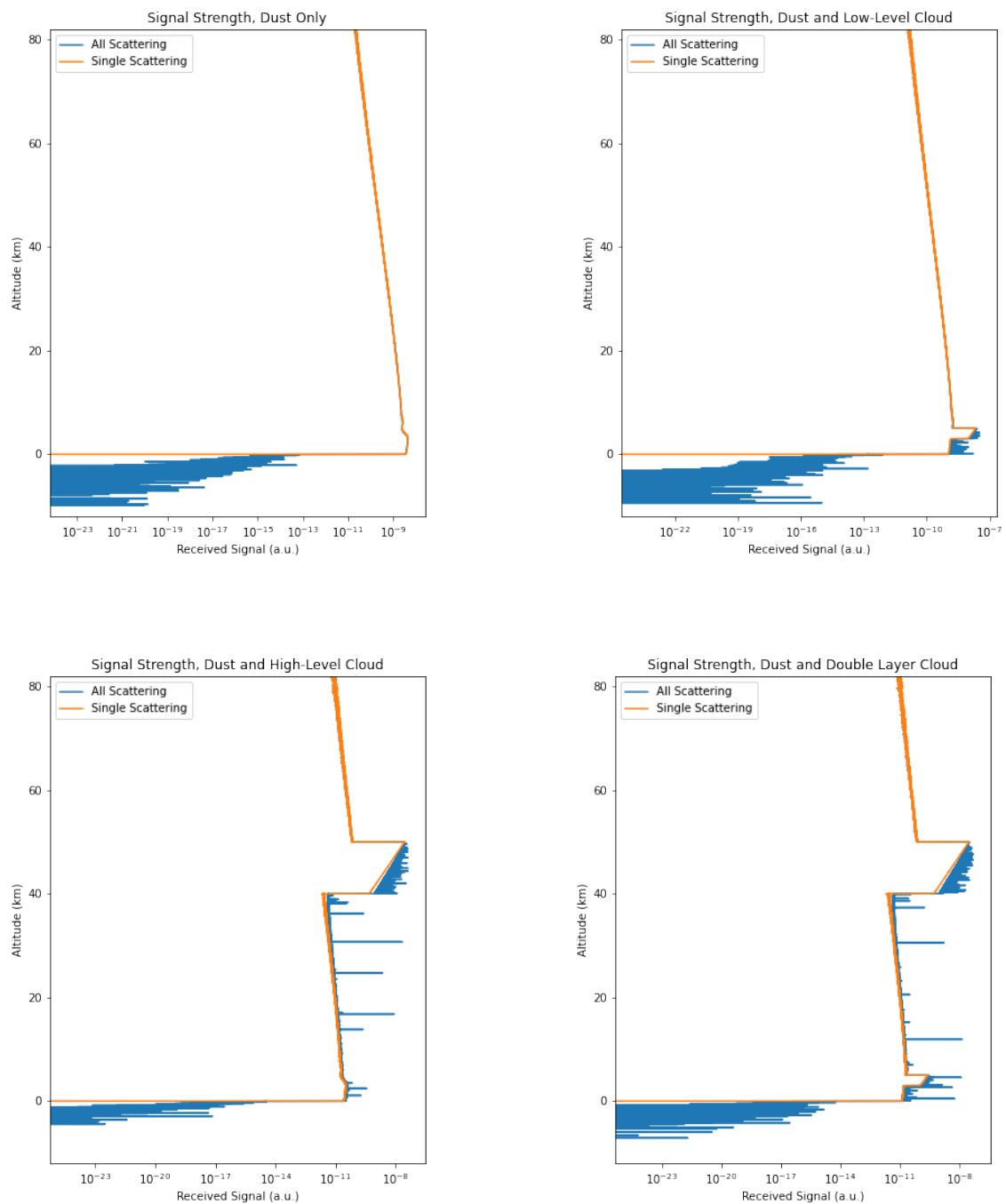


Figure 22: Simulated LIDAR signal for Martian mineral dust and water ice clouds. LIDAR specifications match those of CALIPSO.

Figures 23 and 24 show the simulated LIDAR signals converted to count rate for the cloud-only cases and the dust cases, respectively. The signals include multiple scattering effects from the

Monte Carlo model, background light, and signal noise. The return signals from dust are significantly weaker than the signals from clouds due to the lower extinction coefficient of the dust compared to the clouds. The vertically thick, high altitude cloud attenuates the signal to the point that any features below it are difficult to see. Specifically, the signal coming from the dust below the high altitude cloud in Figure 24 (bottom left) is very weak in comparison to the dust-only and dust and low altitude cloud cases. The same effect is observed in the weakening of the return signal from the low altitude cloud below the high altitude cloud in the double cloud layer case. Dust tends to reduce the strength of return signals from low altitude clouds more than from high altitude clouds. This is due to the inverse relationship between dust extinction coefficient and altitude.

Signal-to-noise ratio plots for the cloud-only cases and the cases that include dust are shown in Figures 25 and 26, respectively. Without dust present, our results indicate that an orbital LIDAR system with the same specifications as CALIPSO is capable of achieving SNR greater than 5 at all points in the measured vertical profile of individual high altitude and low altitude clouds. This applies to measurements made during the day or night. In the case of a double cloud layer, the SNR sometimes dips below one in the low altitude cloud, specifically for daytime measurements in the perpendicular channel. This is not ideal for measuring depolarization, but still indicates good performance given the high level of attenuation caused by the high altitude cloud. In the case of a typical Martian dust profile, our results indicate that an orbital LIDAR system with the specifications of CALIPSO is capable of achieving an SNR greater than 1 up to an altitude of approximately 30 km during the day and up to an altitude of approximately 58 km during the night. When a low altitude cloud is present with the dust, our results indicate that the system is capable of achieving a SNR greater than 1 at all points throughout and below the cloud. When a high altitude cloud is present with the dust, the SNR is greater than 5 at all points within the cloud, but is below 1 throughout much of the dust profile, especially for measurements made during the day. It should be noted that the signal ‘tails’ caused by multiple scattering seem to have SNRs below 1 in all cases, meaning that the ‘tails’ should not pose any problems in determining the height of cloud bases.

