

Structure and Local Properties of Dark Matter Halos
Nonspherical Jeans Modeling of Self-Interacting Dark Matter Halos
and
Quantifying the Impact of the Large Magellanic Cloud
on Local Properties of Dark Matter in Hydrodynamic Simulations

by
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Abstract

The structure and local properties of dark matter (DM) are important input variables for both direct and indirect detection experiments. Common assumptions about DM, such as, its particle nature being cold and collisionless—CDM—and its distribution in halos being isothermal and spherical, and well-modeled by an isotropic Maxwell-Boltzmann velocity distribution—the Standard Halo Model (SHM)—prove to be at odds with astrophysical observations. In this thesis I present work towards elucidating two areas of refinement to the above-mentioned assumptions regarding DM: 1) allowing moderate self-interactions between DM particles (SIDM), extending the semi-analytic Jeans modeling technique for SIDM halos to multiple dimensions and including baryons and nonspherical effects to study the resulting change in the DM halo structure; 2) quantifying the departure from the SHM by analyzing hydrodynamical simulations of Milky Way (MW) like halos to characterize the impact on local properties of DM induced by satellite galaxies like the Large Magellanic Cloud (LMC) and their implications for direct detection experiments.

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Introduction

The first formal evidence for matter beyond the visible was hinted at by the Swiss astrophysicist Fritz Zwicky in 1933 [1] in observing the radial dispersion velocities of the Coma cluster. Zwicky noted that the stars and gas visible within the galaxies did not contain enough mass to provide the gravitational anchoring to hold the cluster together. Zwicky concluded that the cluster must contain significant amounts of unseen mass,¹ which he termed "dark matter."

Incontrovertible evidence for the existence of said dark matter (DM) waited until 1970, when Astronomers Vera Rubin and Kent Ford used the spectra of ionized gas in M31 to accurately measure the orbital velocities as a function of galactic radius, $v(R)$, of stars and gas out to $R = 24\text{kpc}$ [3]. The expectation for $v(R)$ based on Newtonian arguments is, beyond the radius of the stars and therefore the visible mass distribution, orbital speeds should fall off as $R^{-1/2}$ -behavior known as *Keplerian orbits*. In stark contrast, what Rubin and Ford found was that the rotation curve did not fall off but instead remained relatively constant out to large galactic radii. To account for the constant orbital speeds Rubin and Ford postulated the existence of a dark halo of mass that enveloped and extended far beyond the stellar disk, thus providing the extra mass necessary to hold the visible galactic components together.

A more modern approach to studying dark matter halos comes from gravitational lensing. Owing to the large mass distributions of dark halos extending beyond the luminous components of galaxy clusters and galaxies, light emitted from a source object that passes through or near such DM halos will have its path deflected by the local curvature of spacetime, leaving a marked footprint of its mass distribution and structure. Strong and weak lensing theory has since led to some of the strongest estimates for the mass density and structure of dark matter halos [4].

Moreover, dark matter as a theory has been corroborated by cosmological evidence. Observation of the Cosmic Microwave Background (CMB) radiation has led to the standard model of cosmology,

¹Using the virial theorem for a self-gravitating steady-state system, and making various assumptions, the estimate for the mass of the Coma cluster revealed that the combined mass of the visible stars and gas only accounted for 11% of the total mass of the cluster [2].

Λ CDM, where Λ is the cosmological constant associated with dark energy and the expansion of the universe, and CDM stands for cold, collisionless, dark matter [5]. According to Λ CDM, in the early Universe, CDM decoupled from the thermal bath prior to baryonic matter,² forming gravitational pockets, or halos, in the small density fluctuations of the CMB that would grow into the potential wells within which gas would condense to form stars and, by hierarchical aggregation, galaxies and clusters of galaxies. This process is borne out in simulations, where the qualitative structure of the observed cosmic web can be faithfully reconstructed from the Λ CDM model, showing remarkable agreement with observations of large-scale structure that probe the history of the Universe up to the CMB [6].

Under the Λ CDM paradigm the current estimate is that roughly 26.5% of the mass-energy budget of the Universe is dark matter, with only 5% ordinary baryonic matter, and the remaining dark energy. Of the matter content, roughly 85% of mass in the Universe is in the form of dark matter [7].

Despite the abundant astrophysical evidence and the successes of cosmological models, the exact particle nature or manifestation of dark matter remains a mystery. Candidates for dark matter abound, with predicted masses ranging from 10^{-22}eV (ultralight dark matter)³ to $10^5 M_\odot$ (primordial black holes).

The leading particle candidate is the weakly interacting massive particle (WIMP), which is a non-Standard Model particle thought to be thermally produced in the early universe, with predicted mass in the range $\mathcal{O}(1 - 1000 \text{ GeV})$. Extensions to the Standard Model (SM) of particle physics such as supersymmetry naturally predict the existence of candidate particles for WIMP DM that match the currently observed abundance of DM.⁴ Modern state-of-the-art detectors seek to detect WIMP DM via direct interactions with SM particles (direct detection), indirectly through the by-products of DM annihilation or decay (indirect detection), or via the creation of DM particles or its products in high-energy particle collisions (collider searches).

²Protons, neutrons, and their derivatives.

³In natural units, i.e. $c = \hbar = G = 1$.

⁴A coincidence referred to as the *WIMP miracle*.

Both direct and indirect DM searches depend on the theoretical properties of DM to define and calibrate the parameter spaces of their searches. DM mass, interaction strength and annihilation rate, velocity and density distributions, are properties that enter into the calculations for direct and indirect searches, but which are *a priori* unknown. The standard assumptions of WIMP DM used to inform the experimental searches are that DM is collisionless, interacting only via gravity and weakly with SM particles, and that WIMP DM organizes into isothermal spheres with a Maxwell-Boltzmann velocity distribution.

In recent decades, some of the standard assumptions about the properties of DM have come into conflict with observation and simulations. For example, early N-body simulations of CDM-only cosmologies predicted a universal DM density distribution for galaxies and clusters of galaxies that contain a steeply rising core-*cusp*—that scales as $\rho(r) \propto r^{-1}$, well described by an ad hoc fitting function due to Navarro-Frenk-White (the "NFW" profile) [8]. However, observations of rotation curves for spiral, dwarf, and low surface brightness (LSB) galaxies have been shown to disfavor cuspy cores and instead prefer cores with constant density, i.e. $\rho(r) \propto r^0$ [9–12]. This problem is known as the *core-cusp problem*, and figures as one among many small-scale structure issues with CDM. Others include: *the missing satellites problem*, *Too-Big-To-Fail problem*, and *the diversity problem* (see Ref. [13] and Ref. [14] for detailed discussion on the small-scale challenges of Λ CDM). Recent high-resolution hydrodynamical simulations including dissipative baryonic physics such as gas cooling, star formation, and supernova feedback, have pressed in the direction of resolving some of the small-scale structure issues noted above. Yet, baryonic feedback depends strongly on the particular models used and has not been shown to provide a unified explanation of DM structure across observed mass scales. Hence, to what degree the inclusion of baryonic physics can resolve small-scale structural issues remains unclear.

To resolve the tensions between CDM and the observable small-scale structure of DM, Spergel and Steinhardt proposed in 1999 an alternative to the CDM model allowing a small but non-negligible DM-DM self-interaction cross-section, called *self-interacting dark matter* (SIDM) [15]. An attractive feature of SIDM is that it has the potential to soften the tensions between CDM and

observations on small scales while retaining the success of CDM on large scales⁵ [16].

Since scattering is proportional to density, self-interactions in the dense inner halo serve to drive mass into the less dense outer halo, establishing a quasi-equilibrium state with a constant density core as preferred by observations. Moreover, scattering events transport heat from the outer halo to the inner halo, thermalizing the inner halo and leading to a constant velocity dispersion consistent with a Maxwell-Boltzmann distribution, while particle orbits become isotropic, erasing ellipticity and leading to a spherical density distribution.

N-body simulations of galactic formation and evolution are brought to bear to confront the observational consequences of SIDM cosmologies, however, N-body simulations prove to be computationally costly, and, due to being fully dynamical, are unable to model particular observed systems for direct comparisons. An alternative and complementary approach to N-body simulations for studying the observational consequences of SIDM is a semi-analytical method for modeling SIDM halos called the *Jeans model*, whose main advantage is its speed and ability to match the final states of observable systems, but is limited by only being able to test for the weaker condition of consistency of a SIDM system, and not whether such a system will arise in the context of SIDM cosmology.

Recent work has used the Jeans model to confront SIDM predictions and to match observations [10, 17–20], noting the surprising accuracy of the Jeans model to predict density profiles of galaxy and galaxy cluster halos [21]. Moreover, using the Jeans model Refs. [22, 23] showed that models with SIDM and baryons provide a promising avenue to solve the *diversity problem*, that is, halos with similar properties showing a diversity in their rotation curves and density profiles, unexplained by CDM simulations. However, Jeans modeling has so far only been explored in one- and two-dimensions assuming spherical symmetry.

My research explores the possibility of extending the Jeans model into higher dimensions and

⁵For SIDM to impact the structure of a DM halo it must scatter at least once on average per lifetime of a halo. The cross-section can be tuned such that DM remains effectively collisionless on large scales. The typical quoted cross-section per unit mass for DM to remain in accordance with large-scale observations is $\sigma/m < 1\text{cm}^2/\text{g}$.

relaxing the symmetry assumptions. A theoretical framework has been developed alongside a corresponding software implementation that models a SIDM halo with baryons and baryonic physics that has been validated against SIDM+baryons EAGLE-50 simulations from Ref. [21]. I will show that the nonspherical Jeans model can perform equally well as the established spherical Jeans model while providing information about the shape of SIDM halos that can be used to confront observations and potentially discriminate between SIDM and CDM cosmologies where the spherical model is unable to, offering a novel probe for SIDM phenomenology.

My research also focuses on understanding the local DM distribution of CDM halos. Ground-based direct detection experiments search to measure the recoil energy of a target nucleus or electron after a scattering event with a DM particle. Direct detection experiments have so far probed a large part of the DM parameter space, and have set exclusion limits on the DM mass and interaction cross-section. In order to interpret the results of such experiments astrophysical assumptions concerning the properties of DM in the Solar neighborhood need to be made—such as the local DM density and the velocity distribution [24]. The standard astrophysical picture is that galaxies like the Milky Way (MW) are situated within an isothermal sphere of DM assumed to have an isotropic Maxwell-Boltzmann velocity distribution with a peak speed that is equal to the local circular speed, an assumption known as the Standard Halo Model (SHM) [25]. However, hydrodynamical simulations of CDM point to a large halo-to-halo scatter in the local velocity distributions of galaxies [26–31], while astronomical surveys (Gaia survey [32]; Sloan Digital Sky Survey [33]) and kinematic data from the MW point to significant substructures from its non-quiescent formation and merger history [34–38] that lead to departures from the SHM that bear important implications for DM direct detection searches.

Recent work has devoted attention to the largest satellite of the MW—the Large Magellanic Cloud (LMC)—and the impact its recent pericenter (closest) approach has had on the local DM properties of the MW and direct detection limits [39–44]. Specifically, Ref. [40] used an idealized model of the MW and the LMC to determine what impact the latter has on the velocity distribution in the Solar neighborhood, and what implications that has on direct detection exclusion limits derived

therefrom.

My research builds on this by using fully hydrodynamical simulations with MW-LMC analog pairs that match the kinematics of the MW-LMC system, evolving the MW-LMC pairs through their pericenter approach to answer the question of what impact LMC-like halos have on the local DM properties, and therefore direct detection limits, of their MW-like host halos. What's more, I attempt to quantify how various factors such as the particular Sun-LMC geometry in the simulations, fraction of LMC particles in the Solar region, and the gravitational boost to the native DM particles of the host halo due to its satellite, uniquely impact local DM properties and the exclusion limits in a fully cosmological context.

Part I will introduce the theoretical framework for the non-spherical Jeans modeling of a SIDM halo and the details for the numerical approach to solving the nonspherical Jeans equations with the addition of a baryon distribution and baryonic physics. A detailed example of the nonspherical Jeans model will be presented for a MW-mass DM halo to illustrate and interpret the results of the nonspherical Jeans analysis. The nonspherical Jeans model will be compared to simulations to test its effectiveness in inferring the input cross-section as well as modeling density and shape parameters. To conclude I will comment on some future directions for the use of the nonspherical Jeans model while summarizing the key results of the foregoing analyses.

Part II presents hydrodynamical simulations of MW-LMC analogs and an analysis of the impact of the LMC on the local properties of DM in the Solar neighborhood, and how this affects the astrophysical assumptions inherent in direct detection experiments. DM halo properties will be derived directly from cosmological simulations of MW-LMC analog systems and compared to the SHM. The simulation-derived halo properties will then be translated to predictions for near-future direct detection experiments, showing that the exclusion limits become more sensitive to low DM masses in comparison to the SHM when including the effects due to the LMC. In conclusion, I will summarize the key findings and comment on how they inform future dark matter direct detection searches.

Structure of SIDM halos and non-spherical Jeans modeling

1 Overview

Current N-body simulations that model the realistic interplay between SIDM and baryonic processes have alleviated some of the long-standing small-scale structural issues with the Λ CDM paradigm. However, SIDM remains a viable alternative to Λ CDM because it can be tested against astrophysical observations and it provides a potential solution to the diversity observed in galaxy rotation curves [22,23,45]. N-body simulations have been the main route by which SIDM predictions are confronted with observations. However, N-body simulations suffer from being computationally costly and unable to model particular observed systems, since they solve for the full dynamical evolution of a system given an initial condition. The latter is partly resolved by large simulation volumes, wherein many systems with a range of halo masses are simulated. Despite large simulation volumes, there is no guarantee that a baryon profile—which depends sensitively on how the baryonic physics is modeled—will match an observed system.

The Jeans model is a semi-analytic alternative to modeling SIDM halos that is complimentary to the more realistic N-body simulations [20,21,23,46,47]. The key advantages of the Jeans model are its low computational cost and being non-dynamical. By assuming the dark matter halo is in hydrostatic equilibrium, the Jeans model is able to fully specify the final state of a system. Hence, in principle, the Jeans model can match as many observables as desired, enabling direct comparison with observed systems.

However, the Jeans model also has its disadvantages. Being non-dynamical the Jeans model does not account for complex assembly histories or substructure. Moreover, the Jeans model can only test for the lesser condition of whether a system is consistent with SIDM cosmologies, and not whether such a system arises in SIDM. Further, to model a SIDM halo we assume a heuristic that partitions the halo into a self-interacting inner region and an effectively collisionless outer region, which is necessarily a simplification and is not expected to hold for real systems. Despite

these disadvantages, the Jeans model has become a popular tool with which to confront SIDM predictions for astrophysical observations.

The Jeans model has shown great success in modeling the one-dimensional density profiles of DM halos under the assumption of spherical symmetry, where fits to SIDM simulations showed similar levels of agreement as the NFW profile with CDM halos [21]. The Jeans model has also been used to constrain the velocity-dependent cross-section of galaxy groups and clusters including a wide range of systematic effects [20]. However, symmetry assumptions are expected to break down in galaxies with baryon-dominated interiors, where the gravitational potential due to baryons influences the structure of SIDM [46, 48]. Moreover, although SIDM is expected to be more spherical than CDM, in regions of the halo that are effectively collisionless SIDM halos can retain some of the triaxiality of CDM halos. Therefore, we present a theoretical framework for extending the Jeans model beyond spherical symmetry.

Sec. 2 will briefly outline the structural differences between CDM and SIDM halos. Sec. 3 will present the basic setup of the Jeans model for a SIDM halo and Sec. 4 will describe the theoretical framework for extending the Jeans model beyond the assumption of spherical symmetry including details of the numerical method used to solve the Jeans equation in multiple dimensions. Sec. 5 will describe the parameters used to characterize the nonspherical Jeans model algorithm, giving an example of the results for a MW-mass halo and baryon potential, and then compare in detail the nonspherical Jeans model with SIDM simulations from the EAGLE-50 simulation suite. Finally, Sec. 6 will comment on the outlook of the nonspherical Jeans model and future use cases, while Sec. 7 will conclude with a summary of the key results of the nonspherical Jeans model.

2 The structure of DM halos

CDM density profile

The DM density profile for CDM halos from N-body simulations across galaxy and cluster scales is well fit by the Navarro-Frenk-White (NFW) profile [8],

$$\rho_{\text{NFW}} = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2} \quad (1)$$

where ρ_s and r_s are the scale density and radius. An equivalent parameterization of eq. (1) is possible in terms of M_{200} and c , where M_{200} is the enclosed mass within the virial radius r_{200} ,⁶ and c is the concentration parameter defined as $c = r_{200}/r_s$. Further, M_{200} and c (or ρ_s and r_s) can be shown to be highly correlated via the mass-concentration relation [49] so that the density can be characterized by a single parameter.

