Module 6 Practice Problems: Conditional Probability and Bayes' Rule

1. The business of an ice-cream parlour depends (among other things) on the summer weather. For simplicity, assume that the parlour's yearly profits can be either $\$10,000$ or $\$6,000$. If the summer is a hot one, then this parlour's yearly profits will be $\$10,000$ with probability 0.7 and $\$6,000$ with probability 0.3. On the other hand, if the summer is a normal one, then this parlour's yearly profits will be $\$10,000$ with probability 0.5 and $\$6,000$ with probability 0.5. An expert predicts that with probability 0.6, this year's summer will be a hot one.

(a) Depict this situation using a probability tree. Be sure to write the various conditional probabilities.
(b) Based on the expert's prediction, what is the overall probability that the parlour's yearly profit will be $\$10,000$ this year?
(c) Let us now add another possibility: apart from hot or normal, the summer can also be a cold one. If the summer is cold, then this parlour's yearly profits will be $\$10,000$ with probability 0.2 and $\$6,000$ with probability 0.8. The expert's prediction is that this year's summer will be hot with probability 0.6, normal with probability 0.2 and cold with probability 0.2. Now, draw the probability tree and calculate what is the overall probability that the parlour's yearly profit will be $\$10,000$ this year?

2. A DoorDash driver has two deliveries to make. If he is delayed on the first delivery, his probability of being delayed on the second delivery is 0.8. On the other hand, if he makes the first delivery on time, his probability of being delayed on the second delivery is 0.2. His time on the first delivery depends on the particular highway he has to take. If the highway is jammed (which happens 40% of the time), he will be delayed on his first delivery; if the highway is moving well, he will be on time for his first delivery.

(a) Depict this situation using a probability tree.
(b) Is his timing on the first and second delivery independent? Explain.

(c) What is the probability that this driver will be delayed on both of his deliveries?
(d) What is the probability that this driver will be delayed on his second delivery?
(e) What is the probability that this driver will be delayed on exactly one of his deliveries?

3. An incumbent government's chances of getting re-elected are often linked to the state of the economy. Let us assume that if the economy is good, then the incumbent government gets re-elected with probability 0.6, while if the economy is bad, its probability of getting re-elected is 0.3.

Consider a country whose economy is largely based on agriculture, which itself is affected by whether there is too little, too much or normal rainfall. Let us assume that if there is normal rainfall, the probability of the economy being good is 0.8. On the other hand, if there is too much rainfall, the probability of a good economy is 0.5, while if there is too little rainfall, the probability of the economy being good drops to 0.2.

The meteorology office has predicted that this year, the chances of normal rainfall is 0.5, while the chances of too little or too much rainfall is 0.25 each.

(a) Depict this situation using a probability tree.
(b) Conditional on a good economy, what is the probability of the incumbent government getting re-elected?
(c) Conditional on normal rainfall, what is the probability of the incumbent government getting re-elected?
(d) Conditional on too little rainfall, what is the probability of the incumbent government getting re-elected?
(e) What is the overall probability of the incumbent government getting re-elected?

4. Suppose $P(A) = 0.2$, $P(B) = 0.4$, $P(A \cup B) = 0.5$.

(a) Determine $P(A \cap B)$.
(b) Determine $P(B|A)$ and $P(A|B)$.
(c) Are A and B independent?

5. In a group of university students, if one is chosen at random, the probability that he/she has a iPhone is 0.4, while the probability that he/she has a iPad is 0.3. The probability that a randomly chosen student has both a iPhone as well as a iPad is 0.2.

(a) Suppose we pick a random student and find that he/she has a iPhone. Knowing this, what is the probability that this student also has a iPad?

(b) Conversely, suppose a randomly chosen student is found to have a iPad. Knowing this, what is the probability that this student also has a iPhone?

6. A family has 3 children.

(a) What is the probability that all three are girls?
(b) Suppose we know that the youngest of the three is a girl. What is the probability that the remaining two are girls?
(c) Suppose we know that one of the three is a girl. What is the probability that the remaining two are girls? How does this compare with your results in (a) and (b)?

7. A group of 300 children were asked whether they played hockey or basketball. Among the group, 120 said they played hockey, 80 said they played basketball and 40 said they played both hockey and basketball.

(a) What is the probability that a child selected at random from the group plays hockey?
(b) What is the probability that a child selected at random from the group plays hockey, given that he/she plays basketball?
(c) What is the probability that a child selected at random from the group plays hockey, given that he/she plays one game only?

8. The American Community Survey is a regular survey conducted by the United States Census Bureau on various subjects of interest to communities. In a 2019 survey it conducted in the city of Miami, Florida on people who used non-motorized means to get to work, it found the following. Among men, 7390 walked and 792 biked to work. Among women, it found that 4309 walked and 454 biked to work.

(a) Use a two-way frequency table to depict this data.
(b) Among people who use non-motorized means to get to work in Miami, what is the probability that a randomly chosen person is a woman who bikes to work?
(c) Among women who use non-motorized means to get to work in Miami, what is the probability that a random woman bikes to work? How does this compare with the probability among men?
(d) What is the probability that among those who bike to work, a randomly chosen person is female?

9. Let us reconsider the data from Problem 1 in the Module on Probability Basics. During the 2000 US election between Joe Biden and Donald Trump, Edison Research conducted exit polls in which they interviewed 15,590 voters outside polling places or early-voting sites or by phone. Of these voters, 8069 said they had voted for Joe Biden, 9198 did not have a college degree, and 4145 voters without a college degree said they had voted for Joe Biden. The remaining voted for Donald Trump.

(a) Use a two-way frequency table to depict this data.
(b) Conditional on having a college degree, what is the probability that a random voter voted for Donald Trump?
(c) Conditional on not having a college degree, what is the probability that a random voter voted for Donald Trump? How does this compare with (b)?
(d) Conditional on having voted for Donald Trump, what is the probability that a random voter has a college degree?

10. It is estimated that 30% of emails are spam emails. A certain detection software claims that it can detect 98% of spam emails, and the probability of a false positive (it classifying a non-spam email as spam) is 5%.

You decide to install this software for filtering your emails. You wonder what fraction of the emails the software has detected as spam are actually not spam. Can you use Bayes' rule to figure out this probability?

11. A particular company's revenues is significantly tied to the state of the Chinese economy. If the Chinese economy expands, there is a 90% probability that the company's revenues rises; but if the Chinese economy does not expand, the probability that the company's revenues rise is 20%. At the start of the year, experts said that there is a 25% probability that the Chinese economy will expand this year.

Suppose the company's CEO announces that its revenues have risen. Given this information, what is the probability that the Chinese economy has expanded?

12. In a research study on smoking and mortality, 10% of the participants reported as being heavy smokers when young, 20% as light smokers, and 70% as non-smokers. In the five-year study, it was determined that the death rate of heavy smokers was four times that of non-smokers, while the death rate among light smokers was two times that of nonsmokers. Suppose a randomly selected participant dies over the period of the study. What is the probability that this person had been (i) a heavy smoker when young, (ii) a non-smoker when young?

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