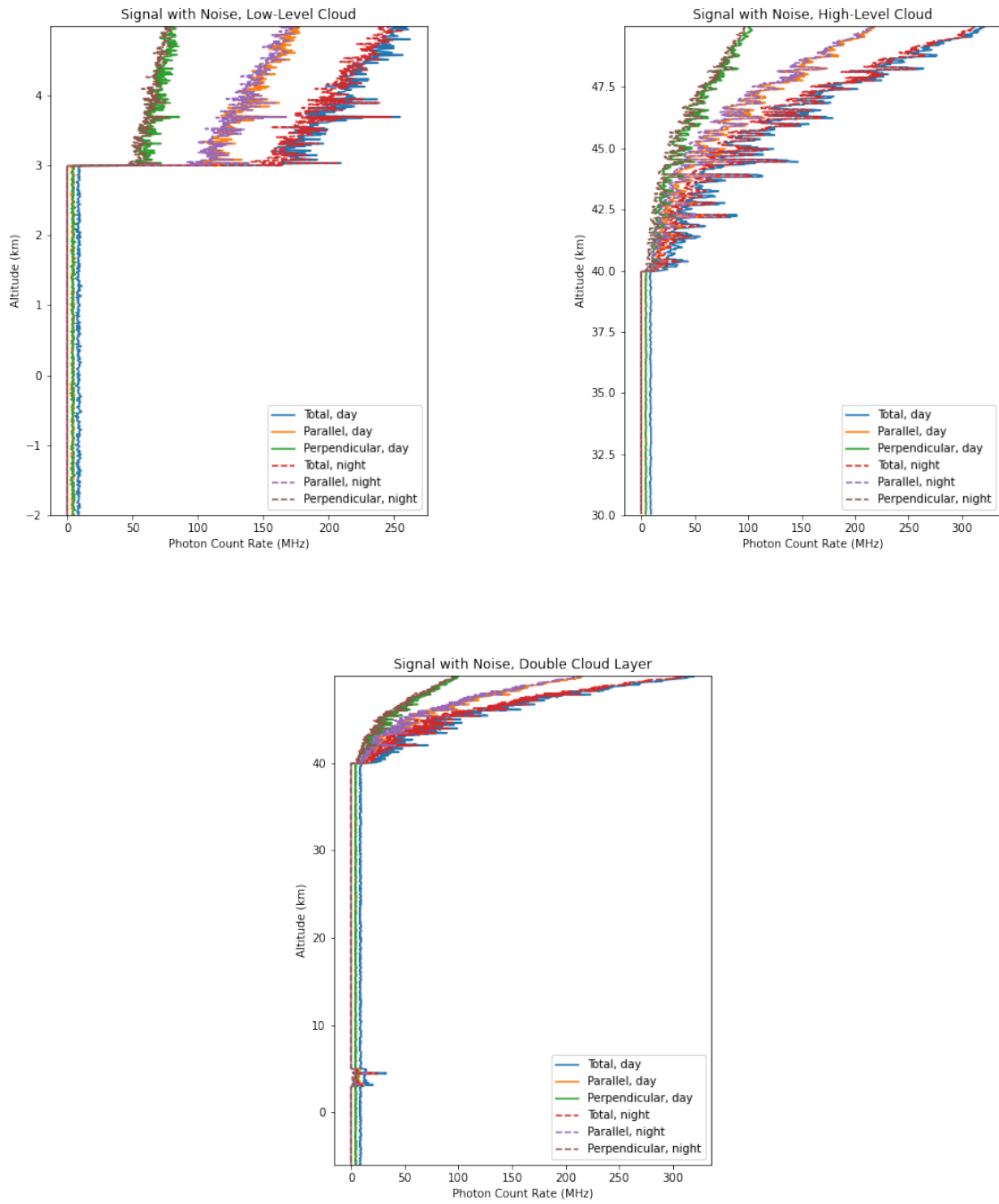


Figure 23: Simulated LIDAR signals converted to count rate for cloud-only test cases. The signals include background light and simulated noise. Respective day and night measurements often overlap.