The NFW density profile is characterized by a shallow inner slope (cusp), where the logarithmic slope $\alpha \sim -1$,⁷ transitioning to $\alpha \sim -2$ near the center and up to $\alpha \sim -3$ for large radii (see Fig. 1).

CDM halo shapes

The shapes of NFW halos are approximately ellipsoidal, or triaxial, consistent with numerical simulations of the gravitational collapse and hierarchical structure formation of CDM halos [50]. The prediction that CDM halos are triaxial is an important fact, as it offers a further observational handle beyond the density profiles with which to discriminate between CDM and SIDM halos.

SIDM density profile

One of the attractive features of SIDM is its ability to resolve the core-cusp problem. SIDM naturally provides the mechanism by which a cusp can become a core, that is, scattering events in the inner, dense, regions serve to thermalize the DM, redistributing kinetic energy and achieving a state of

⁶The virial radius is defined as the radius at which the mean enclosed density is 200 times the critical density,

$\rho_{\text{crit.}}$

⁷The logarithmic slope is defined as $\alpha = d \ln \rho_{\text{dm}} / d \ln r$

quasi-equilibrium wherein a constant density core is formed. In terms of the logarithmic slope, SIDM density profiles have an inner region where $\alpha \sim 0$ and transition to $\alpha \sim -2$ or $\alpha \sim -3$ in outer regions where the rate of collisions is low enough to be considered effectively collisionless (see Fig. 1). The inclusion of baryons affects this picture by deepening the gravitational potential well in the core where self-interactions are in equilibrium, driving the DM to have increased core densities in comparison to SIDM-only predictions [47].

SIDM halo shapes

Scattering events randomize DM particle velocities, leading to the velocity dispersion being closer to isotropic and therefore halo shapes that are rounder than their CDM counterparts. However, since SIDM is intimately tied to baryons, SIDM halo shapes can deviate significantly depending on the particular baryon distribution in the core. This becomes especially important in galaxies with high baryon-mass fractions and or highly asymmetric baryon distributions [18, 23, 48].

The following section presents the basic setup for modeling an SIDM halo using the Jeans model including the effect of baryons and baryonic physics.

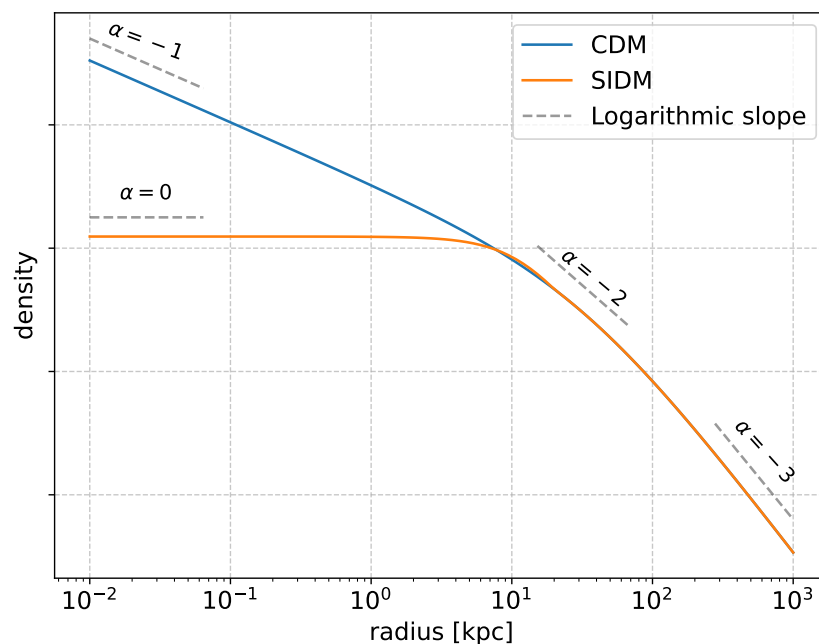


Figure 1: Logarithmic slopes for CDM and SIDM spherically averaged DM density profiles.

3 The Jeans model for SIDM halos

The Jeans model for an isothermal DM halo is derived starting from the collisional Boltzmann equation [51],

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla \Phi_{\text{tot}} \cdot \frac{\partial f}{\partial \mathbf{v}} = \mathcal{C}[f], \quad (2)$$

where f is the DM distribution function, $\nabla \Phi$ is the total gravitational potential of the system, and $\mathcal{C}$ is the self-interaction term. Self-interactions drives DM towards being approximately isotropic and isothermal with a Maxwellian distribution function, $f \propto \exp(-\frac{1}{2}|\mathbf{v}|^2/\sigma_0^2)$, where the one-dimensional velocity dispersion, σ_0 , can be taken to be constant. Taking the first moment of eq. (2) and assuming kinetic equilibrium we arrive at the time-independent Jeans equation,

$$\sigma_0^2 \nabla(\rho_{\text{dm}}) = -\rho_{\text{dm}} \nabla \Phi_{\text{tot}}, \quad (3)$$

where ρ_{dm} is the one-dimensional DM density distribution. Eq. (3) is equivalent to treating the DM as an ideal gas in hydrostatic equilibrium with an equation of state, $p_{\text{dm}} = \sigma_0^2 \rho_{\text{dm}}$. Solving eq. (3)

for ρ_{dm} gives,

$$\rho_{\text{dm}}(\mathbf{r}) = \rho_0 \exp\left(-\frac{\Phi(\mathbf{r}) - \Phi(\mathbf{0})}{\sigma_0^2}\right). \quad (4)$$

The potential, Φ , for the system is described by Poisson's equation for DM and baryonic density components, i.e.,

$$\nabla^2 \Phi(\mathbf{r}) = 4\pi G(\rho_{\text{dm}}(\mathbf{r}) + \rho_{\text{b}}(\mathbf{r})) \quad (5)$$

where G is Newtons gravitational constant and ρ_{b} is the baryonic density. Solving eq. (5) with suitable boundary conditions enables a solution for ρ_{dm} via eq. (4), which gives an isothermal DM density profile.

In order to model a realistic DM halo with self-interactions we adopt the useful heuristic that partitions the halo into an inner isothermal halo modified by frequent self-interactions and an outer, effectively collisionless, CDM halo. To define a transition radius at which self-interactions are effectively turned off we define the rate equation,

$$\rho_{\text{dm}}(r_{\text{m}}) \frac{\sigma}{m} v_{\text{rel}} t_{\text{age}} = 1, \quad (6)$$

where σ/m is the velocity independent cross-section⁸ per unit DM mass for DM-DM self-interactions, v_{rel} is the average relative velocity between DM particles (assuming a velocity-independent cross-section), and t_{age} is the age of the halo. The rate equation defines the radius, r_{m} , at which the DM scatters on average once per lifetime (t_{age}) of the halo. Inside r_{m} self-interactions occur on average $\mathcal{O}(\geq 1)$, assumed to be sufficient to keep the DM in thermal equilibrium, while outside r_{m} DM can be well-approximated as collisionless.

Together, the density profile of the SIDM halo is described by the piecewise function,

$$\rho_{\text{dm}}(\mathbf{r}) = \begin{cases} \rho_{\text{Jeans}} & r < r_{\text{m}} \text{ (self-interacting)} \\ \rho_{\text{NFW}} & r > r_{\text{m}} \text{ (collisionless)}, \end{cases} \quad (7)$$

where ρ_{Jeans} comes from the solution to the isothermal Jeans equation (eq (3)) and ρ_{NFW} is the NFW profile defined in eq (1) used to model a CDM halo.

⁸For a velocity-dependent cross-section take the ensemble average, i.e. $\langle \sigma v_{\text{rel}} \rangle$, weighted by the DM velocity distribution function.

In order to match the inner SIDM profile to the outer CDM profile, the density and mass are required to match at r_m . In the spherically symmetric case we may impose $\rho_{\text{Jeans}}(r_m) = \rho_{\text{NFW}}(r_m)$ and $M_{\text{Jeans}}(r_m) = M_{\text{NFW}}(r_m)$ simultaneously.

Along with suitable boundary conditions, the isothermal profile for a SIDM halo can be solved and matched to the outer NFW profile. However, since the inner halo is affected by contributions to the total potential at all radii, changes in the outer halo can impact the inner isothermal solutions. Hence, we consider cases where the outer halo is itself nonspherical and cases where the baryonic component alters the halo via adiabatic contraction by modifying the NFW profile.

3.1 Adiabatic contraction

Adiabatic contraction (AC) refers to the process whereby DM contracts in response to a deepening of the gravitational potential well due to the cooling and condensation of baryons in the central regions of a DM halo. Hydrodynamical N-body simulations of DM with baryons show that AC is required to model the realistic interplay between DM and baryonic physics in DM halos [52].

In response to condensed baryonic matter, the DM halo increases in concentration, leading to higher densities in the core, and altering the enclosed mass profile with a more pronounced effect for smaller radii. This effect on the DM halo competes with those due to SIDM, which serves to flatten and lower central densities. Therefore, to obtain a comprehensive picture of the interplay between baryons and SIDM on halo structure it is important to include AC in our model for SIDM.

Effects due to AC are included in the Jeans model for a SIDM halo, eq. (7), by modifying the enclosed mass profile of the outer NFW halo according to the prescription laid out in Ref. [52]. By comparing the enclosed mass ratio of dark matter only (DMO) simulations and fully hydrodynamical simulations including baryons⁹ to the total mass ratio, Ref. [52] found a deterministic relationship, the fitting of which enables a prediction of enclosed mass as a function of radius for a contracted DM halo.

⁹For MW mass halo, i.e. $\sim 1 \times 10^{12} M_\odot$.

4 Jeans model beyond spherical symmetry

The one-dimensional Jeans model for a SIDM halo assuming spherical symmetry has been discussed extensively in the literature [18, 20, 21, 23, 47, 53, 54]. The following describes a novel procedure for solving the n-dimensional Jeans equation while relaxing the assumptions of spherical symmetry.

We begin by separating the total gravitational potential into its DM and baryonic components,

$$\nabla^2 \Phi_{\text{dm}} = 4\pi G \rho_{\text{dm}}, \quad \nabla^2 \Phi_b = 4\pi G \rho_b. \quad (8)$$

By the superposition principle both Φ_{dm} and Φ_b satisfy Poisson's equation. Redefining the Φ_{dm} and Φ_b to be the dimensionless variables,

$$\phi_{\text{dm}}(\mathbf{r}) = \frac{\Phi_{\text{dm}}(\mathbf{r}) - \Phi_{\text{dm}}(0)}{\sigma_0^2}, \quad \phi_b(\mathbf{r}) = \frac{\Phi_b(\mathbf{r}) - \Phi_b(0)}{\sigma_0^2}, \quad (9)$$

we can rewrite the formal solution to the Jeans equation, eq. (4), as,

$$\rho_{\text{dm}}(\mathbf{r}) = \rho_0 \exp\left(-\frac{\Phi(\mathbf{r}) - \Phi_0}{\sigma_0^2}\right) = \rho_0 e^{-\phi_{\text{dm}}(\mathbf{r}) - \phi_b(\mathbf{r})}, \quad (10)$$

where ρ_0 and Φ_0 are the DM density and total potential at the origin. Poisson's equations, eq. (5), can now be written,

$$\nabla^2 \phi_{\text{dm}}(\mathbf{r}) = \left(\frac{4\pi G \rho_0}{\sigma_0^2}\right) e^{-\phi_{\text{dm}}(\mathbf{r}) - \phi_b(\mathbf{r})}. \quad (11)$$

Solving eq. (11) for ϕ_{dm} enables a solution for the full DM density profile via eq. (10).

To accommodate nonsphericity we decompose ϕ_{dm} into a sum of spherical harmonics. In scenarios where the anisotropy in ϕ_{dm} is subdominant to the spherical symmetry, a relatively low-order truncation in harmonics is expected to yield a satisfactory approximation. Since ϕ_{dm} is a real function we expand in a series of real-valued spherical harmonics, i.e. *tesseral harmonics* $Z_{\ell m}$ (see Appendix D), yielding,

$$\phi_{\text{dm}}(r, \theta, \varphi) = \sum_{\ell, m} \phi_{\ell m}(r) Z_{\ell m}(\theta, \varphi). \quad (12)$$

Substituting eq. (12) into eq. (11) and using the orthogonality of the tesseral harmonics, we arrive at eq. (11) projected into harmonic modes ℓ and m ,

$$\frac{\partial^2 \phi_{\ell m}}{\partial r^2} + \frac{2}{r} \frac{\partial \phi_{\ell m}}{\partial r} - \frac{\ell(\ell+1)}{r^2} \phi_{\ell m} = \left(\frac{4\pi G \rho_0}{\sigma_0^2} \right) \int d\Omega Z_{\ell m}(\theta, \varphi) e^{-\phi_b(\mathbf{r})} e^{-\phi_{\text{dm}}(\mathbf{r})}. \quad (13)$$

However, since this equation is nonlinear in ϕ_{dm} we may not solve it via direct integration. Here we take recourse to the *relaxation method*, a numerical method for solving finite non-linear differential equations described generally in Appendix A.

To set up the problem to be solved we first define,

$$\mu_{\ell m}(r) = r^2 \frac{\partial \phi_{\ell m}}{\partial r} \quad (14)$$

to restate eq. (13) as the finite system of coupled first-order differential equations,

$$\frac{\partial \phi_{\ell m}}{\partial r} + \mathcal{G}_{\ell m}(\mathbf{y}, r, \mathbf{p}) = 0 \quad (15)$$

$$\frac{\partial \mu_{\ell m}}{\partial r} + \mathcal{F}_{\ell m}(\mathbf{y}, r, \mathbf{p}) = 0, \quad (16)$$

where $\mathcal{G}_{\ell m}$ and $\mathcal{F}_{\ell m}$ are functions defined on the input vectors,

$$\mathbf{y} = \begin{pmatrix} \phi_{00} \\ \phi_{10} \\ \vdots \\ \mu_{00} \\ \mu_{10} \\ \vdots \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} r_0^2 \\ \sigma_0^2 \end{pmatrix}, \quad (17)$$

where $r_0 = \sigma_0 / \sqrt{4\pi G \rho_0}$. Following eq. (13) the functions $\mathcal{G}_{\ell m}$ and $\mathcal{F}_{\ell m}$ are themselves defined,

$$\mathcal{F}_{\ell m}(\mathbf{y}, r, \mathbf{p}) = -\ell(\ell+1)\phi_{\ell m} - \frac{r^2}{r_0^2} \int d\Omega Z_{\ell m}(\theta, \varphi) e^{-\phi_b(\mathbf{r})} e^{-\phi_{\text{dm}}(\mathbf{r})} \quad (18)$$

$$\mathcal{G}_{\ell m}(\mathbf{y}, r, \mathbf{p}) = -\frac{\mu_{\ell m}}{r^2}. \quad (19)$$

Therefore, we now have $2n_{\ell m}$ functions, where $n_{\ell m}$ is the number of ℓ and m modes, and two parameters, r_0 and σ_0 , for which to solve. In total, we need $2n_{\ell m} + 2$ boundary conditions.

4.1 Boundary conditions

Since solving for the inner isothermal profile depends on the unknowns r_0 and σ_0 , we have freedom in defining boundary conditions. We may set r_0 and σ_0 in the inner profile and proceed to solve for the inner profile, fitting M200 and c (or ρ_s and r_s) at the boundary to define the outer NFW halo. This method is called *inside-out* matching and has the benefit of being simple to solve numerically but has been found to be biased toward selecting large cross-section solutions [20, 21]. However, we may also set the outer halo parameters M200 and c first and solve for r_0 and σ_0 iteratively given suitable boundary conditions, called *outside-in* matching. This method comes at a computational cost but has the advantage of potentially being free of the bias inherent in inside-out matching and allowing the freedom to alter the outer NFW profile to include effects on the enclosed mass profile due to adiabatic contraction.

Following the outside-in approach we fix the outer halo, ρ_{NFW} , and solve eqs. (15) and (16) as a boundary value problem. At the origin, $r = 0$, we impose that $\rho_{\text{dm}} = \rho_0$ and $M_{\text{dm}} = 0$, which in terms of ℓ and m modes is,

$$\phi_{00}(0) = 0, \quad r^2 \frac{\partial \phi_{00}}{\partial r} \Big|_{r=0} = 0. \quad (20)$$

We also require that the function $\phi_{\ell m}$ is well-behaved at the origin and infinity by imposing,

$$\phi_{\ell m}(0) = \phi_{\ell m}(\infty) = 0. \quad (21)$$

To generalize the boundary conditions at r_m mentioned in Sec. 3 for the spherically symmetric case we match rather the mean density and mass enclosed on a sphere of radius r_m .¹⁰ The condition on the mass enclosed remains the same as in the spherical case, i.e.,

$$M_{\text{Jeans}}(r_m) = M_{\text{NFW}}(r_m). \quad (22)$$

¹⁰Note that although the mass is continuous at r_m the density is not. Matching the isodensity contours at r_m leads to nonphysical boundary conditions, where the potential is divergent as $r \rightarrow \infty$ from additional homogeneous terms r^ℓ .

Noting that μ_{00} is related to the mass by,

$$\mu_{00} = \frac{\sqrt{4\pi}GM_{\text{dm}}(r)}{\sigma_0^2}, \quad (23)$$

the boundary condition on the enclosed mass at the origin and at $r = r_m$ can then be written,

$$\mu_{00}(0) = 0, \quad \mu_{00}(r_m) = \frac{\sqrt{4\pi}G}{\sigma_0^2}M_{\text{NFW}}(r_m). \quad (24)$$

The condition on the mean density now becomes,

$$\int \frac{d\Omega}{4\pi} \rho_{\text{Jeans}}(r_m, \Omega) = \int \frac{d\Omega}{4\pi} \rho_{\text{NFW}}(r_m, \Omega). \quad (25)$$

Considering the mean density on a sphere of radius r_m updates the rate equation so that it now reads,

$$\int \frac{d\Omega}{4\pi} \rho_{\text{dm}}(r_m, \Omega) \frac{\sigma}{m} v_{\text{rel}} t_{\text{age}} = 1. \quad (26)$$

And for the mean density at r_m , i.e. ρ_m ,

$$\int \frac{d\Omega}{4\pi} \rho_0 \exp\left(-\sum_{\ell m} \phi_{\ell m}(r_m) Z_{\ell m}(\Omega) - \phi_b(r_m, \Omega)\right) = \rho_m = \int \frac{d\Omega}{4\pi} \rho_{\text{NFW}}(r_m, \Omega). \quad (27)$$

To get the remaining boundary conditions at $r = r_m$ for non-zero ℓ and m modes we note that the solutions to Poisson's equation in terms of spherical harmonics can be written [51],

$$\phi_{\ell m}(r) = -\frac{4\pi G}{\sigma_0^2} \frac{1}{2\ell+1} \left(\frac{1}{r^{\ell+1}} \int_0^r dx x^{\ell+2} \rho_{\ell m}(x) + r^\ell \int_r^\infty dx x^{1-\ell} \rho_{\ell m}(x) \right) \quad (28)$$

$$\mu_{\ell m}(r) = \frac{4\pi G}{\sigma_0^2} \frac{1}{2\ell+1} \left(\frac{\ell+1}{r^\ell} \int_0^r dx x^{\ell+2} \rho_{\ell m}(x) - \ell r^{\ell+1} \int_r^\infty dx x^{1-\ell} \rho_{\ell m}(x) \right), \quad (29)$$

where $\rho_{\ell m}(r) = \int d\Omega Z_{\ell m}(\theta, \varphi) \rho_{\text{dm}}(\mathbf{r})$. Eqs. (28) and (29) are well-behaved at the origin and at infinity as required. To derive a boundary condition for $r = r_m$ we take a linear combination of eqs. (28) and (29),

$$\mu_{\ell m}(r_m) + (\ell+1)r_m \phi_{\ell m}(r_m) = -\frac{4\pi G}{\sigma_0^2} r_m^{\ell+1} J_{\ell m}, \quad (30)$$

where $J_{\ell m} = \int_{r_m}^\infty dx x^{1-\ell} \rho_{\ell m}(x)$, and is defined for $r > r_m$ and is therefore fixed by ρ_{NFW} . Different boundary conditions lead to solutions that vary up to homogeneous terms $Ar^\ell + B/r^{\ell+1}$, which do not respect the expected asymptotic behavior. For the $\ell = m = 0$ mode eqs. (28) and (29) provide boundary conditions that can be shown to be redundant with Poisson's equation.

Therefore, eqs. (21), (24), (27), and (30) provide the $2n_{\ell m} + 2$ boundary conditions required to fully specify the system of equations eqs. (15) and (16).

4.2 Relaxation method for the SIDM profile

Equipped with a system of differential equations and suitable boundary conditions we can proceed to solving the system using the relaxation method. Following Appendix A we specialize to the system defined by eqs. (15) and (16).

The first step is to discretize our solution space by defining a grid $r_i = r_1, \dots, r_N$ where $r_1 = 0$ and $r_N = r_m$. We then define our unknown functions on the grid in r_i , i.e.,

$$\phi_{\ell m}^i = \phi_{\ell m}(r_i), \quad \mu_{\ell m}^i = \mu_{\ell m}(r_i). \quad (31)$$

To solve, we begin by using the solutions to the spherical Jeans model without baryons with $\ell = 0$ (and $m = 0$ by default) as an initial guess (see Appendix B), i.e. for ϕ_{00}^i and μ_{00}^i , increasing the ℓ modes to $\ell_{max} > 0$. The relaxation algorithm follows the same procedure described in Appendix A for the system described by eqs. (15) and (16). We use the finite difference method to define a solution vector, $\mathbf{E}$ (see eq. (A.13)). The input vectors $\mathbf{y}$ and $\mathbf{p}$ defined by eq. (17) are the unknowns to be solved for. Since $\mathbf{E}$ is nonlinear we can not solve by matrix inversion, so we first linearize $\mathbf{E}$ and compute the shifts, $\Delta\mathbf{y}$ and $\Delta\mathbf{p}$, in the input parameters (eq.(A.4)). Constructing a linear and invertible matrix, $\mathbf{M}$, from $\mathbf{E}$ and associated boundary conditions we arrive at the matrix equation for the shifts,

$$\mathbf{M}\Delta\mathbf{Y} = -\mathbf{E}, \quad (32)$$

where $\Delta\mathbf{Y}$ is a vector composed of the shifts in the input parameters to be solved. Since linearizing is only approximate we iterate this process until the true solution is converged upon, i.e. $\mathbf{E} = 0$.

Definitions for the components of $\mathbf{M}$, $\mathbf{E}$, and constituent matrices are stated here for the case of the nonspherical Jeans model.

Recall that the $\mathbb{A}$ and $\mathbb{B}$ matrix blocks come from the derivatives of the functions $\mathcal{F}_{\ell m}$ and $\mathcal{G}_{\ell m}$

with respect to $\mathbf{y}$ and $\mathbf{p}$. For $\mathbb{A}$ we have,

$$\mathbb{A}_i = \frac{1}{2} (r_{i+1} - r_i) \begin{pmatrix} & & \frac{\partial \mathcal{G}_{00}}{\partial \mu_{00}} & 0 & 0 \\ & \mathbb{0}_{n_{\ell m} \times n_{\ell m}} & 0 & \frac{\partial \mathcal{G}_{10}}{\partial \mu_{10}} & 0 \\ & & 0 & 0 & \ddots \\ \hline \frac{\partial \mathcal{F}_{00}}{\partial \phi_{00}} & \frac{\partial \mathcal{F}_{00}}{\partial \phi_{10}} & \cdots & & \\ \frac{\partial \mathcal{F}_{10}}{\partial \phi_{00}} & \frac{\partial \mathcal{F}_{10}}{\partial \phi_{10}} & \cdots & & \\ \vdots & \vdots & \ddots & & \end{pmatrix} \begin{matrix} \\ \\ \\ \mathbb{0}_{n_{\ell m} \times n_{\ell m}} \\ \\ \end{matrix} \Big|_{\substack{\mathbf{y}=\bar{\mathbf{y}}_i \\ r=\bar{r}_i}} \quad (33)$$

where,

$$\frac{\partial \mathcal{F}_{\ell m}}{\partial \phi_{\ell' m'}} \Big|_{\substack{\mathbf{y}=\bar{\mathbf{y}}_i \\ r=\bar{r}_i}} = -\ell(\ell+1) \delta_{\ell \ell'} \delta_{m m'} + \frac{\bar{r}_i^2}{r_0^2} \int d\Omega Z_{\ell m}(\Omega) Z_{\ell' m'}(\Omega) e^{-\phi_b(\bar{r}_i, \Omega) - \bar{\phi}_{\text{dm}}^i(\Omega)} \quad (34)$$

$$\frac{\partial \mathcal{G}_{\ell m}}{\partial \mu_{\ell' m'}} \Big|_{\substack{\mathbf{y}=\bar{\mathbf{y}}_i \\ r=\bar{r}_i}} = -\frac{1}{\bar{r}_i^2} \delta_{\ell \ell'} \delta_{m m'}, \quad (35)$$

and the overbars denote the midpoint value evaluated on the grid, i.e.,

$$\bar{\phi}_{\ell m}^i = \frac{\phi_{\ell m}^{i+1} + \phi_{\ell m}^i}{2}, \quad \bar{\mu}_{\ell m}^i = \frac{\mu_{\ell m}^{i+1} + \mu_{\ell m}^i}{2}, \quad \bar{r}_i = \frac{r_{i+1} + r_i}{2}, \quad \text{and} \quad \bar{\phi}_{\text{dm}}^i(\Omega) = \sum_{\ell, m} Z_{\ell m}(\theta, \varphi) \bar{\phi}_{\ell m}^i. \quad (36)$$

For $\mathbb{B}$,

$$\mathbb{B}_i = (r_{i+1} - r_i) \begin{pmatrix} & \mathbb{0}_{n_{\ell m} \times 2} \\ \hline \frac{\partial \mathcal{F}_{00}}{\partial r_0^2} & \frac{\partial \mathcal{F}_{00}}{\partial \sigma_0^2} \\ \frac{\partial \mathcal{F}_{10}}{\partial r_0^2} & \frac{\partial \mathcal{F}_{10}}{\partial \sigma_0^2} \\ \vdots & \vdots \end{pmatrix} \Big|_{\substack{\mathbf{y}=\bar{\mathbf{y}}_i \\ r=\bar{r}_i}} \quad (37)$$

where,

$$\frac{\partial \mathcal{F}_{\ell m}}{\partial r_0^2} \Big|_{\substack{\mathbf{y}=\bar{\mathbf{y}}_i \\ r=\bar{r}_i}} = \frac{\bar{r}_i^2}{r_0^4} \int d\Omega Z_{\ell m}(\Omega) e^{-\bar{\phi}_b(\bar{r}_i, \Omega) - \bar{\phi}_{\text{dm}}^i(\Omega)} \quad (38)$$

$$\frac{\partial \mathcal{F}_{\ell m}}{\partial \sigma_0^2} \Big|_{\substack{\mathbf{y}=\bar{\mathbf{y}}_i \\ r=\bar{r}_i}} = -\frac{\bar{r}_i^2}{r_0^2 \sigma_0^2} \int d\Omega Z_{\ell m}(\Omega) \bar{\phi}_b^i e^{-\bar{\phi}_b(\bar{r}_i, \Omega) - \bar{\phi}_{\text{dm}}^i(\Omega)}. \quad (39)$$

The solution vector $\mathbf{E}$ then takes the form,

$$\mathbf{E}_i = \begin{pmatrix} \phi_{00}^{i+1} - \phi_{00}^i + (r_{i+1} - r_i)\mathcal{G}_{00}(\bar{\mathbf{y}}_i, \bar{r}_i, \mathbf{p}) \\ \phi_{10}^{i+1} - \phi_{10}^i + (r_{i+1} - r_i)\mathcal{G}_{10}(\bar{\mathbf{y}}_i, \bar{r}_i, \mathbf{p}) \\ \vdots \\ \hline \mu_{00}^{i+1} - \mu_{00}^i + (r_{i+1} - r_i)\mathcal{F}_{00}(\bar{\mathbf{y}}_i, \bar{r}_i, \mathbf{p}) \\ \mu_{10}^{i+1} - \mu_{10}^i + (r_{i+1} - r_i)\mathcal{F}_{10}(\bar{\mathbf{y}}_i, \bar{r}_i, \mathbf{p}) \\ \vdots \end{pmatrix}. \quad (40)$$

The boundary conditions compose the $\mathbf{H}(\mathbf{y}_1, \mathbf{y}_N, \mathbf{p})$ vector in analogy to eq. (A.8), yielding,

$$\mathbf{H}(\mathbf{y}_1, \mathbf{y}_N, \mathbf{p}) = \begin{pmatrix} \phi_{00}^1 \\ \phi_{10}^1 \\ \vdots \\ \hline \int \frac{d\Omega}{4\pi} \rho_{\text{dm}}^N(\Omega) - \rho_m \\ \hline \vdots \\ \mu_{\ell m}^N + r_m(\ell+1)\phi_{\ell m}^N + \frac{4\pi G}{\sigma_0^2} r_m^{\ell+1} J_{\ell m} \\ \vdots \\ \hline \mu_{00}^1 \\ \mu_{00}^N - \frac{\sqrt{4\pi G}}{\sigma_0^2} M_m \end{pmatrix}, \quad (41)$$

$\left. \begin{matrix} \phi_{00}^1 \\ \phi_{10}^1 \\ \vdots \end{matrix} \right\} n_{\ell m} \text{ rows}$
 $\left. \int \frac{d\Omega}{4\pi} \rho_{\text{dm}}^N(\Omega) - \rho_m \right\} 1 \text{ row}$
 $\left. \begin{matrix} \vdots \\ \mu_{\ell m}^N + r_m(\ell+1)\phi_{\ell m}^N + \frac{4\pi G}{\sigma_0^2} r_m^{\ell+1} J_{\ell m} \\ \vdots \end{matrix} \right\} \begin{matrix} (n_{\ell m} - 1) \text{ rows} \\ \ell > 0 \text{ only} \end{matrix}$
 $\left. \begin{matrix} \mu_{00}^1 \\ \mu_{00}^N - \frac{\sqrt{4\pi G}}{\sigma_0^2} M_m \end{matrix} \right\} 2 \text{ rows}$

which includes the $2n_{\ell m} + 2$ boundary conditions described in Sec. 4.1, and where,

$$\rho_{\text{dm}}^N(\Omega) = \rho_0 \exp \left(- \sum_{\ell m} \phi_{\ell m}^N Z_{\ell m}(\Omega) - \phi_b(r_N, \Omega) \right) \quad (42)$$

stands for the DM density at $r_N = r_m$. The $\mathbb{C}$ and $\mathbb{D}$ matrix blocks follow from eq. A.12 as the

derivatives of the $\mathbf{H}$ with respect to $\mathbf{y}$ and $\mathbf{p}$. For $\mathbb{C}$ we get,

$$\mathbb{C}_1 = \left(\begin{array}{c|ccc} \mathbb{1}_{n_{\ell m}} & \mathbb{O}_{n_{\ell m} \times n_{\ell m}} & & \\ \hline \mathbb{O}_{n_{\ell m} \times n_{\ell m}} & \mathbb{O}_{n_{\ell m} \times n_{\ell m}} & & \\ \hline \mathbb{O}_{2 \times n_{\ell m}} & 1 & 0 & \dots & 0 \\ & 0 & 0 & \dots & 0 \end{array} \right), \quad \mathbb{C}_N = \left(\begin{array}{c|ccc} \mathbb{O}_{n_{\ell m} \times n_{\ell m}} & \mathbb{O}_{n_{\ell m} \times n_{\ell m}} & & \\ \hline \mathbb{L}_{n_{\ell m} \times n_{\ell m}} & \mathbb{K}_{n_{\ell m} \times n_{\ell m}} & & \\ \hline \mathbb{O}_{2 \times n_{\ell m}} & 0 & \dots & 0 & 0 \\ & 1 & 0 & \dots & 0 \end{array} \right), \quad (43)$$

where the sub-blocks $\mathbb{L}$ and $\mathbb{K}$ are,

$$\mathbb{L}_{n_{\ell m} \times n_{\ell m}} = \left(\begin{array}{c|ccc} -\frac{\rho_{00}^N}{4\pi} & \dots & -\frac{\rho_{\ell m}^N}{4\pi} & \dots \\ \hline 0 & & & \\ \vdots & & & \\ 0 & & & \end{array} \right), \quad \mathbb{K}_{n_{\ell m} \times n_{\ell m}} = \left(\begin{array}{c|ccc} 0 & 0 & \dots & 0 \\ \hline 0 & & & \\ \vdots & & & \\ 0 & & & \end{array} \right), \quad (44)$$

$\underbrace{r_m \text{diag}(\ell+1)}_{\ell > 0 \text{ only}}$

with $\rho_{\ell m}^N = \int d\Omega Z_{\ell m}(\Omega) \rho_{\text{dm}}^N(\Omega)$. The $\mathbb{D}$ block is,

$$\mathbb{D} = \left(\begin{array}{c|ccc} \mathbb{O}_{n_{\ell m} \times 2} & & & \\ \hline -\int \frac{d\Omega}{4\pi} \rho_{\text{dm}}^N(\Omega)/r_0^2 & \int \frac{d\Omega}{4\pi} \rho_{\text{dm}}^N(\Omega) \frac{1+\phi_b(r_N, \Omega)}{\sigma_0^2} & & \\ \hline 0 & \vdots & & \\ \vdots & -\frac{4\pi G}{\sigma_0^4} r_m^{\ell+1} J_{\ell m} \quad (\ell > 0 \text{ only}) & & \\ 0 & \vdots & & \\ \hline 0 & 0 & & \\ 0 & \frac{\sqrt{4\pi G}}{\sigma_0^4} M_{\text{NFW}}(r_m) & & \end{array} \right). \quad (45)$$

Plugging the definitions for the above matrix blocks into the definition for the matrix, $\mathbf{M}$ (analogous to eq. (A.10)), and $\mathbf{E}$ (analogous to eq. (A.13)), enables the solution of the matrix equation

(eq. (A.14)) for the shifts. Iterating this process until a solution is converged upon gives the profile for the SIDM halo.