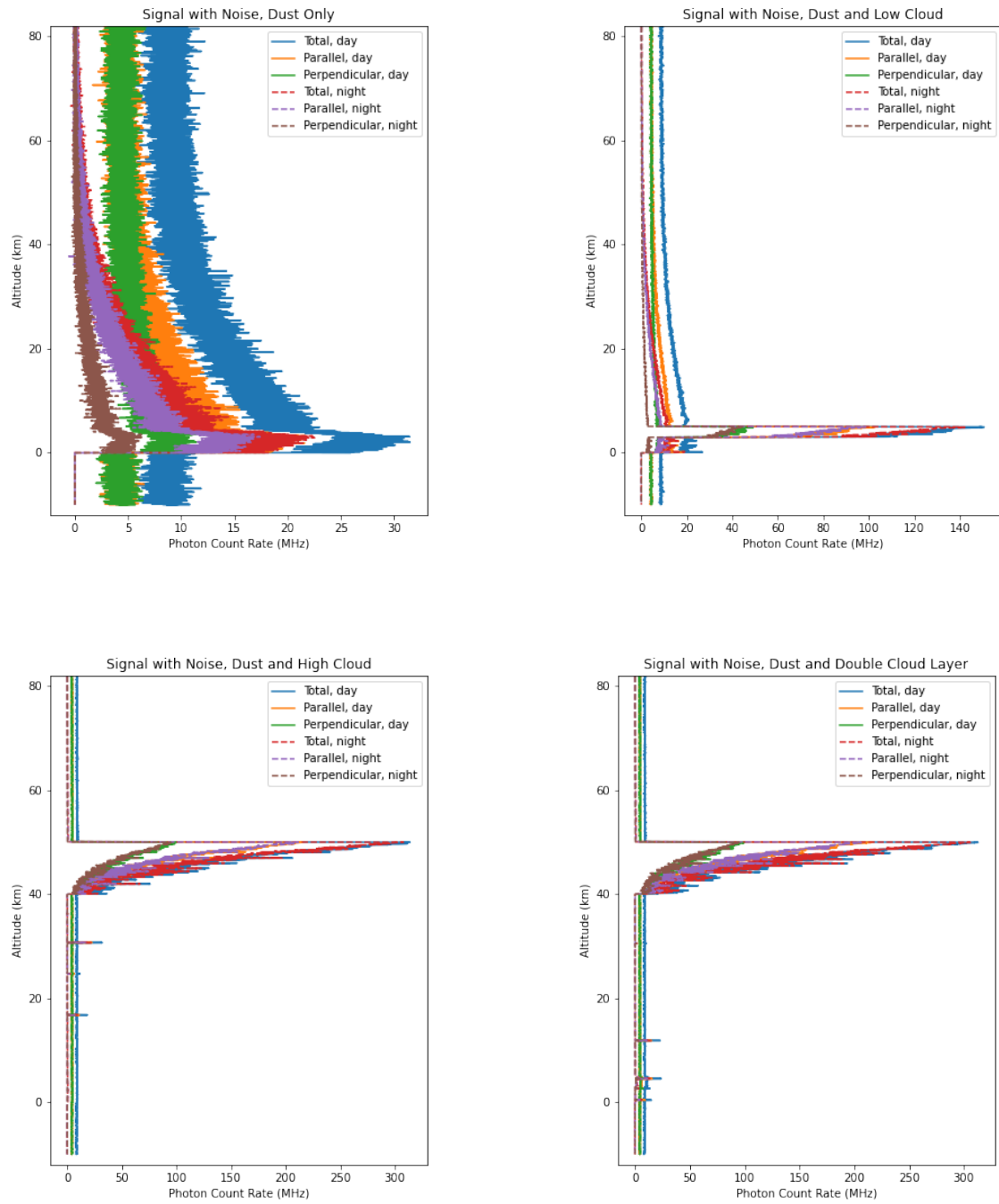


Figure 24: Simulated LIDAR signals converted to count rate for test cases that include dust. The signals include background light and simulated noise. Respective day and night measurements often overlap.

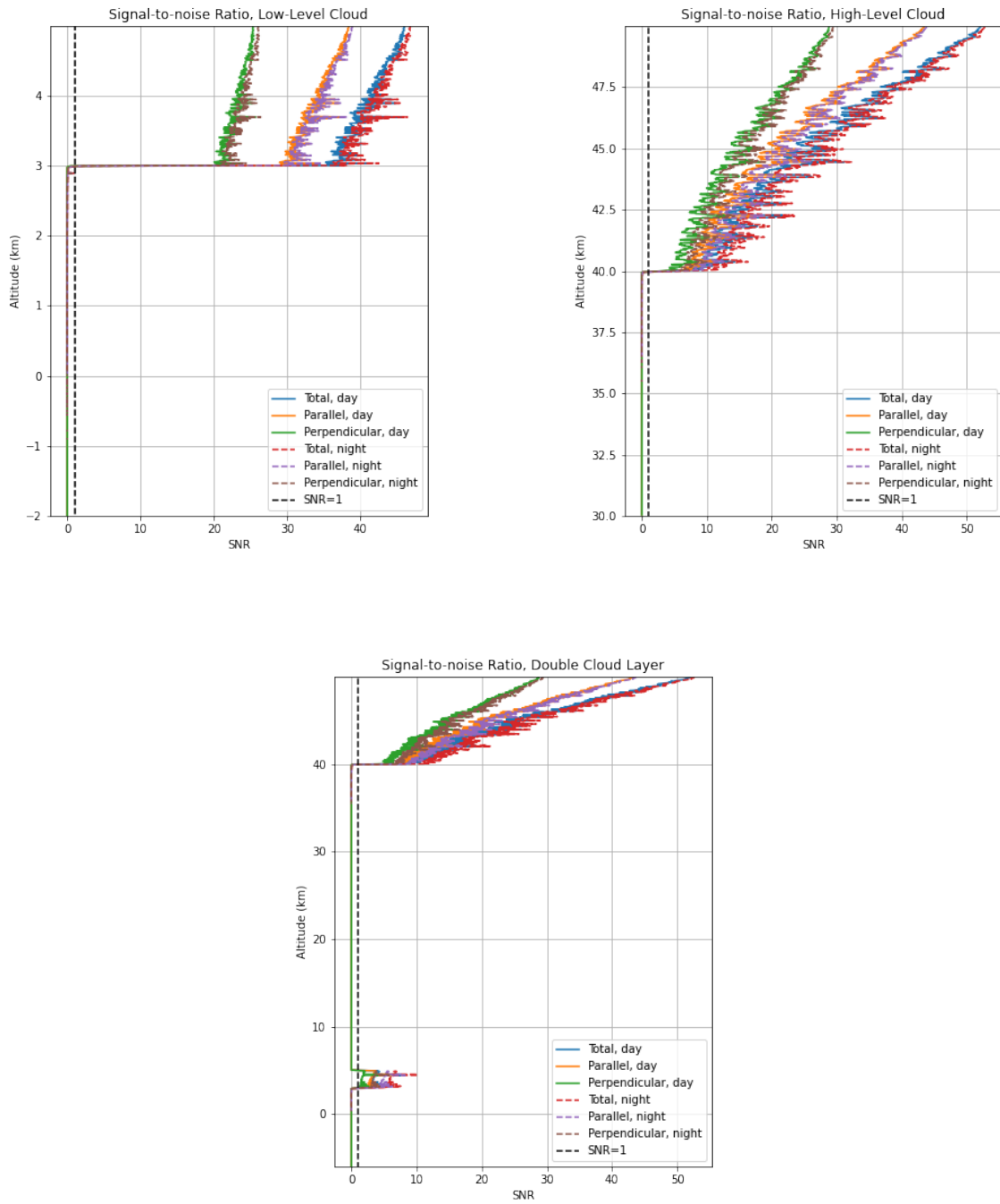


Figure 25: Simulated signal-to-noise ratio for cloud-only test cases. SNR=1 is marked by a vertical line for reference. Respective day and night measurements often overlap.

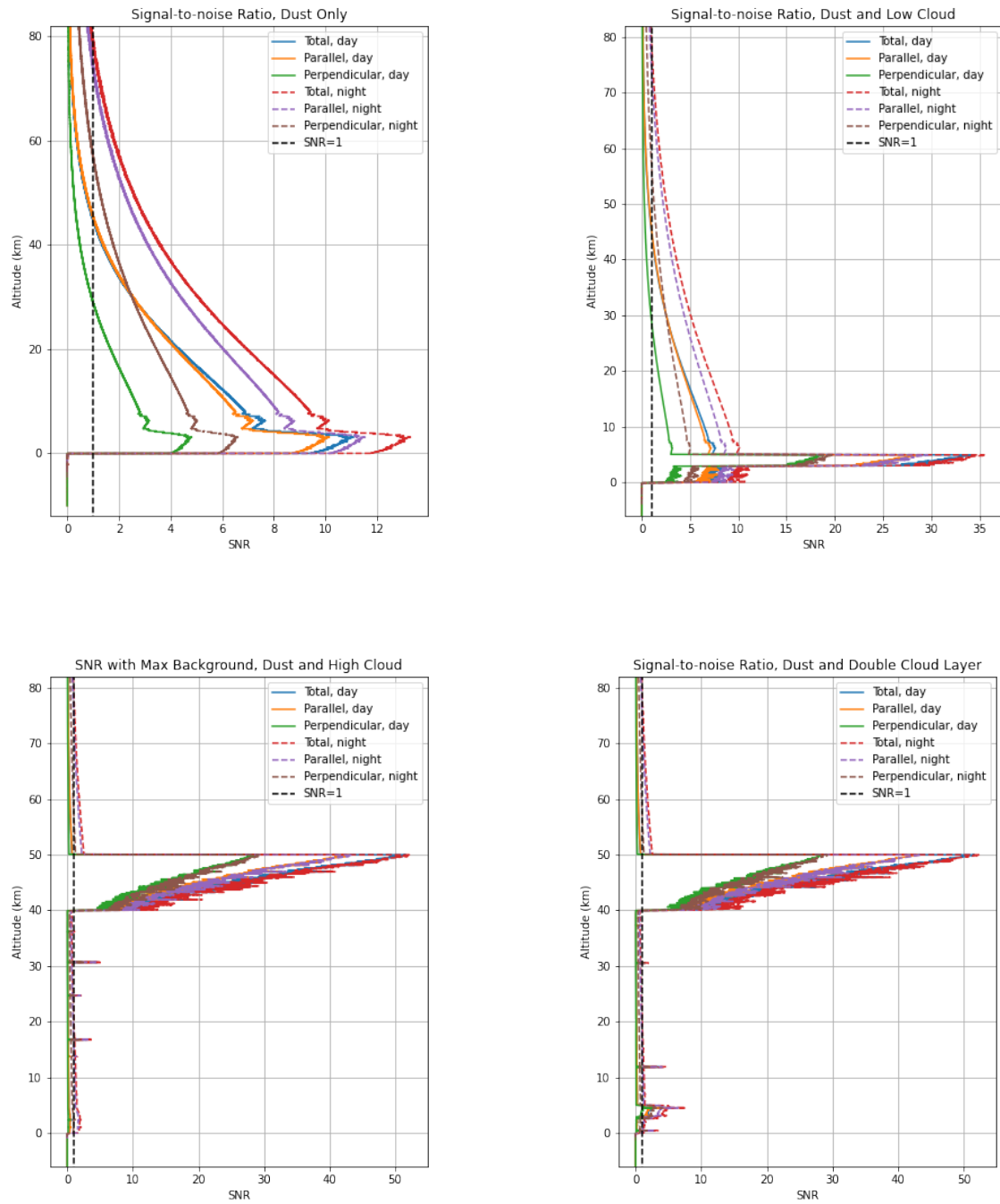


Figure 26: Simulated signal-to-noise ratio for test cases that include dust. SNR=1 is marked by a vertical line for reference. Respective day and night measurements often overlap.

Although our results indicate that an orbital LIDAR system equipped with the same specifications as CALIPSO would perform fairly well in measuring clouds and dust in the atmosphere of Mars,

this system is probably unrealistic for a Mars satellite. CALIPSO is a very expensive system with a large receiver telescope of 1 m diameter. The cost and size would make it difficult to justify for a Mars mission. The large receiver telescope makes it difficult to design a satellite that includes multiple instruments in addition to the LIDAR system. A smaller telescope receiver would make the instrument significantly more practical. We will now lower the diameter to 0.5 m and analyze the effects on the signal. The signal-to-noise ratio for test cases that include dust are shown in Figure 27 for a LIDAR system with a telescope receiver of 0.5 m diameter. The results indicate that a SNR greater than 1 can be achieved across all measurement channels in all cloud regions, excluding the case of a low altitude cloud that is positioned below a thick, high altitude cloud. In the dust-only case, the system is capable of achieving a SNR greater than 1 up to approximately 18 km during the day and up to approximately 38 km during the night. These results indicate that using a telescope receiver of 0.5 m diameter versus 1 m diameter would not have a drastic impact on signal quality but may still lead to some signal strength issues, especially in the case of multiple cloud layers.