5 Nonspherical SIDM halo from the Jeans model

In this section, the SIDM halo for the Jeans model described in Sec. 3 and arrived at via the relaxation algorithm outlined in Sec. 4 will be presented. First, the input parameters for the Jeans algorithm will be presented along with special cases thereof. Second, input variables for a MW mass halo will be entered into the Jeans model for various conditions to illustrate the behavior of the relaxation algorithm and to compare the results with SIDM expectations. Third, the nonspherical Jeans model will be compared with simulations from the EAGLE-50 SIDM1b and CDMb suite. The nonspherical Jeans model will be validated by inferring the correct input cross-section as well as fitting to density and shape data from the simulations. Lastly, a few remarks will be made about the future use cases for the nonspherical Jeans model.

5.1 Jeans model parameters

Tab. 1 summarizes the input variables for the Jeans model relaxation algorithm. r_m is the boundary matching radius for the inner and outer halos defined by eq. (6), M_{200} and c specify the NFW profile, AC modifies the enclosed mass of the NFW profile to account for adiabatic contraction, and q determines the nonsphericity of the outer halo. Φ_b is the potential due to the baryon distribution. In lieu of the baryon potential, M_b , the baryon mass profile, may also be inputted. Finally, L_{list} and M_{list} specify the list of angular modes up to which we truncate in the harmonic expansion. As of writing the code only considers axisymmetric models, and thus M_{list} is set to 0 by default.

If $r_m = 0$ the Jeans model returns a full NFW (CDM) profile. $r_m = 0$ and $\Phi_b = 0$ will return a CDM only profile. A non-zero r_m and $\Phi_b = 0$ returns an SIDM-only profile while a non-zero r_m and Φ_b return an SIDM profile with baryons. If $L_{\text{list}} = 0$ only the spherically symmetric term in the spherical harmonic expansion is included. If no L_{list} is specified the relaxation algorithm

Variable	Description
r_m	Matching radius
M_{200}	Virial halo mass
c	Concentration parameter
AC	Adiabatic contraction
q	Outer halo shape
Φ_b	Baryon potential
M_b	Baryon mass profile
L_{list}	Truncated ℓ harmonics
M_{list}	Truncated m harmonics

Table 1: Jeans model relaxation algorithm code input variables and description.

will attempt to add modes until relative convergence is achieved. Moreover, if $L_{\text{list}} = 0$ and Φ_b is spherically averaged the relaxation algorithm simplifies to the one-dimensional spherically averaged Jeans model.

5.2 MW-mass halo example

To illustrate the Jeans model we take the example case of a MW-mass halo, i.e. $M_{200} = 1 \times 10^{12} M_\odot$. M_{200} sets the concentration parameter via the mass-concentration relation [49] such that $c \approx 8$ for a MW mass halo.¹¹ The MW-mass range is chosen to correspond with the simulated halo mass ranges presented in Sec. 5.3. The MW-mass range is also unique in that spherically averaged density profiles for CDM and SIDM simulations are not easily distinguishable, and due to the stellar mass fraction (M_*/M_{200}) peaking in this range [55–57], the effects of baryons are expected to be prominent, providing an advantageous scale for analysis of halo shapes and SIDM with baryons.

For the baryon potential, we choose the Miyamoto and Nagai (MN) disk which represents the

¹¹Here c is fixed by the mass-concentration relation for illustrative purposes but in principle may vary freely, so long as the outer halo has an appropriate matching condition.

potential for an axisymmetric disk. The potential for the MN disk is given by,

$$\Phi_{\text{MN}}(R, z) = \frac{-GM_{\text{d}}}{\sqrt{R^2 + (a + \sqrt{z^2 + b^2})^2}}, \quad (46)$$

where (R, z) are the radial and axial cylindrical coordinates, M_{d} the disk mass, and a and b are the scale lengths in R and z respectively. For $a = b$ the disk is spherical, for $a > b$ oblate, and $a < b$ prolate. In the following example we take $a = 3$ and $b = 1$, signifying a oblate disk flattened in the vertical direction and elongated in the radial direction, in analogy with the potential for a spiral galaxy, and we set the stellar mass to $M_{\text{d}} = 6 \times 10^{10} M_{\odot}$, in analogy with the MW stellar mass, M_{*} , estimates [58].

Fig. 2 shows the spherically averaged density profiles for the Jeans model for five representative cases. The first case is the CDM (pure NFW) profile (black solid curve) for the example MW-mass halo in the absence of an MN disk. The second is a CDM profile with the addition of an MN disk (with $a = 3$ and $b = 1$) and AC.¹² The third profile is a SIDM-only profile (blue solid curve). The final two profiles are SIDM with an MN disk, with AC (orange dashed curve) and without AC (orange solid curve). All SIDM profiles are generated with an input r_{m} of 20 kpc.¹³ The nonspherical Jeans model allows the outer halo to be nonspherical, however, the spherically averaged density profiles only vary at the 10% level for extreme values of q , hence, for illustration we choose the outer halo to be spherical, i.e. $q = 1$.

The spherically averaged density profiles for SIDM halos in comparison with the CDM halos are consistent with the expectation that self-interactions bring about cored profiles in comparison to their CDM counterparts. The SIDM-only profile (blue solid curve) shows a core with a logarithmic slope $\alpha \sim 0$ out to nearly 10 kpc, compared to the CDM-only profile (black solid curve) which has a slope $\alpha \sim -1$ down to sub kpc. Moreover, the addition of the MN disk can be seen to increase the density of the core by nearly an order of magnitude for the SIDM profile (orange solid). While

¹²Note that CDM with the MN disk and no AC produces a curve exactly matching the CDM only curve and is therefore not shown.

¹³Note that σ/m varies due to the dependence of eq. (26) on v_{rel} , which is proportional to σ_0 —a free parameter being solved for by the relaxation algorithm

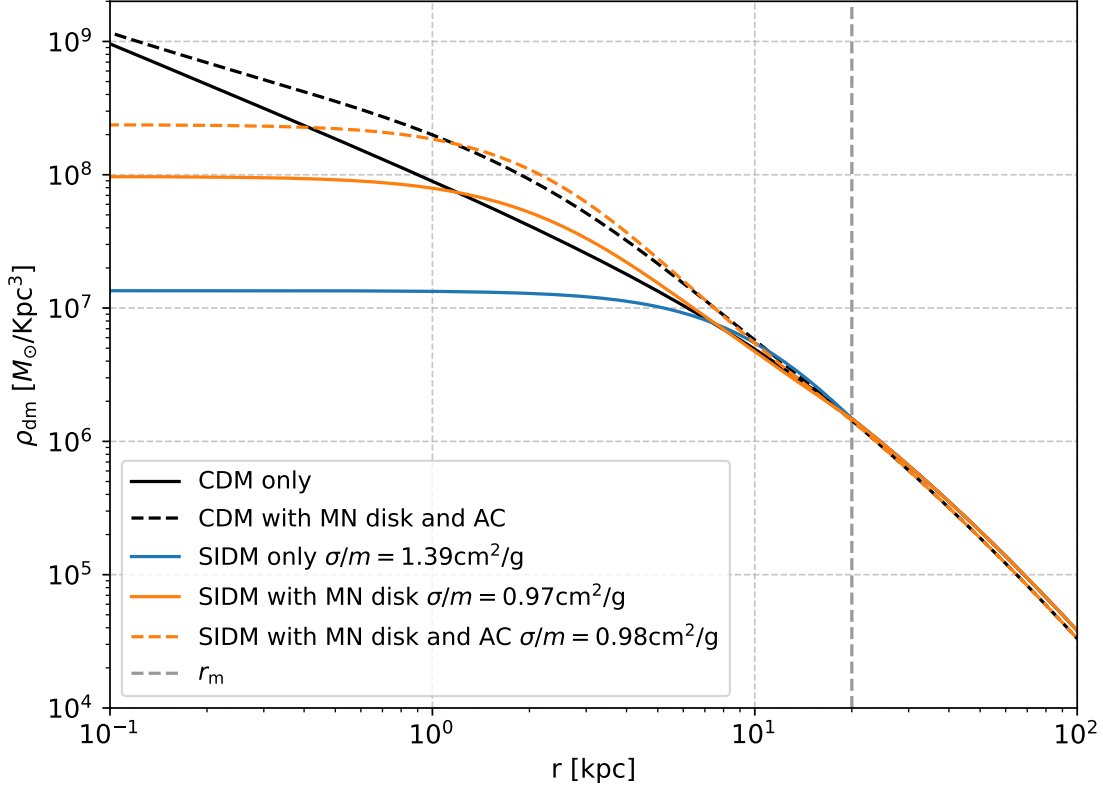


Figure 2: spherically averaged DM density profiles from the nonspherical Jeans model for MW-mass DM halos with $M_{200} = 1 \times 10^{12} M_{\odot}$ with and without an MN disk, where the disk potential is described by a stellar mass, $M_d = 6 \times 10^{10} M_{\odot}$, and shape parameters, $a = 3$, and $b = 1$. The profiles shown are: CDM (black solid curve), a pure NFW profile; CDM with an MN disk and AC (black dashed curve); SIDM only (blue solid curve); SIDM with MN disk (orange solid curve); and SIDM with an MN disk and AC (orange dashed curve). All profiles have a spherical outer halo, i.e. $q = 1$, and all SIDM profiles are generated with $r_m = 20\text{ kpc}$. Note that $r_m = 20\text{ kpc}$ is plotted as a vertical dashed line and σ/m computed from the final solution for ρ_{dm} via eq. (26) is added to the legend for SIDM halos.

the addition of AC can be seen to increase the core density for the SIDM profile still more (orange dashed). For the CDM halo AC has the effect of concentrating mass toward the core, $\mathcal{O}(1\text{ kpc})$,

while forming a similar cusp to CDM in the inner region. For SIDM AC can also be seen to reduce the core radius when compared to the SIDM-only case. Both the SIDM profiles with MN disks (orange curves) have significantly smaller core radii than the SIDM-only profile (blue)—a result that is consonant with the fact that SIDM density profiles respond sensitively to the stellar potential, especially in galaxies with baryon-dominated inner regions [18].

Since the relaxation algorithm for the nonspherical Jeans model solves for the DM density in two dimensions, we also have access to halo shape data. In the literature on halo shapes [59], the quoted value of interest is the axis ratio of the semi-minor to semi-major axes, c/a . Given the DM density from the Jeans model as a function of radius and angle, $\rho_{\text{dm}}(r, \theta)$, we define thin spheroidal shells at effective radii, for which we compute the moment-of-inertia tensor taking the ratio of the diagonal elements to get c/a as a function of radius, which hereafter is referred to as $q(r)$. The details for calculating $q(r)$ from the Jeans model DM density are found in Appendix C.

Fig. 3 displays the halo shapes, $q(r)$, for the Jeans modeled SIDM halos for spherical (left panel) and nonspherical (right panel) MW-mass halos with the oblate MN disk defined above. The profiles plotted are: SIDM only (blue solid line), SIDM with baryons (orange solid curve), and SIDM with baryons and AC (orange dashed curve). All halos are generated with $r_{\text{m}} = 20 \text{ kpc}$, shown as the vertical dashed grey line. In the spherical case when $r > r_{\text{m}}$ $q(r)$ is trivially 1 due to the matching condition that the outer halo be spherical, i.e. $q = 1$. The right panel shows a slightly nonspherical outer halo, i.e. $q = 0.88$, to which $q(r)$ converges when $r > r_{\text{m}}$.

Fig. 3 illustrates that the shape of the isothermal profile inside of r_{m} can deviate significantly from spherical symmetry in the presence of a baryon-dominated core, with both the SIDM with baryons and SIDM with baryons and AC starting as low as $q = 0.6$ for $\mathcal{O}(\text{kpc})$ radii. However, as the density of the baryons drops off the effect of SIDM is to drive $q(r)$ towards spherical symmetry. This can also be seen by the addition of AC to SIDM, which systematically increases $q(r)$ toward spherical symmetry at radii where the density of SIDM with baryons and AC is higher than SIDM with baryons alone (see Fig. 2). Since the shape of the SIDM halo depends on the baryon fraction, a higher SIDM density implies a lower relative fraction of baryons and hence will drive $q(r)$ to unity.

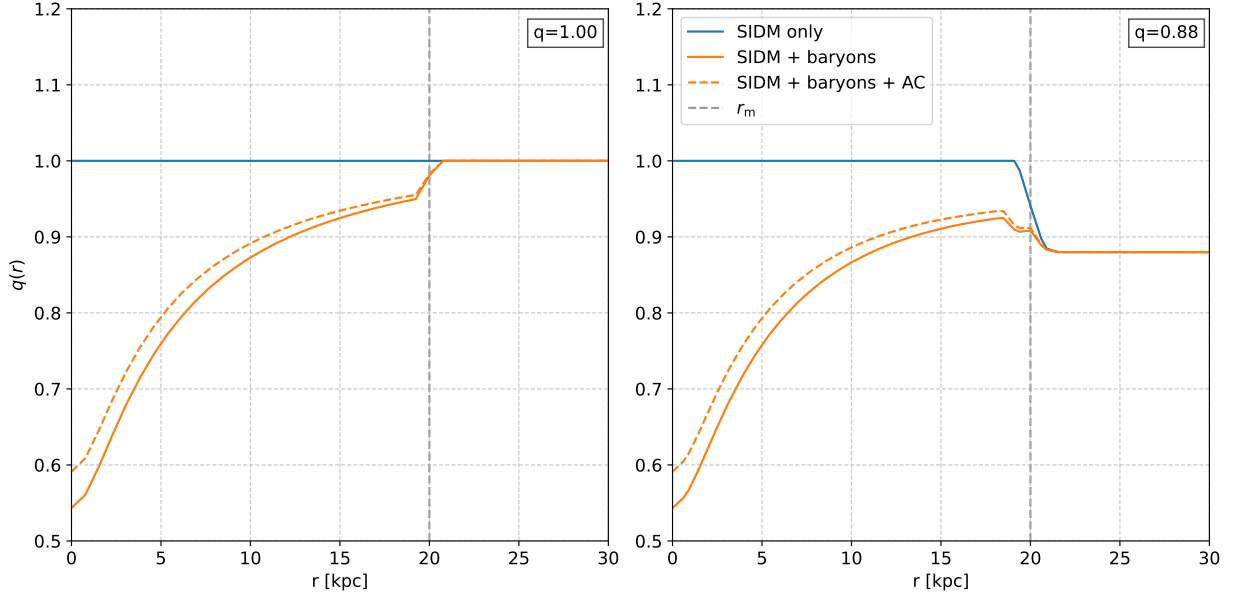


Figure 3: Halo shape $q(r)$ for SIDM profiles generated for a MW-mass halo with an axisymmetric baryon potential described by an oblate MN-disk with $M_{\text{disk}} = 6 \times 10^{10} M_{\odot}$ and scale lengths $a = 3$ and $b = 1$. The outer halo is spherical (left panel) with $q = 1$, or slightly oblate (right panel) with $q = 0.88$. Plotted are the DM halo shapes for SIDM only (blue solid line), SIDM with baryons (orange solid curve), and SIDM with baryons and AC (orange dashed curve). All profiles are generated with $r_m = 20$ kpc, shown as the vertical dashed grey line.