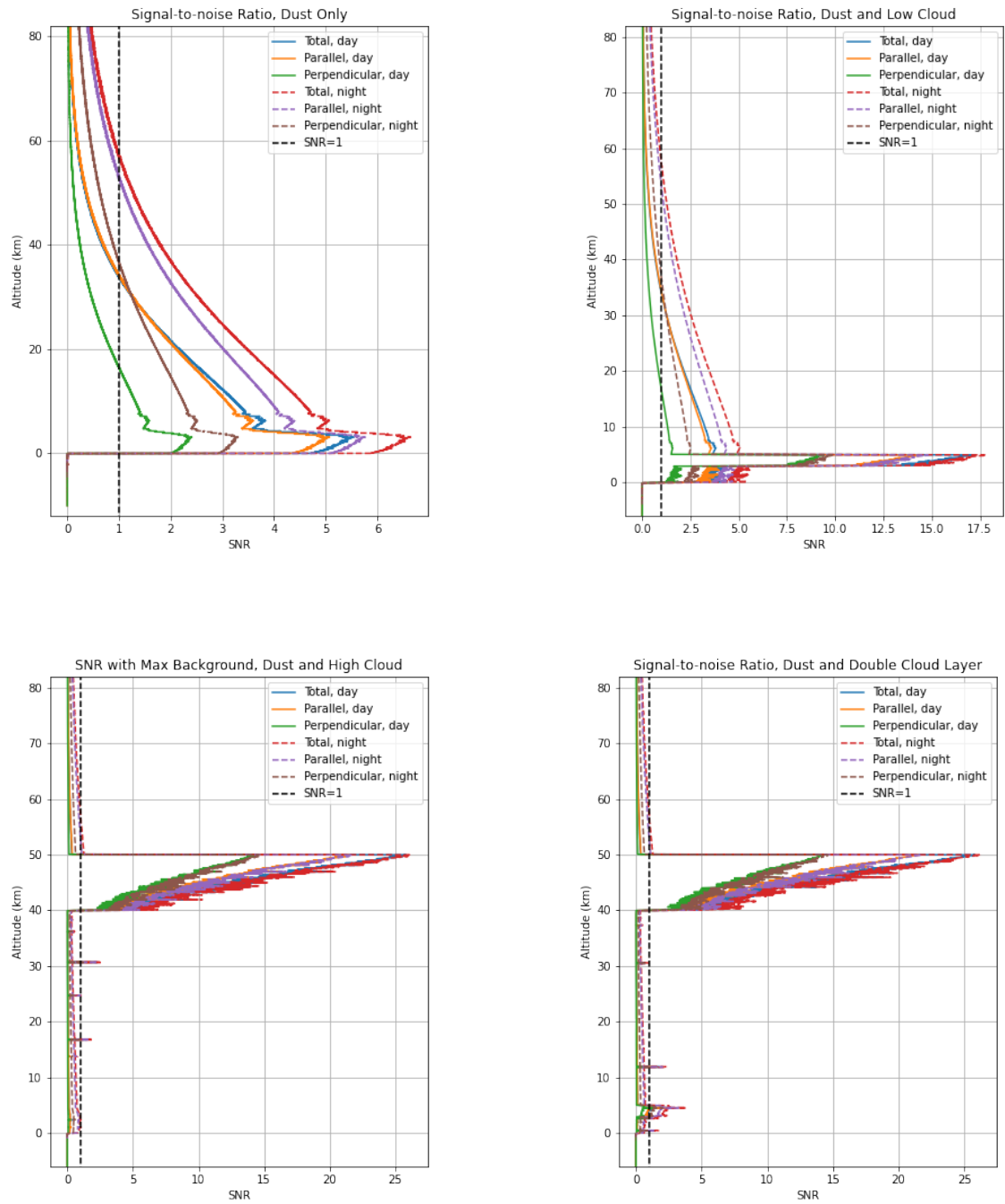


Figure 27: Same as Figure 26 but with a telescope receiver diameter of 0.5 m. Respective day and night measurements often overlap.

5.3 Depolarization Ratio

As discussed previously, the linear depolarization ratio is given by the ratio of the perpendicular component of the signal to the parallel component of the signal. The depolarization ratio, computed directly from Monte Carlo model output, is shown in Figures 28 and 29. The depolarization ratio plots show that multiple scattering has a significant impact on the measured depolarization ratio, increasing it significantly compared to the single scattering case. This effect is significantly stronger in Martian water ice clouds than in Martian mineral dust. This is due to the clouds having a larger extinction coefficient than the dust, leading to more multiple scattering and the accompanying depolarization of scattered photons. Figure 28 shows that, when no dust is present, multiple scattering from clouds can lead to measured depolarization ratios close to 1 below the cloud base. However, it should be noted that our results in Section 5.2 indicate that the signal-to-noise ratio is typically too low below these cloud bases for this effect to be detected. Figure 29 shows that the depolarization ratio in the dust regions is consistently lower than the depolarization ratio in the cloud regions, regardless of multiple scattering effects. However, it should be noted that multiple scattering causes the depolarization ratio to increase significantly in the dust regions directly below the cloud bases. In the case of the thick high altitude cloud, multiple scattering causes the depolarization ratio directly below the cloud to be almost equal to the single scattering depolarization ratio from the cloud. This indicates that it would be difficult to use the depolarization ratio to distinguish between the base of the high altitude cloud and the dust directly beneath it. Overall, these results indicate that an orbital LIDAR system around Mars would have the capability of differentiating clouds from dust based on the depolarization ratio but may have difficulty in distinguishing the cloud base from dust when the clouds are optically thick. It should additionally be noted that, in the presence of a thick high altitude cloud, the signal-to-noise ratio in the dust regions will likely be too low to properly measure the depolarization ratio.