In contrast to CDM, which is not as responsive to the distribution of baryons, SIDM shapes can deviate from expected spherical symmetry in response to the particular distribution of baryons. Hence, halo shapes can be used as a key discriminant between CDM and SIDM cosmologies in astrophysical observations, or conversely, as a key constraint in the SIDM cross-section space. Thus, the Jeans model can be a cost-effective way to model such halo shape variations and determine the consistency and robustness of halo shape-derived constraints.

This section illustrated that the nonspherical Jeans model produces reasonable results based on prior expectations for SIDM morphology. To further establish the results of the nonspherical Jeans model, the next section validates results by comparison with EAGLE-50 SIDM simulation data.

5.3 Comparing SIDM halos from the Jeans model to simulations

In this section, the nonspherical Jeans model is confronted with SIDM simulations. Using baryon distributions directly from SIDM simulations as an input, we evaluate whether the nonspherical Jeans model can model well the one-dimensional and two-dimensional DM density distributions, the halo shapes, and correctly infer the input cross-sections across intermediate-mass galaxies from the EAGLE-50 simulation suite.

Simulations

We consider the high-resolution cosmological simulations from the EAGLE-50 project for two DM models: SIDM with $\sigma/m = 1 \text{ cm}^2/\text{g}$ and baryons (SIDM1b) and CDM with $\sigma/m = 0 \text{ cm}^2/\text{g}$ [21] and baryons (CDMb). These simulations are derived from the SIDM versions of the EAGLE simulations [57, 60], simulated with higher resolution and a box volume with side length 50 Mpc. The simulations include DM and baryons with particle masses of $9.7 \times 10^6 M_\odot$ and $1.8 \times 10^6 M_\odot$ respectively. The simulations use the EAGLE galaxy formation model including subgrid baryonic physics such as gas cooling, stellar feedback, black hole mergers, AGN feedback, etc. Self-interactions are modeled by hard-sphere scattering with a differential cross-section that is velocity-independent and isotropic in scattering angle. Scattering probabilities are described by a Monte Carlo algorithm given in Ref. [61], with its implementation in the EAGLE code explicated in Ref. [62].

We consider 250 of the most massive halos spanning halo viral masses $M_{200} = [3.53 \times 10^{11} - 4.43 \times 10^{13}] M_\odot$, in the intermediate mass range including MW-mass (spiral galaxy) halos and massive elliptical galaxy halos. Since the nonspherical Jeans model is implemented in two dimensions assuming azimuthal symmetry, and the simulations are in three dimensions, to enable comparison the simulations are first azimuthally averaged. We do so by defining the most gravitationally bound particle as the center of the halo and then defining the z-axis in the direction of the angular momentum of the stellar distribution, about which we azimuthally average. For each population—DM, star, and gas—we define the two-dimensional density map on a grid in the cylindrical coordinates,

i.e. $\rho(R, z)$, for the regions $0 \leq R \leq 100 \text{ kpc}$, $-100 \leq z \leq 100 \text{ kpc}$. To ensure the correct density for each pixel in the (R, z) grid we sum the mass of the particles in a pixel and divide by the 3D volume of a pixel which is $\approx 2\pi R \Delta R \Delta z$ where $\Delta R = \Delta z = 1 \text{ kpc}$.

Spherical and nonspherical Jeans models compared to simulations

It has been shown that the one-dimensional Jeans model does a surprisingly good job at modeling spherically averaged SIDM density profiles from the EAGLE-50 simulation suite [21]. We now turn to the question of whether the nonspherical Jeans model in two dimensions is equally as successful in modeling the two-dimensional azimuthally averaged density profiles from SIDM simulation. In order to characterize a goodness-of-fit for the Jeans model comparisons to simulations, we define a log-likelihood (up to a normalization constant), $\ln \mathcal{L} \propto -1/2\chi^2$.¹⁴ We define two χ^2 , one for comparing the one-dimensional spherically averaged density profiles, $\rho(r)$, and one for comparing the two-dimensional azimuthally averaged density maps, $\rho(R, z)$. For the one-dimensional model, we have the log-likelihood,

$$\ln \mathcal{L}_{\rho(r)} \propto -\frac{1}{2}\chi_{\rho(r)}^2 = -\frac{1}{2} \sum_{r=3 \text{ kpc}}^{R200} \frac{(\rho_{Jeans}(r) - \rho_{Sim}(r))^2}{\delta\rho(r)^2}, \quad (47)$$

where we take the difference between the model and simulation density between the radii $3 \text{ kpc} \leq r \leq R200$ where $R200$ is the virial radius of the halo. We assume a fractional error of 10% the density, such that $\delta\rho(r) = 0.1 \times \rho(r)$. For the two-dimensional model, we define the weighted log-likelihood,

$$\ln \mathcal{L}_{\rho(R,z)} \propto -\frac{1}{2}\chi_{\rho(R,z)}^2 = -\frac{1}{2} \sum_{R,z}^{R_{max}, z_{max}} \frac{(M_{Jeans}(R, z) - M_{Sim}(R, z))^2}{\delta M(R, z)^2} w(r), \quad (48)$$

where we take the difference between the mass per pixel in (R, z) over the range of R and z defined for the simulation density maps above. We assume a fractional error on the mass of 10% such

¹⁴Since the Jeans model is necessarily simplistic—assuming an abrupt transition from self-interacting to collisionless and neglecting substructure and assembly histories—the goodness-of-fit metrics outlined here represent systematic differences in the appearance of the distributions, and not random error or particle noise in the simulations.

that $\delta M(R, z) = 0.1 \times M(R, z)$. $w(r)$ is a weighting factor defined as $w(r) = r_0/r$, where r is Euclidean distance to a point (R, z) on the density map, and $r_0 = 10$ kpc such that the (R, z) pixels interior to r_0 are given a weight greater than one and the pixels outside of r_0 are given a weight that is proportional to r^{-1} . This weighting prescription is used to emphasize the performance of the Jeans model in the core where the self-interactions are modeled by the isothermal profile and where the baryonic mass profile dominates the gravitational potential. The choice of fitting range and fractional errors are somewhat arbitrary but sufficient to test the Jeans model's robustness in fitting simulation densities and inferring the input cross-sections.¹⁵

To verify the Jeans model we run a spherical and nonspherical scan¹⁶ of input parameters (r_m, c, q) ¹⁷ for the first 100¹⁸ of the 250 galaxies on an equal log-spaced grid within the ranges: $r_m = 1 - 600$ kpc, $c = 2 - 16$, and $q = 0.5 - 1.8$. The baryon distribution and $M200$ are fixed by the simulations.¹⁹ We then evaluate our log-likelihoods (eqs. (47) and (48)) on the resulting grid of Jean's model profiles. Fig. 4 shows the histogram for the inferred cross-sections from the Jeans model, chosen from the profiles that minimize the one-dimensional log-likelihood (orange) for the spherical Jeans model and the two-dimensional log-likelihood (blue) for the nonspherical Jeans model. The same-color dashed vertical lines show the 50th percentile (median) and 16th and 84th percentiles (representing one standard deviation from the mean) for both distributions.

For the 100 galaxy parameter scan the median inferred cross-sections for the spherical and nonspherical Jeans model are $\sigma/m = 0.81^{+1.30}_{-0.50} \text{ cm}^2/\text{g}$ and $\sigma/m = 0.49^{+1.41}_{-0.37} \text{ cm}^2/\text{g}$, respectively. Both the spherical and nonspherical Jeans models accurately infer the input cross-section for the simulations to within an order of magnitude.

To illustrate further the effectiveness of the Jeans model we focus on two galaxies from the simulations in particular: galaxy ID 1 (G1) and galaxy ID 99 (G99). The virial mass and radius for G1 and G99 for the SIDM1b and CDMb models are summarized in Tab. 2.

¹⁵Alternate definitions for the log-likelihoods were tested with similar results being observed between them.

¹⁶Note that adiabatic contraction is included in all following Jeans models.

¹⁷For the spherical scan $q = 1$ for the outer halo.

¹⁸The halo masses range from $M200 = [8.50 \times 10^{11} - 4.43 \times 10^{13}] M_\odot$.

¹⁹For the spherical Jeans model the input baryon potential is first spherically averaged.

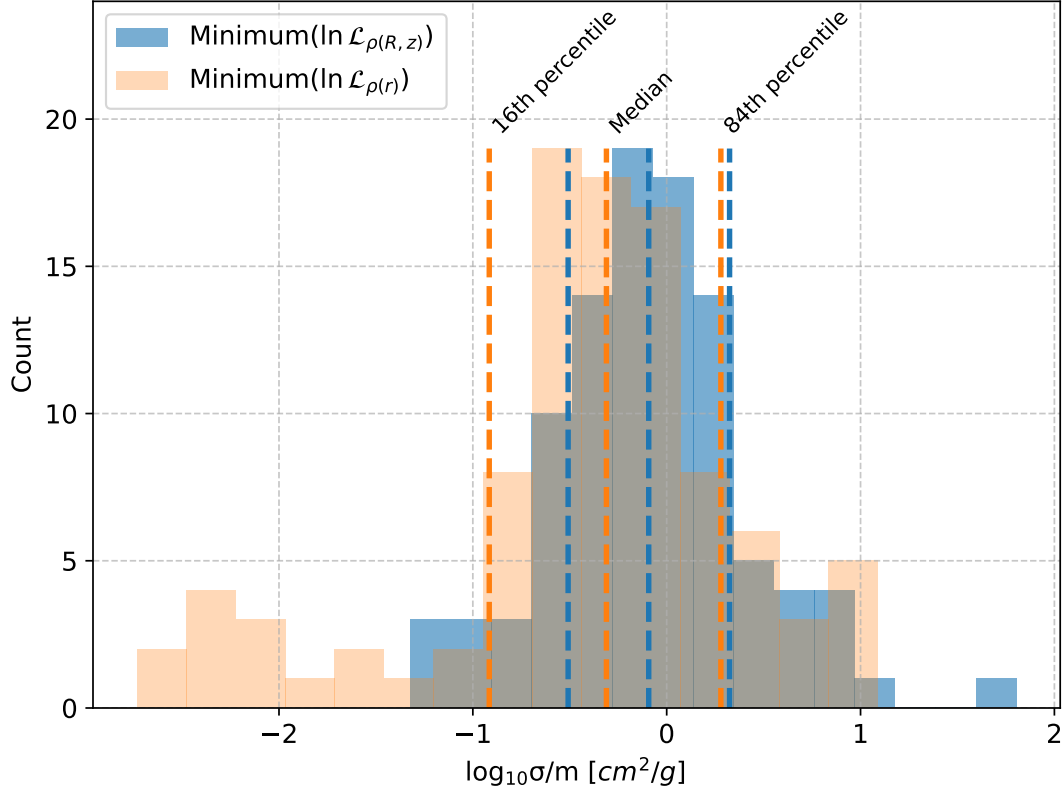


Figure 4: Histogram for the cross-section ($\log_{10}\sigma/m$) inferred from the minimum log-likelihood evaluated on a grid of Jeans models for 100 simulated SIDM1b galaxies. The nonspherical model (blue-shaded bands) is evaluated using the two-dimensional log-likelihood, eq. (48), and the spherical model (orange shaded bands) is evaluated using the one-dimensional log-likelihood, eq. (47). The vertical dashed lines correspond to the distribution of the same color and show the median and 16th and 84th percentiles for the distributions of inferences on the cross-section.

We choose a single profile from the parameter grid in (r_m, c, q) , corresponding to the choice of parameters whose Jeans model minimizes the log-likelihood function. The resulting Jeans models from this procedure are called the *best-fit* profiles. The parameters for the best-fit Jeans models are summarized in Tab. 3. Note that M_{200} is the same for spherical and nonspherical Jeans models since we are fitting to the same SIDM1b simulation. Note that $q = 1.0$ for the spherical model by

Galaxy ID	Model	$M_{200} [M_{\odot}]$	$R_{200} [\text{kpc}]$
G1	CDMb	1.76×10^{13}	548
G1	SIDM1b	1.78×10^{13}	550
G99	CDMb	1.93×10^{12}	262
G99	SIDM1b	1.88×10^{12}	260

Table 2: Description of example simulated SIDM1b and CDMb galaxies G1 and G99.

Galaxy ID	Jeans Model	$\sigma/m [\text{cm}^2/\text{g}]$	$r_m [\text{kpc}]$	$M_{200} [M_{\odot}]$	c	q
G1	spherical	0.99	56.8	1.78×10^{13}	4.26	1.0
G1	nonspherical	1.0	56.8	1.78×10^{13}	4.26	0.89
G99	spherical	0.01	1.0	1.88×10^{12}	7.51	1.0
G99	nonspherical	0.81	20.7	1.88×10^{12}	7.51	0.63

Table 3: Summary of the best-fit parameters for the spherical and nonspherical Jeans models for G1 and G99.

construction.

In Fig. 5 we plot the spherically averaged density profiles from the SIDM1b (black points) and CDMb (red points) simulation data and the spherical (green dashed curve) and nonspherical (green solid curve) best-fit Jeans models for the SIDM1b versions G1 and G99.

Fig. 5 shows that for G1 (left panel) the spherical and nonspherical best-fit Jeans models do a good job at matching the spherically averaged density distributions, only deviating slightly in the core. For G99 (right panel) the spherical and nonspherical best-fit Jeans models fit well to the data but deviate substantially in the core. This is owing to the fact that for the spherical Jeans model (orange dashed curve) the fit is equivalent to a sum of least squares with the data (black squares) and therefore fits well at all radii, while the nonspherical Jeans model (orange solid curve) is fit to a two-dimensional density map, and need not correspond exactly to the features of the

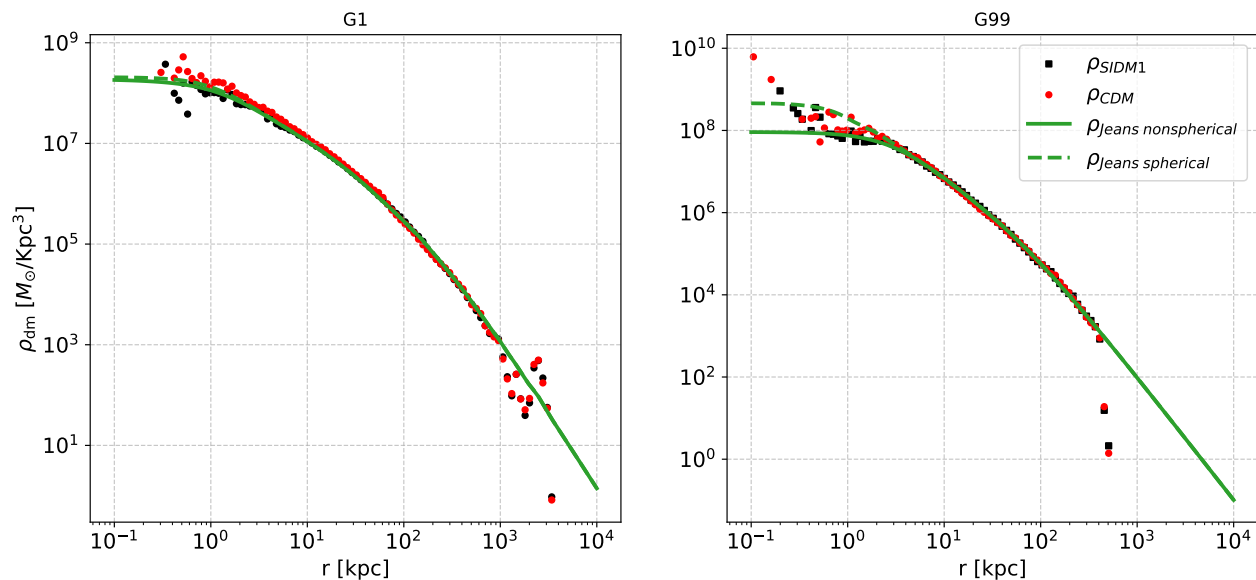


Figure 5: spherically averaged density profiles for G1 (left panel) and G99 (right panel) for the SIDM1b and CDMb simulations (black and red points) and the best-fit spherical and nonspherical Jeans models (green dashed and solid curves).

spherically averaged density data. Moreover, note that for the spherical fit to G99 $r_m = 1$ kpc (see Tab. 3), which is an edge case for the parameter grid in r_m , indicating that the model did not find a sufficient fit for the inner SIDM halo within the available parameter space. The nonspherical best-fit parameter for q is equal to 0.63 for G99, indicating that the DM density is asymmetric out to r_m . If the DM density interior to r_m is highly asymmetric, the spherical model is bound to be a poor fit no matter how r_m or c are varied.²⁰ In cases such as this one, the nonspherical Jeans model may provide a better model than the spherical Jeans model.