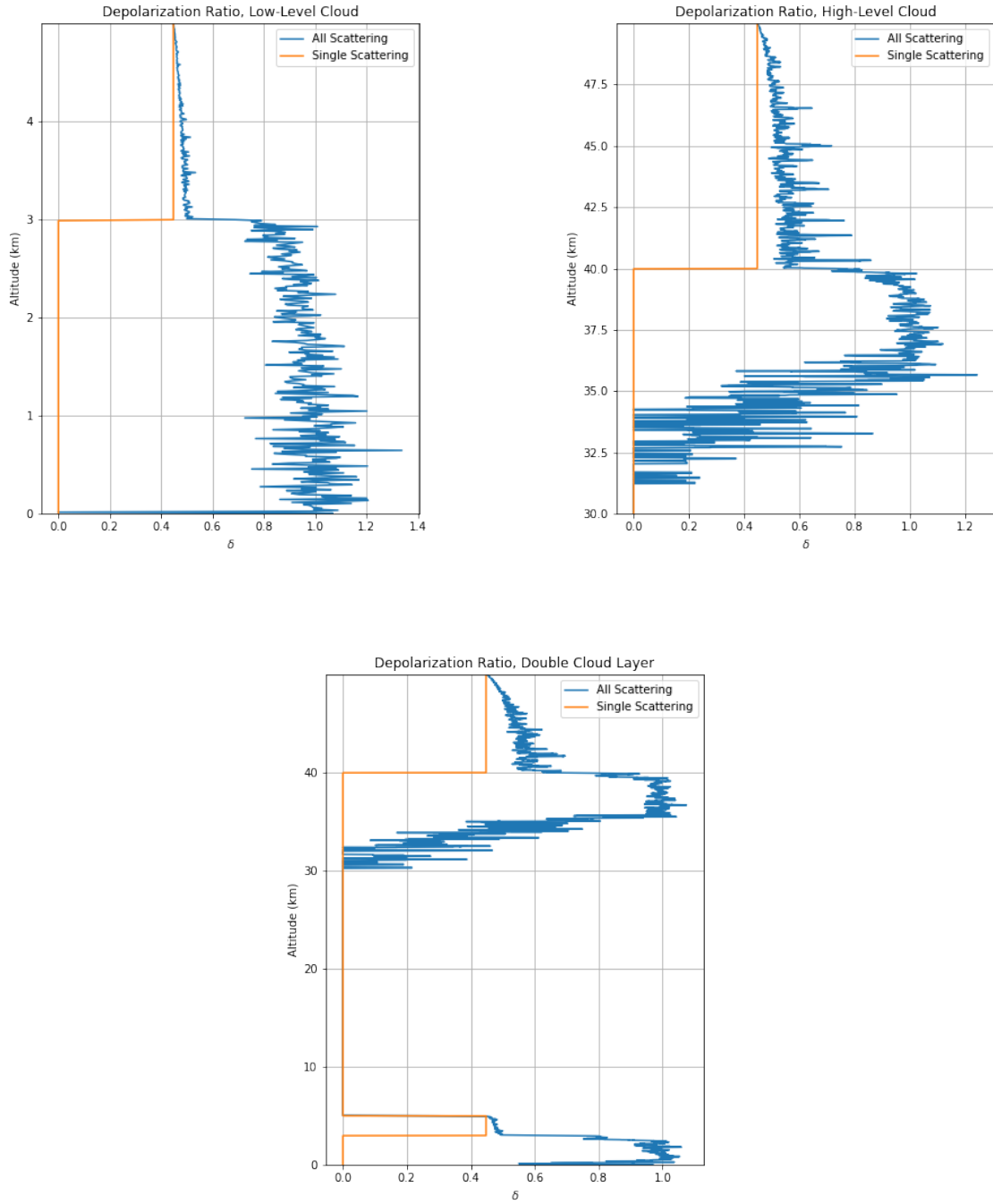


Figure 28: Simulated linear depolarization ratio for cloud-only cases. Results computed using single scattering and all orders of scattering are plotted separately.