Moreover, we also note that in the case of G1 and G99 the SIDM1b and CDMb data do not deviate appreciably in the core,²¹ indicating that the spherically averaged density profile may be insufficient to differentiate between SIDM and CDM cosmologies. To discriminate between SIDM and CDM in intermediate mass ranges we take recourse to the two-dimensional data for G1 and

²⁰Recall the baryon distribution is spherically averaged for the spherical model, and hence, if the baryons induce asymmetry in the DM the spherical model would be insensitive to this feature.

²¹This is true in general for the SIDM1b and CDMb galaxies in the EAGLE-50 simulation suite for galaxies less than $M_{200} = 1 \times 10^{13} M_\odot$, further corroborated in Ref. [63] for intermediate mass SIDM halos.

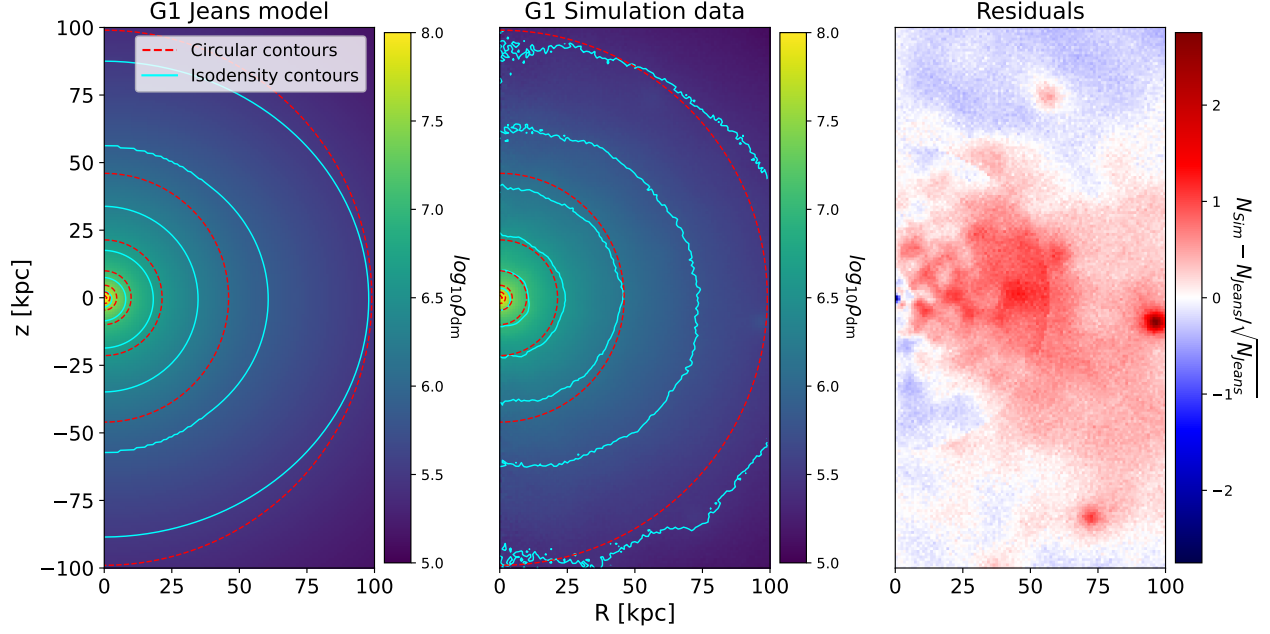


Figure 6: Two-dimensional density maps for G1 from the Jeans model (left panel), the simulation data (middle panel), and the residuals. The cyan dashed contours are the isodensity contours for the DM density equally spaced on a log grid. The red dashed contours signify the circular contours equally log-spaced for reference with the isodensity contours.

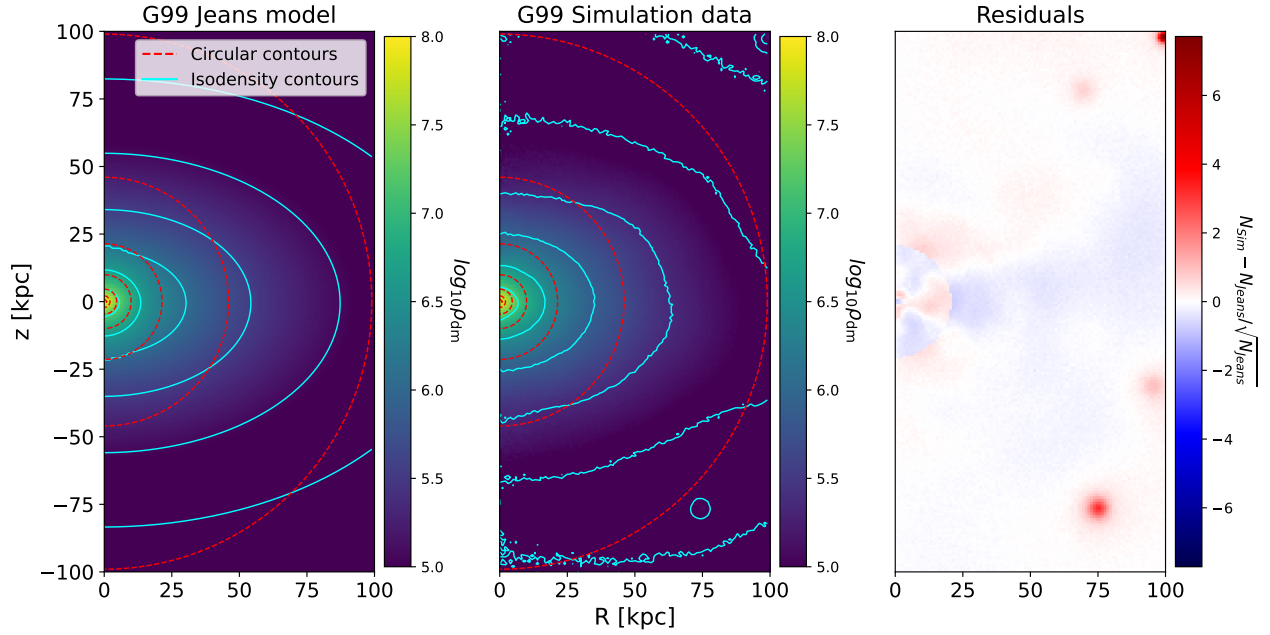


Figure 7: Same as Fig. 6 but for G99.

G99 and the nonspherical Jeans models for them.

Figs. 6 and 7 show the azimuthally averaged simulation data for galaxy G1 and G99 (middle panels), along with the best-fit nonspherical Jeans model (left panels) and a residual showing the relative difference per pixel (right panel) between simulation and model. In both the Jeans model and simulation panels two contours are drawn on top of $\rho_{\text{dm}}(R, z)$: isodensity contours spaced equally in the $\log_{10}\rho_{\text{dm}}$ space (cyan contours), and circular contours spaced equally in the $\log_{10}r$ space for comparison to the isodensity contours.

For G1, Fig. 6, the isodensity contours for the best-fit nonspherical Jeans model and the simulation are moderately spherical across all radii, which accounts for the similarity in the spherical and nonspherical Jeans model density profiles in Fig. 5. However the residual plot²² shows large over- and under-densities. This could be due to perturbations and substructures born of mergers and a complex assembly history for which the Jeans model does not account. Notwithstanding the two-dimensional fit, in the case where the two-dimensional density is approximately spherical, the nonspherical model performs equally well at matching the one-dimensional density profile as seen in Fig. 5.

For G99, Fig. 7, the isodensity contours are elongated in the R directions, signifying an oblate halo shape. The best-fit nonspherical Jeans model does an excellent job capturing the shape of the isodensity contours; and a good job fitting the density map, as seen by the small deviations in the residual plot. Recall that in Fig. 5 the spherical and nonspherical Jeans models deviated substantially in the core. Here, however, it is clear that the nonspherical Jeans model is doing a better job at capturing the features of the simulated density, including in the isothermal region where $r < r_{\text{m}} = 20.7 \text{ kpc}$.

To see more clearly the ability of the Jeans model to capture the asymmetric shape of the DM density in the simulations, we further plot the isodensity contours in Fig. 8 for G1 (left panel) and G99 (right panel) for the SIDM1b (black dashed contours) and CDMb (red dashed contours)

²²Note that the discontinuity visible in the residual plot is a result of the discontinuous boundary condition for ρ_{dm} at r_{dm} where we match the mean density at r_{m} and not $\rho_{\text{dm}}(R, z)$.

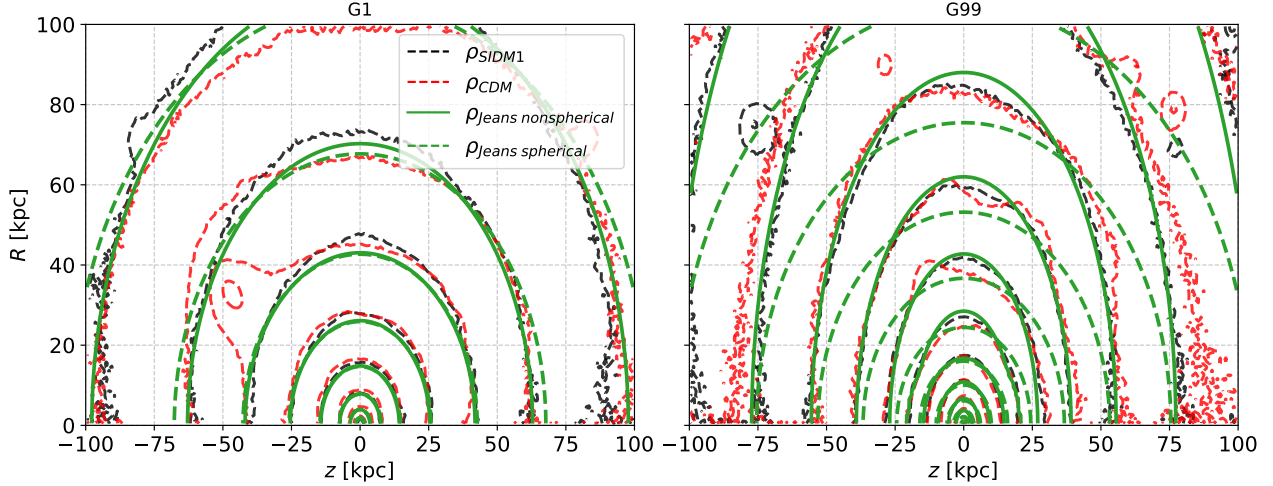


Figure 8: Density contours for G1 (left panel) and G99 (right panel) for the SIDM1b and CDMb simulations (black and red dashed contours) and the best-fit spherical and nonspherical Jeans models (green dashed and solid contours).

data and the nonspherical (green solid contour) and spherical (green dashed contour) best-fit Jeans models.

In Fig. 8 for G1 the SIDM1b and CDMb densities can be seen to be approximately spherical out to the boundaries, with the nonspherical Jeans model approximately tracking the contours. Meanwhile, for G99 both SIDM1b and CDMb are elongated in the R direction (oblate). The nonspherical Jeans model provides a remarkably good fit to the isodensity contours of the SIDM1b data, as the solid green curve tracks the SIDM1b contour from small out to large radii. However, there is little variation between the isodensity contours between SIDM1b and CDMb in both G1 and G99, implying that the two-dimensional density is equally unable to differentiate between CDM and SIDM at the intermediate mass scales.

This begs the question of whether the nonspherical Jeans model can be used to model halo shapes accurately enough to be used as a discriminant between CDM and SIDM cosmologies in instances where the spherically averaged and azimuthally averaged profiles are indistinguishable. However, the shapes of the simulated SIDM1b and CDMb models arrived at by a similar integration in thin spheroidal shells of the density distribution contain a considerable degree of noise, making direct

comparison with the Jeans model predictions statistically challenging. However, owing to a large simulation volume we can instead compare general trends in halo shapes between the full ensemble of 250 galaxy simulations and the nonspherical Jeans model predictions for them.

To that end, we perform a new parameter scan where we allow r_m and c ²³ to vary and we fix M_{200} and q to the data. To fix q we take $q_{data}(r = r_m)$ from the shape data for the simulations, ensuring that the shape of the Jeans model halo matches the simulation at the boundary, outside of which we model the shape as a constant. We then take the best-fit profiles by minimizing the two-dimensional log-likelihood as before. The results of this process are shown in Fig. 9 where we have plotted the 90th percentile distributions for c/a ²⁴ for CDMb (grey band), SIDM1b (orange band) and the best-fit nonspherical Jeans models (blue band) for galaxies sorted by $\log M_{200}$ (upper left corner of each panel).

What we see in Fig. 9 is that the CDMb halo shapes are broadly distributed across all radii. This is expected since the shape of a CDMb is not (as) affected by baryonic physics or driven towards being spherical by self-interactions as with SIDM. In other words, no systematic trends in CDM halo shapes are expected. In contrast, across all mass ranges SIDM can be seen to be on average more asymmetric in the baryon-dominated interiors and more spherical in the region where DM begins to dominate over baryons, $\mathcal{O}(\sim 10 \text{ kpc})$. This is a feature of SIDM responding to the baryons in the inner regions where baryons dominate the potential, and relaxing to the isothermal assumption of sphericity in regions where the baryon fraction has fallen off. The best-fit nonspherical Jeans models show a similar feature, where the shape distribution tightens about $c/a = 1.0$ in the isothermal region of the halo, i.e. $r < r_m$. A sharp deviation is seen at $r = r_m$ where the Jeans model is fit to the simulation data, after which the model for the shape is a constant value, i.e. $q_{data}(r_m)$. The region around which c/a tightens about 1 can also be seen to grow with increasing halo mass. This is due larger values of r_m being preferred for larger mass halos, and therefore, the isothermal region in between where the baryon fraction is diminished ($\mathcal{O}(10 \text{ kpc})$) and where the

²³Over the same quoted log-spaced ranges as for the above-mentioned parameter scan.

²⁴Defined as q if $q < 1$ and $1/q$ if $q > 1$.

model transitions to collisionless (r_m) is dominated by SIDM and hence is driven toward the SIDM only expectation of sphericity. This intermediate region where SIDM drives the halo shape toward spherical could provide an important observational probe for discriminating SIDM from CDM where the density profiles are indistinguishable.

The nonspherical Jeans model does not show the same broadening in c/a in the inner region that SIDM1b does. Since there is a similar broadening apparent in the CDMb distribution, we may question whether the asymmetry in the SIDM1b is due to baryons or low particle number in the inner region and therefore higher statistical noise in the shape calculation. Moreover, the degree to which SIDM1b approaches $c/a = 1$ is not as dramatic as for the nonspherical Jeans model. This is due to the Jeans model being an idealized model. In a real SIDM halo, the transition from self-interacting to effectively collisionless would not be as abrupt as the Jean model predicts. Moreover, the simulations account for mergers and assembly histories that can affect the halo-to-halo scatter.

Irrespective of the difference between SIDM1b and the nonspherical Jeans model, it is clear that the broad features of SIDM are captured by the nonspherical Jeans model. In order to conclude with greater certainty that the nonspherical Jeans model produces reasonable predictions for halo shapes we would need simulation data with higher resolution on shapes in the inner kpc where baryons significantly impact SIDM shapes.

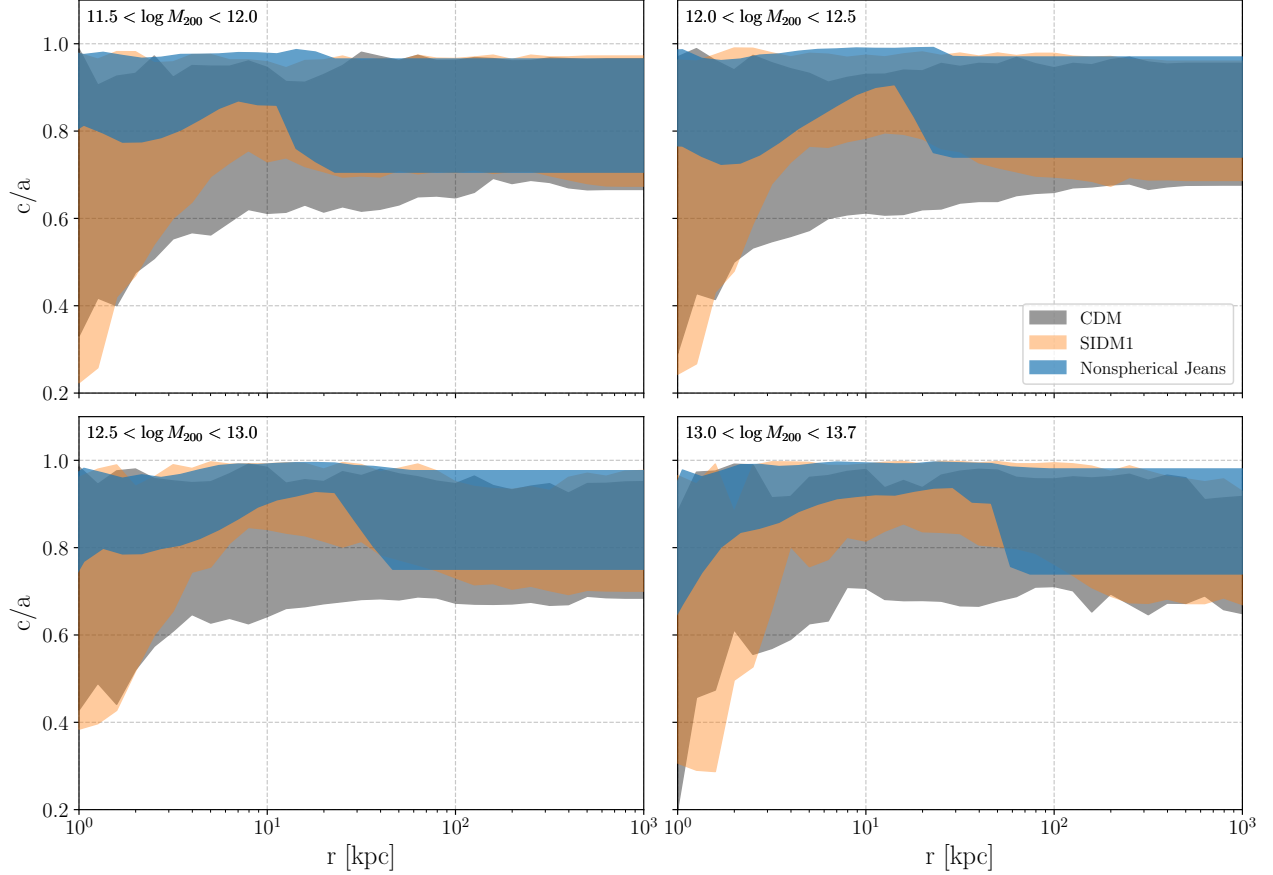


Figure 9: Shape distributions for all 250 simulated galaxies for CDMb (grey band), SIDM1b (orange band), and the best-fit nonspherical Jeans models (blue band) for them. The bands represent the 90th percentile for the distributions of shapes among all 250 galaxies.

6 Comments on future use of the nonspherical Jeans model for SIDM halos

There are several intriguing avenues of exploration open to the nonspherical Jeans model. The first of which is to extend the Jeans analysis to a broader range of halo masses, from dwarf to cluster scales. As noted, cosmological simulations of CDM and SIDM with baryons in the intermediate mass range can result in similar density profiles, making it difficult to differentiate CDM and SIDM cosmologies. Conversely, dwarf and cluster scales, having lower baryon mass fractions, may provide propitious environments to study the nonspherical Jeans model's ability to model SIDM phenomenology.

Velocity-independent cross-sections have been constrained by observational probes, with the strongest constraint coming from ellipticity measurements at cluster scales that rule out $\sigma/m > 0.1 \text{ cm}^2/\text{g}$. An alternative is to allow the cross-section to vary with relative velocities, which scale as a function of increasing halo mass. Astrophysical observation across galaxy and cluster scales have shown a preference for a mildly velocity-dependant cross-section that can accommodate $\sigma/m = 0.1 \text{ cm}^2/\text{g}$ for clusters and $\sigma/m \approx 1 \text{ cm}^2/\text{g}$ on dwarf scales, while simultaneously maintaining the mechanisms to rectify small-scale issues. In conjunction with extending the nonspherical Jeans model to new mass scales, it would be worthwhile to include velocity dependence in the nonspherical Jeans analysis to confront the strongest constraints at differing mass scales.

In practice, the Jeans model can also be applied directly to observables, like rotation curves and gravitational lensing data, to infer halo properties like the DM density and halo shape. In principle, given information about the baryon distribution an observed system can be modeled with the nonspherical Jeans model as a consistency check within the SIDM framework. As mentioned, observed systems can be modeled in as much detail as required to confront the predictions of SIDM against observation. A key advantage of the Jeans model is that large parameter scans over the SIDM parameter space can be accomplished with relatively low computational cost, complimenting N-body simulations where, for example, only a few cross-sections are tested at a time.

7 Conclusion

The theoretical framework for a nonspherical Jeans model has been explicated along with the numerical methods employed to solve the Jeans equation in multiple dimensions while relaxing the assumptions of symmetry. The nonspherical Jeans model was then implemented for the example of a MW-mass DM halo, showing that the results—densities and shapes—accord well with the expectations for SIDM halos. To validate the nonspherical Jeans model further, we used the baryon distributions from EAGLE-50 SIDM1b simulation as an input to the model, showing that the nonspherical Jeans model correctly inferred the input cross-section from the simulation while modeling well the

one-dimensional spherically averaged and two-dimensional azimuthally averaged density profiles for the simulations.

It was pointed out that the spherically averaged profiles alone can be insufficient to discriminate between CDM and SIDM cosmologies. To discriminate CDM from SIDM we turned to the two-dimensional density maps, for which we showed that the nonspherical Jeans model could provide a qualitatively good fit to the data in instances where halos are highly asymmetric. However, CDM and SIDM can have similar density structures, evading differentiation by means of density maps alone. We then turned to halo shapes, but owing to noise in individual halo shape data that precluded useful comparison with models we instead probed the average differences in the shapes of the entire simulation ensemble, seeing that, in contrast to CDM, SIDM halo shapes tighten about $c/a = 1.0$ in regions where SIDM dominated the mass distribution, with this effect being more pronounced for higher mass halos. The nonspherical Jeans model correctly models this behavior, with further verification required to validate the results of the Jeans model as a reasonable probe for halo shapes.

The LMC and local properties of dark matter

8 Overview

In Part I we assumed that the SIDM velocity distribution was isotropic and described by a Maxwellian distribution which enabled equilibrium solutions to the Jeans equation. However, the exact phase space distribution for a cosmological system may depart from this assumption significantly. For example, different models for DM, baryonic effects, formation history, mergers, and substructures all significantly impact the phase space of a cosmological halo.

The study of the phase space of DM halos bears important consequences on models for the direct detection of DM, which depend sensitively on the assumptions used for the local DM velocity and density. High-resolution hydrodynamical simulations of galaxy formation and evolution have shown that the Standard Halo Model (SHM) provides a good fit to CDM velocity distributions of simulated Milky Way (MW) like halos, although there exists a large halo-to-halo scatter in the distributions that leads to large uncertainties in the interpretation of direct detection results [26–31, 64, 65]. Moreover, recent analysis of SIDM hydrodynamical simulations from EAGLE-50 have shown that SIDM is consistent with the SHM as well as estimates of the local DM density [66].

However, in light of data from the Gaia mission [32] and the Sloan Digital Sky Survey (SDSS) [33] there is mounting evidence that the MW DM halo contains kinematically distinct substructures due to its formation and merger history [34–38, 67–69] that can significantly alter the local velocity distribution and density of the MW halo. Efforts have been made to refine the SHM to include kinematic structures like the radially anisotropic component, the *Gaia sausage*, of the MW [34, 70].

In addition to substructures, recent attention has been paid to the largest satellite galaxy of the MW, the Large Magellanic Cloud (LMC), since it is believed to be the largest perturber of the MW DM [40, 71] and near its first Pericenter passage of the MW [72] (although a recent paper calls this into question [73]). A study of an idealized simulation fitting the exact kinematic properties of the MW-LMC system has revealed that high-speed DM particles that come unbound from the LMC

in its near-orbit can end up in the MW Solar neighborhood, increasing the high-speed tail of the DM velocity distribution [40]. Additionally, the gravitational influence of the LMC in its orbit of the MW can also boost the native MW DM particles in response to the reflex motion of the LMC at infall [39, 74, 75]. These combined effects can significantly impact the tails of the DM velocity distribution in the Solar region, which has implications for direct detection searches, especially for low DM mass models which are sensitive to small changes in the tails.

In this Part, we extend the study of the LMC from idealized simulations, which fail to account for the formation history of fully cosmological halos with multiple accretion events, to the case of fully hydrodynamical CDM simulations of MW-LMC analog systems. Choosing an ensemble of MW-LMC analogs with varied assembly histories provides insight into the halo-to-halo scatter of true cosmological halos and is necessary to characterize the signatures of the LMC impact on the local properties of DM, and hence how robust are the constraints inferred from direct detection searches. What is more, we use the freedom conferred by the full cosmological simulation in defining the position of the Sun to answer the question of the importance of the exact Sun-LMC geometry concerning its impact on the local DM velocity distribution.

Sec. 9 briefly describes the theoretical background of direct detection experiments and the important astrophysical parameters that figure in their calculations. Sec. 10 discusses the simulations and the criteria for defining MW-LMC analog systems and how we match the Sun-LMC geometry. Sec. 11 will be concerned with elucidating the simulation results by defining local DM properties and derived quantities, showing how the LMC quantitatively impacts the local DM. Sec. 12 will show how the impact of the LMC on local DM properties translates to implications for near-future direct detection parameter spaces. And Sec. 13 will conclude by reiterating the findings for the impact of the LMC on the DM distribution and direct detection limits.

9 Direct detection of dark matter

DM direct detection experiments attempt to detect the signal of a DM interaction with SM particles. The WIMP model of DM allows for small and infrequent interactions of DM with SM particles that are theorized to be distinguishable from background events in state-of-the-art detectors. Currently, no detection of WIMP DM has been made [76] and a large portion of the WIMP mass and interaction cross-section parameter space has been ruled out. Next-generation direct detection experiments will attempt to push exclusion limits beyond the irreducible background of coherent neutrino scattering [77].

In the event that a DM particle of mass m_χ collides and scatters elastically with a target nucleus of mass m_T , depositing a nuclear recoil energy in the detector of E_R , the energy differential event rate is given by,

$$\frac{dR}{dE_R} = \frac{\rho_\chi}{m_\chi} \frac{1}{m_T} \int_{v>v_{\min}} d^3v \frac{d\sigma_T}{dE_R} v \tilde{f}_{\text{det}}(\mathbf{v}, t), \quad (49)$$

where ρ_χ is the local dark matter density, σ_T is the DM-nucleus scattering cross-section, and $\tilde{f}_{\text{det}}(\mathbf{v}, t)$ is the DM velocity distribution in the detector rest frame normalized to unity, where $\mathbf{v}$ is the relative velocity between DM and nucleus. The integral is taken for velocities greater than $v_{\min}$, the minimum velocity required to deposit recoil energy, E_R , to the detector, and is defined by,

$$v_{\min}(E_R) = \sqrt{\frac{m_T E_R}{2\mu_{\chi T}^2}}, \quad (50)$$

where $\mu_{\chi T}$ is the reduced mass of the DM-nucleus pair. Assuming a spin-independent cross-section with equal couplings to protons and neutrons, the differential DM-nucleus cross-section takes the functional form,

$$\frac{d\sigma_T}{dE_R} = \frac{m_T A^2 \sigma_{\chi N}^{\text{SI}}}{2\mu_{\chi N}^2 v^2} F^2(E_R), \quad (51)$$

where A is the atomic mass number, $\sigma_{\chi N}^{\text{SI}}$ is the spin-independant DM-nucleon cross-section, and $F(E_R)$ is the spin-independent nuclear form factor.²⁵ Taking the velocity-independent terms outside

²⁵In Sec. 12.1 we adopt the Helm form factor from Ref. [78].

of the integral in eq. (49) we can express the event rate as,

$$\frac{dR}{dE_R} = \frac{A^2 \sigma_{\chi N}^{\text{SI}}}{2\mu_{\chi N}^2} F^2(E_R) \frac{\rho_\chi}{m_\chi} \eta(v_{\min}, t), \quad (52)$$

where η is the *halo integral*,²⁶ defined,

$$\eta(v_{\min}, t) \equiv \int_{v > v_{\min}} d^3v \frac{\tilde{f}_{\text{det}}(\mathbf{v}, t)}{v}. \quad (53)$$

Along with the local DM density, ρ_χ , the halo integral parameterizes the astrophysical dependence of the differential event rates, and for which the standard assumptions come from the SHM [24].

Similarly, in the event that the DM scatters with an electron in a crystal target the energy differential event rate is written [79],

$$\frac{dR}{d \ln E_e} = N_{\text{cell}} \frac{\rho_\chi}{m_\chi} \frac{\bar{\sigma}_e \alpha m_e^2}{\mu_{\chi e}^2} \int d \ln q \frac{E_e}{q} \left[|F_{\text{DM}}(q)|^2 |f^{\text{crystal}}(E_e, q)|^2 \eta(v_{\min}(q, E_e)) \right], \quad (54)$$

where N_{cell} is the number of cells per unit mass in the crystalline target, $\bar{\sigma}_e$ is the DM-electron cross-section, α the fine structure constant, $\approx 1/137$, E_e the energy deposited to the electron, q the momentum transfer from DM to electron, m_e the electron mass, and $\mu_{\chi e}$ the DM-electron reduced mass. Here there are two form factors, $F_{\text{DM}}(q)$, the DM form factor, and, f^{crystal} , the crystal form factor. The latter describes the electronic structure of the target material while the former describes the momentum transfer of the interaction, taking the following forms: $F_{\text{DM}}(q) = 1$ for contact interactions with a heavy mediator, $F_{\text{DM}}(q) = \alpha m_e / q$ for electron dipole coupling, and $F_{\text{DM}}(q) = (\alpha m_e / q)^2$ for long-range interactions via exchange of an ultralight or massless mediator [79].

$v_{\min}$ now denotes the minimum speed for a DM particle to deposit the energy E_e with momentum transfer q to the electron, described by,

$$v_{\min}(E_e, q) = \frac{E_e}{q} + \frac{q}{2m_\chi}. \quad (55)$$

Note that the DM density figures in eqs. (52) and (54) as a normalization factor, while the DM velocity distribution enters via the halo integral—an integral over velocities of DM in the detector.

²⁶Note that this is sometimes referred to as the *mean inverse speed* in the literature.

Since the lower bound of the integral in the halo integral is set by $v_{\min}$, and both eqs. (50) and (55) $v_{\min}$ are proportional to $1/m_\chi$, the event rates for low-mass DM models become sensitive to small changes in the high-velocity tails of the halo integrals, having a pronounced effect on exclusion limits discussed more in Sec. 12.

Moreover, the time dependence of eq. (52) and eq. (54) enters through the time dependence of the DM velocity distribution in the detector rest frame. This time dependence is due to the velocity of Earth with respect to the Sun (and leads to an annual modulation signal for DM that is not explored in this paper).

In order to determine $\tilde{f}_{\text{det}}(\mathbf{v}, t)$ one has to boost the DM particle velocities in the Galactic reference frame by, first, the Sun's velocity, $\mathbf{v}_s$, with respect to the Galactic Center, then by the Earth's velocity, $\mathbf{v}_e(t)$, with respect to that of the Sun, such that,

$$\tilde{f}_{\text{det}}(\mathbf{v}, t) = f_{\text{gal}}(\mathbf{v} + \mathbf{v}_s + \mathbf{v}_e(t)) \quad (56)$$

The Sun's velocity with respect to the Galactic Center is the sum of the Sun's circular velocity and the Sun's peculiar velocity with respect to the Local Standard of Rest, i.e. $\mathbf{v}_s = \mathbf{v}_c + \mathbf{v}_{\text{pec}}$, where $\mathbf{v}_{\text{pec}} \approx (11.10, 12.24, 7.25) \text{ km/s}$ [80].