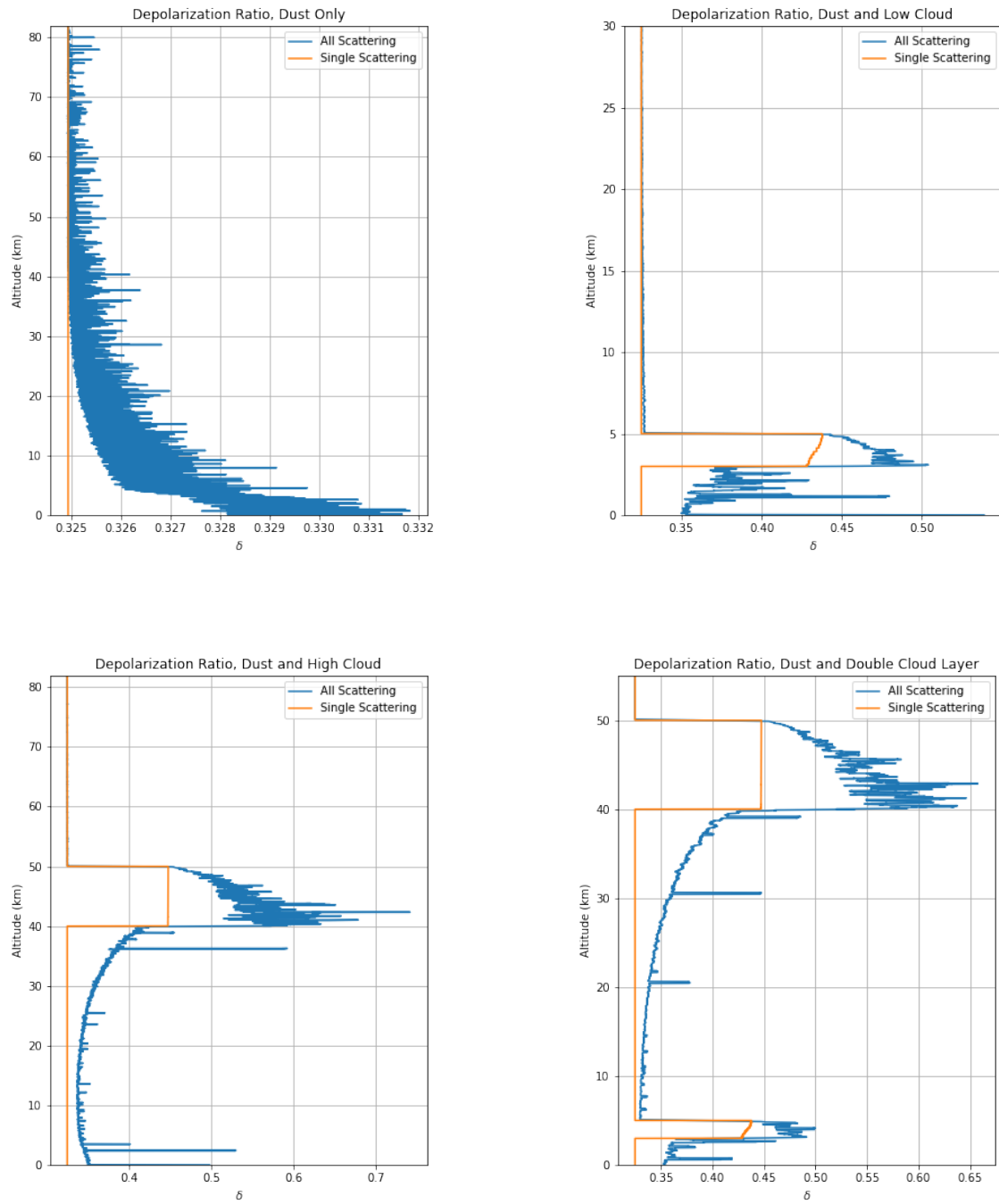


Figure 29: Simulated linear depolarization ratio for cases that include dust. Results computed using single scattering and all orders of scattering are plotted separately.

5.4 Extinction Coefficient

We will now use the simulated signals to retrieve extinction profiles, as explained in Section 4.3. The retrieved extinction profiles for the high cloud, low cloud, and dust-only cases are shown in Figure 30. CALIPSO system specifications were used for all examples shown in Figure 30. Error limit bars are included on the original extinction profiles in Figure 30. These bars represent $\pm 30\%$ of the original extinction coefficient values and serve as an accuracy gauge for the retrieved extinction coefficient. Figure 30 shows that the extinction coefficient can be determined accurately when measuring dust only. The extinction coefficient retrieval becomes less accurate in the case of clouds. This is especially true for the vertically thick high altitude cloud, for which the retrieved extinction coefficient is often outside of the $\pm 30\%$ error limit. This is because the thick high altitude cloud has the greatest amount of multiple scattering, which makes it more difficult to retrieve the extinction coefficient. These results indicate that an orbital LIDAR system around Mars would likely be able to accurately retrieve the extinction coefficient within $\pm 30\%$ error when measuring dust only or low altitude clouds with depths of only a few kilometers. However, thicker clouds may require multiple scattering corrections (Kunkel and Weinman, 1976) in order to accurately determine the extinction coefficient.

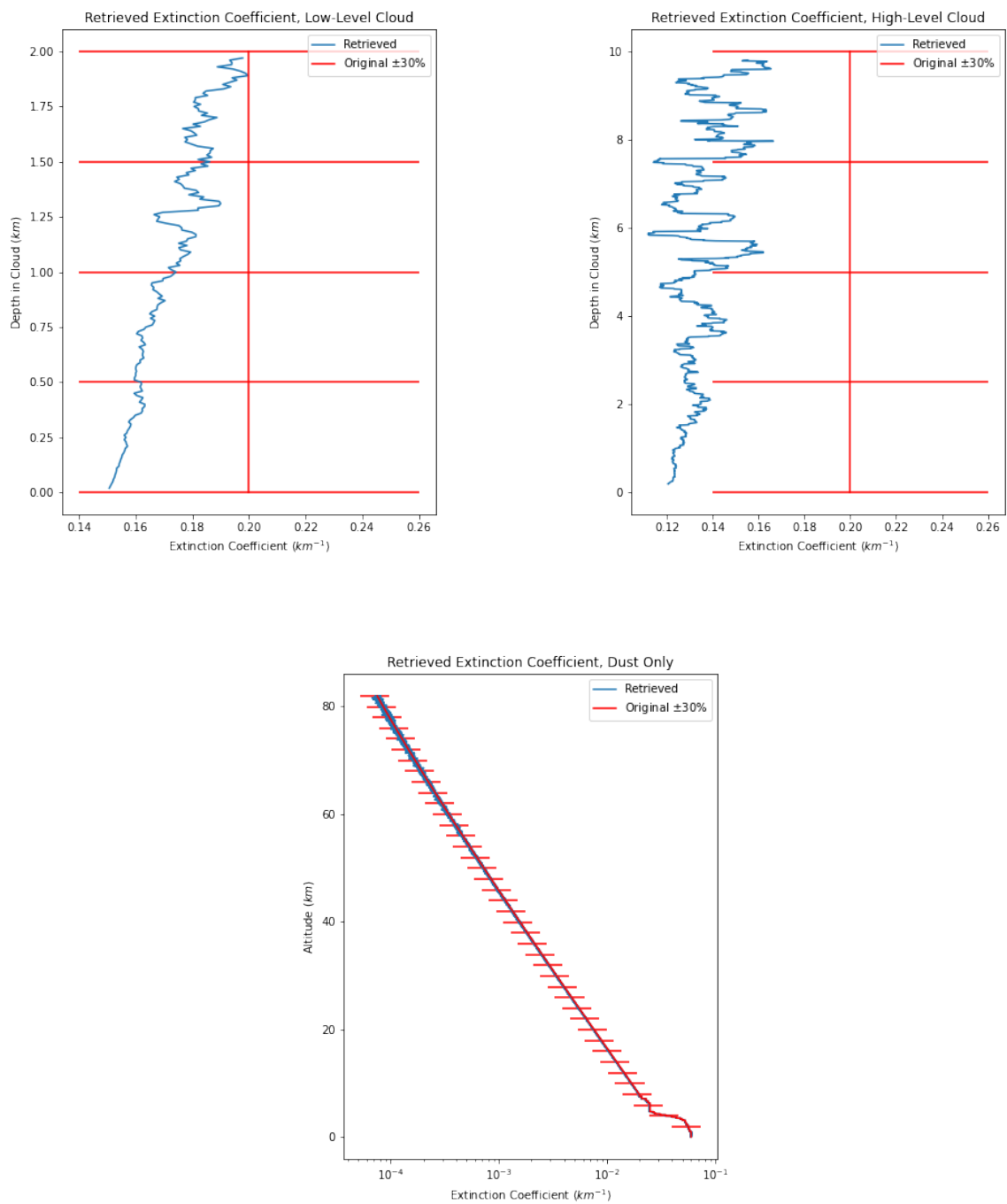


Figure 30: Extinction profiles retrieved from simulated LIDAR signals. The original extinction profiles are also shown with 30% error bars. *Top Left*: Low altitude cloud. *Top Right*: High altitude cloud. *Bottom*: Dust only (retrieved and original often overlap).

The extinction coefficient detection lower limit can be determined by calculating the extinction coefficient as the signal-to-noise ratio approaches a value of 1. The results of this analysis can be found in Table 3. Background sky light was not included in the lower limit calculations. As is the case for all simulations described in this report, the orbit altitude is 400 km, and the vertical resolution is 10 m. No dust is included in these test cases.

The Phoenix LIDAR measured a cloud within an altitude range of 4 - 6 km on Sol 95 at 02:45 local true solar time (Dickinson et al., 2010). The reported measured extinction coefficient within the cloud is $\sim 0.1 \text{ km}^{-1}$ (Dickinson et al., 2010). As shown in Table 3, for a cloud between 4 - 6 km, an orbital LIDAR system would have an extinction coefficient detection lower limit of $\sim 0.0095 \text{ km}^{-1}$ with the 1 m diameter telescope and $\sim 0.038 \text{ km}^{-1}$ with the 0.5 m diameter telescope. These results indicate that the 1 m diameter and 0.5 m diameter orbital LIDAR configurations would both be able to measure the 4 - 6 km cloud measured by the Phoenix LIDAR.

Modeling of the Martian atmosphere was carried out to model water ice clouds at various altitudes, including clouds within an altitude range of 40 - 50 km (Frank Daerden, Personal Communication). The modeled extinction coefficient within the 40 - 50 km clouds is $\sim 0.03 \text{ km}^{-1}$ (Frank Daerden, Personal Communication). As shown in Table 3, for a cloud between 40 - 50 km, an orbital LIDAR system would have an extinction coefficient detection lower limit of $\sim 0.0075 \text{ km}^{-1}$ with the 1 m diameter telescope and $\sim 0.03 \text{ km}^{-1}$ with the 0.5 m diameter telescope. These results indicate that the 1 m diameter orbital LIDAR configuration would be able to measure the 40 - 50 km clouds from the model. The 0.5 m diameter orbital LIDAR configuration would require more spatial averaging to accurately detect the 40 - 50 km clouds from the model.

Receiver Diameter (m)	Cloud Altitude (km)	Minimum Detectable Extinction Coefficient (km^{-1})
1	40-50	~ 0.0075
1	4-6	~ 0.0095
0.5	40-50	~ 0.03
0.5	4-6	~ 0.038

Table 3: Lower detection limit of the extinction coefficient for high altitude (40 - 50 km) and low altitude (3 - 5 km) Martian water ice clouds. Results are shown for 0.5 m and 1 m receiver diameters. Background sky light was not included.

6 Conclusion

We have successfully developed a Monte Carlo model that is capable of simulating multiple scattering and tracking the polarization state of atmospheric LIDAR signals. This model was used to test the viability of using an orbital LIDAR system to study water ice clouds and mineral dust in the Martian atmosphere. It was determined that an orbital LIDAR system with the same specifications as the very successful Earth-based CALIPSO satellite would be able to detect clouds and dust in the atmosphere of Mars in most cases. Although, it would likely be difficult to detect anything below a thick high altitude cloud. Due to the large distance of the satellite from the scattering medium, large amounts of multiple scattering are observed, especially in clouds. This multiple scattering leads to increases in measured depolarization ratios and errors in extinction coefficient retrieval. The retrieved extinction coefficients are within $\pm 30\%$ of the expected values without correcting for multiple scattering in the low altitude cloud and dust-only cases. High altitude clouds with vertical depths of ~ 10 km may require multiple scattering corrections to determine the extinction coefficient within 30% error. Results of depolarization ratio modeling indicate that the depolarization ratio can be used to distinguish between cloud and dust, despite the increased depolarization due to multiple scattering. However, it will be difficult to use the depolarization ratio to distinguish the base of a thick, high altitude cloud from dust. It was determined that the telescope receiver size could be reduced to half that of CALIPSO (0.5 m diameter versus 1 m diameter) and still yield detectable signals of clouds and dust in most test cases.

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