9.1 The Standard Halo Model

The SHM assumes that in the Galactic reference frame the DM velocity distribution is well-modeled by a Maxwellian distribution with peak speed equal to the local circular speed, $v_0 = v_c$,²⁷ and, truncated by the escape speed from the Galaxy, v_{esc} [81],

$$f_{\text{gal}}(\mathbf{v}) = \begin{cases} \frac{4\mathbf{v}^2}{N_{\text{esc}}\sqrt{\pi}v_c^3} \exp(-\mathbf{v}^2/v_c^2) & \mathbf{v} < v_{\text{esc}} \\ 0 & \mathbf{v} > v_{\text{esc}} \end{cases} \quad (57)$$

²⁷It is assumed that the MW's rotation curve has reached its asymptotic value at the Solar radius, $v_c \equiv v_c(R_0)$, where $R_0 \approx 8 \text{ kpc}$, and is therefore equivalent to the peak speed.

where $N_{\text{esc}} = \text{erf}(z) - 2z\exp(-z^2)/\pi^{1/2}$, with $z \equiv v_{\text{esc}}/v_c$. The analytic halo integral for the SHM can similarly be derived [81],

$$\eta(v_{\text{min}}, \mathbf{v}) = \begin{cases} \frac{1}{v_0 y} & z < y, \ x < |y - z| \\ \frac{1}{2N_{\text{esc}} v_0 y} [\text{erf}(x + y) - \text{erf}(x - y) - \frac{4}{\sqrt{\pi}} y e^{-z^2}] & z > y, \ x < |y - z| \\ \frac{1}{2N_{\text{esc}} v_0 y} [\text{erf}(z) - \text{erf}(x - y) - \frac{2}{\sqrt{\pi}} (y + z - x) e^{-z^2}] & |y - z| < x < y + z \\ 0 & y + z < x, \end{cases} \quad (58)$$

where $x \equiv v_{\text{min}}/v_c$, $y \equiv v_E(t)/v_c$, with $v_E(t) = |\mathbf{v}_s + \mathbf{v}_e(t)|$.

In the case of the MW the local circular speed is commonly taken to be 220 km/s and the escape speed 544 km/s. The fiducial value for the DM density is taken to be $\rho_\chi = 0.3 \text{ GeV/cm}^3$, although local methods tracing stellar population and global methods using mass modeling of the MW have estimated the range to be within $\rho_\chi = [0.2 - 0.8] \text{ GeV/cm}^3$ [82–99].

However, uncertainties on the local DM density, the circular speed, escape speed, and the DM velocity distribution result in important uncertainties in the direct detection signals, including the energy, time, and directional dependence of the expected event rate [100].

In this paper, the DM velocity distributions and halo integrals are calculated directly from hydrodynamic simulations (presented in Sec. 10) via DM particle positions and velocities (see Sec. 11), with comparisons made to the SHM expectations in Secs. 11.5 and 12.

10 Hydrodynamical simulations

We use the magneto-hydrodynamical simulations of MW mass halos from the AURIGA project [101]. The Auriga simulations consist of 30 high-resolution cosmological zoom-in simulations of MW-mass dark halos generated using the moving mesh code AREPO [102] and selected from a 100^3 Mpc^3 periodic cube (L100N1504) from the EAGLE project [103, 104]. The Auriga simulations use collisionless DM dynamics and a comprehensive model of subgrid baryonic physics that reproduces present-day observables, such as the observed stellar mass, rotation curves, star formation rates, and metallicities

of MW-mass galaxies. Note that Auriga uses the PLANCK-2015 [105] cosmological parameters for all simulations with standard resolution (Level 4), DM particle mass $m_{\text{DM}} \sim 3 \times 10^5 M_\odot$, baryonic mass element $m_b = 5 \times 10^4 M_\odot$, and the Plummer equivalent softening $\epsilon = 370 \text{ pc}$ [106, 107].

10.1 MW-LMC analog selection criteria

We select MW mass halos that have satellite halos that match the mass, stellar mass, and kinematics of the LMC. The LMC had its first Pericenter pass of the MW $\sim 50 \text{ Myr}$ ago at a distance of $\sim 48 \text{ kpc}$ [72], with a velocity with respect to the MW center of $340 \pm 19 \text{ km/s}$ [108]. Presently, the LMC is at a distance of $\sim 50 \text{ kpc}$ with a speed of $321 \pm 24 \text{ km/s}$ with respect to the MW [109, 110], and has a stellar mass of $\sim 2.7 \times 10^9 M_\odot$ [111].

Owing to the small simulation ensemble size of 30 halos, to increase the probability of matching we allow matching at lookback times up to a redshift of $z = 1$, equivalent to 8 Gyr before present-day. The specific criteria with which we identify MW-LMC analogs are 1) a stellar mass for the LMC analog sub-halo $> 5 \times 10^8 M_\odot$, and 2) the sub-halo distance at Pericenter from its host halo in the range $[40, 60] \text{ kpc}$. With this matching criteria, we identify 15 MW-LMC analog candidate systems. Presented in Tab. 4 are the properties of the 15 MW-LMC candidates, from the leftmost column to the rightmost: Halo ID, Auriga simulation volume ID, the distance of the sub-halo to its host, r_{LMC} , the lookback time, t_{LB} , the stellar mass of the MW analogs, M_*^{MW} , the virial mass of the MW analogs, M_{200}^{MW} , and the stellar mass of the LMC analog, M_*^{LMC} .

Low time resolution between snapshots, i.e. average of $\sim 150 \text{ Myr}$, presents a difficulty in obtaining present-day snapshots of the MW-LMC analogs. To study the impact of the LMC on the MW at present-day we choose Halo 13 (Au-25 in Tab. 4)²⁸ to resimulate in finer time-steps, on average $\sim 10 \text{ Myr}$ between snapshots. From the resimulated Halo 13, we isolate four snapshots in time summarized in Tab. 5. Each snapshot is a representative time in the orbit of the LMC analog of the MW analog: *Iso.*, represents the simulation proxy of the MW analog in the absence of the

²⁸Halo 13 was chosen for resimulation due to having an infall mass close to the upper limit of that from observations, $3.2 \times 10^{11} M_\odot$, which lead to large values for κ_{LMC} (see Tab. 4).

influence of the LMC analog;²⁹ *Peri.*, the closest point of approach to the MW analog along the orbit of the LMC analog; *Pres.*, the present-day equivalent of the analog MW-LMC system; and *Fut.*, a proxy for the analog MW-LMC system at a time in the future. At present-day the resimulated Halo 13 has a distance from MW-LMC analog distance of 50.6 kpc and a speed of 317 km/s, which are in good accordance with the observed values.

The following section describes the procedure for matching the Sun-LMC geometry to observables.

10.2 Matching the Sun-LMC geometry

The observed Sun-LMC geometry shows that the LMC is moving roughly anti-parallel with respect to the motion of the Sun, leading to large relative velocities of particles originating from the LMC. High relative speeds between the LMC DM particles lead to a boost in the tail of the DM velocity distribution to which the direct detection searches are sensitive. Therefore, *a priori*, defining the Sun-LMC geometry should have an impact on the direct detection limits. To explore this point in detail we define a set of criteria to match the simulation analog geometry to the observed Sun-LMC geometry to define a solar position in the simulations.

In Galactocentric coordinates the origin is in the Galactic Center with the x-coordinate, x_g , pointing from the Sun to the Galactic Center, with the y-coordinate, y_g , in the direction of the rotation of the Sun. x_g and y_g define the Sun's orbital plane, and the z-coordinate, z_g , points in the direction of the North Galactic Pole, orthogonal to the orbital plane. Fig. 10 shows the observed Sun-LMC geometry in Galactocentric coordinates. The Sun's position vector, $\mathbf{r}_{\text{Sun}}$, and velocity vector, $\mathbf{v}_{\text{Sun}}$, are plotted alongside the LMC's position vector, $\mathbf{r}_{\text{LMC}}$, and velocity vector, $\mathbf{v}_{\text{LMC}}$, and the LMC's orbital angular momentum, $\mathbf{L}_{\text{LMC}}$, as vector arrows originating from the Galactic Center. The Sun and LMC orbital planes are plotted as yellow and orange planes for reference. Moreover, α and β represent the angles between the Sun's velocity vector with the LMC's position and velocity

²⁹Here we use the earliest snapshot, when the LMC analog and the MW analog are maximally separated, i.e. at first apocenter prior to infall.

Halo ID	Auriga ID	r_{LMC} [kpc]	t_{LB} [Gyr]	M_{*}^{MW} [$10^9 \text{ M}_{\odot}$]	M_{200}^{MW} [$10^{11} \text{ M}_{\odot}$]	M_{*}^{LMC} [$10^9 \text{ M}_{\odot}$]
1	Au-1	53.1	5.1	15	4.0	0.66
2	Au-7	49.2	4.2	23	9.3	0.56
3	Au-12	49.4	4.6	33	11	0.79
4	Au-13	45.8	6.7	26	9.5	2.4
5	Au-13	56.7	7.4	16	7.2	3.1
6	Au-14	45.6	6.7	37	13	3.3
7	Au-14	49.9	2.3	93	16	0.99
8	Au-17	54.0	7.1	50	8.9	0.85
9	Au-19	40.9	6.2	18	6.6	1.6
10	Au-19	50.8	5.4	21	12	9.1
11	Au-21	55.5	3.3	67	17	4.8
12	Au-23	41.0	5.9	60	16	2.5
13	Au-25	43.2	1.0	37	12	15
14	Au-27	58.9	6.3	56	16	1.0
15	Au-30	56.0	6.3	73	9.7	2.5

Table 4: Properties of 15 MW-LMC analog candidate halos within the range of the matching criteria described in Sec. 10.1. The first two columns are the halo ID number and the Auriga simulation volume reference ID. Columns three to seven describe the properties of the MW-LMC system, in order from left to right: distance of the LMC analog from its host halo, r_{LMC} , lookback time, t_{LB} , stellar mass of the MW analogs, M_{*}^{MW} , virial mass of the MW analogs, M_{200}^{MW} , and the stellar mass of the LMC analogs, M_{*}^{LMC} .

vectors, respectively, which are important for finding the Sun’s position, discussed presently.

In the simulations, the center of the MW and LMC analogs are found by finding the particle

Snapshot	Description	$t - t_{\text{Pres.}}$ [Gyr]	r_{LMC} [kpc]
<i>Iso.</i>	Isolated MW analog	−2.83	384
<i>Peri.</i>	LMC’s first Pericenter approach	−0.133	32.9
<i>Pres.</i>	Present-day MW-LMC analog	0	50.6
<i>Fut.</i>	Future MW-LMC analog	0.175	80.3

Table 5: Halo 13 representative snapshots along the LMC analog orbit of the MW analog. The first two columns give the shorthand used throughout this paper and a short description of the time step, respectively. The last two columns give the time with respect to the present-day and the distance of the LMC analog from the MW analog.

with the lowest gravitational potential. The velocity of each simulation particle is then defined with respect to the center of mass velocity of the bound particles in each halo via the SUBFIND algorithm [113]. Positions and velocities for the LMC are then found with respect to the MW analog center of mass.

With position and velocities for the MW and LMC analogs, we proceed to match the Sun-LMC geometry by imposing the condition that the stellar disk orientation has an orbital angular momentum that lies nearly perpendicular to the LMC analog’s orbital plane. We then solve for the direction of the stellar disk’s angular momentum. Since we have an angular degree of freedom, we end up with solutions that vary on a nearly full circle, representing multiple allowed disk orientations. The disk orientation need not align with the stellar disk in the simulation, but this has little impact on the DM distribution [29, 114], so we consider all allowed orientations that match the Sun-LMC geometry.

To find the position of the Sun with respect to the MW analog for each allowed disk orientation we match the Sun’s position and velocity vectors with the LMC’s orbital angular momentum, resulting in a Sun position for each allowed disk orientation.

We further require that the angles between the Sun’s velocity vectors and the LMC’s position

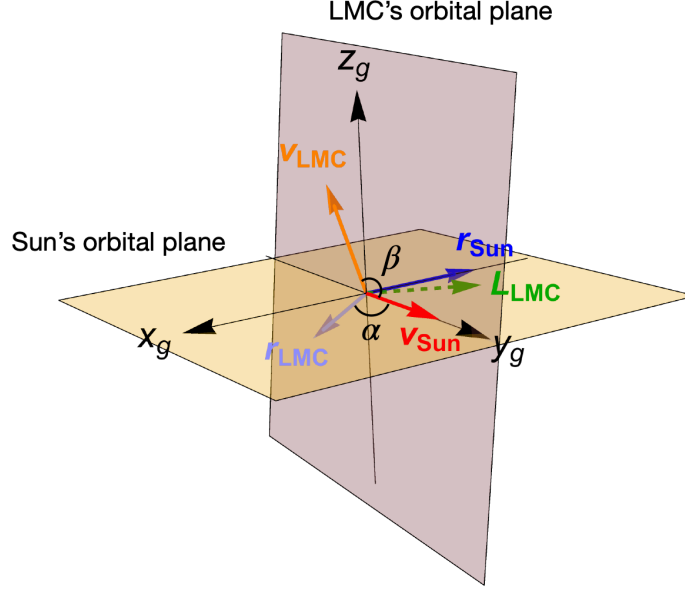


Figure 10: Observed Sun-LMC geometry diagram. Plotted are the Sun's position vector, $\mathbf{r}_{\text{Sun}}$ (blue arrow), the Sun's velocity vector, $\mathbf{v}_{\text{Sun}}$ (red arrow), the LMC's position vector, $\mathbf{r}_{\text{LMC}}$ (lavender arrow), the LMC's velocity vector, $\mathbf{v}_{\text{LMC}}$ (orange arrow), the LMC's orbital angular momentum vector, $\mathbf{L}_{\text{LMC}}$ (dashed green arrow), the Sun's orbital plane (yellow pane), and the LMC's orbital plane (grey pane). Note that the Sun's orbital plane is nearly perpendicular to the LMC's. α and β are the angles between the Sun's velocity vector with the LMC's position and velocity vectors, respectively. From Ref. [112].

and velocity vectors³⁰ match the observed values. To match this condition we introduce the *cosine angles*,

$$\begin{aligned}\cos \alpha &\equiv \hat{\mathbf{v}}_{\text{Sun}}^{\text{sim}} \cdot \hat{\mathbf{r}}_{\text{LMC}}^{\text{sim}} , \\ \cos \beta &\equiv \hat{\mathbf{v}}_{\text{Sun}}^{\text{sim}} \cdot \hat{\mathbf{v}}_{\text{LMC}}^{\text{sim}} ,\end{aligned}\tag{59}$$

where $\hat{\mathbf{v}}_{\text{Sun}}^{\text{sim}}$ is the velocity vector of the Sun in the simulation with respect to the center of the MW analog and $\hat{\mathbf{r}}_{\text{LMC}}^{\text{sim}}$ and $\hat{\mathbf{v}}_{\text{LMC}}^{\text{sim}}$ are the position and velocity vectors of the LMC in the simulation with respect to the MW analog center respectively. (See Fig. 10 for a diagram of the vector and the

³⁰The Sun's position vector is nearly parallel with the LMC's angular momentum vector and therefore does not vary significantly between allowed disk orientations.

cosine angles.) The observed values for the *cosine angles* are,

$$\begin{aligned}\hat{\mathbf{v}}_{\text{Sun}}^{\text{obs}} \cdot \hat{\mathbf{r}}_{\text{LMC}}^{\text{obs}} &= -0.835, \\ \hat{\mathbf{v}}_{\text{Sun}}^{\text{obs}} \cdot \hat{\mathbf{v}}_{\text{LMC}}^{\text{obs}} &= -0.709.\end{aligned}\tag{60}$$

To choose an orientation for the Sun’s velocity vector in the simulations that best matches the observed values in eq. (60) we select a $\cos\alpha$ and $\cos\beta$ from the allowed disk orientations that minimize the sum of the errors squared between the simulation and observed cosine angles. The resulting $\cos\alpha$ and $\cos\beta$ specify a unique position for the Sun that will be hereafter referred to as the *best-fit* Sun position. It is important to note that the halo-to-halo scatter in the allowed disk orientation may not provide a good fit to the observed values.³¹

11 Local dark matter distribution in the simulations

In order to extract the DM velocity distribution we first need to define a volume corresponding to the Solar region. The resolution of the simulation requires that we choose a volume large enough to include a statistically significant number of LMC DM particles while still being restricted enough to be sensitive to changes in the best-fit Sun position. The Sun position is determined following the procedure described in Sec. 10.2. With the Sun at a Galactocentric distance of 8 kpc, we begin by defining two concentric spherical shells with radii 6 and 10 kpc centered on the MW Galactic Center. We then project a cone with an opening angle $\pi/4$ with its vertex at the MW Galactic Center and its axis of symmetry aligned with the position of the Sun. The overlapping volume of the concentric shells and the projected cone constitute the Solar region (shown as the blue shaded volume in Fig. ??).

For the 15 isolated MW-LMC analog systems at Pericenter from the Auriga simulation described in Sec. 10.1, we find the best-fit Sun position and extract information about the number of MW

³¹The allowed cosine angles make a ring in the cosine angle parameter space that can be roughly circular, or ellipsoidal along an arbitrary axis, leading to the best-fit cosine angle being the closest point in the quadrant where $\cos\alpha$ and $\cos\beta$ are negative.

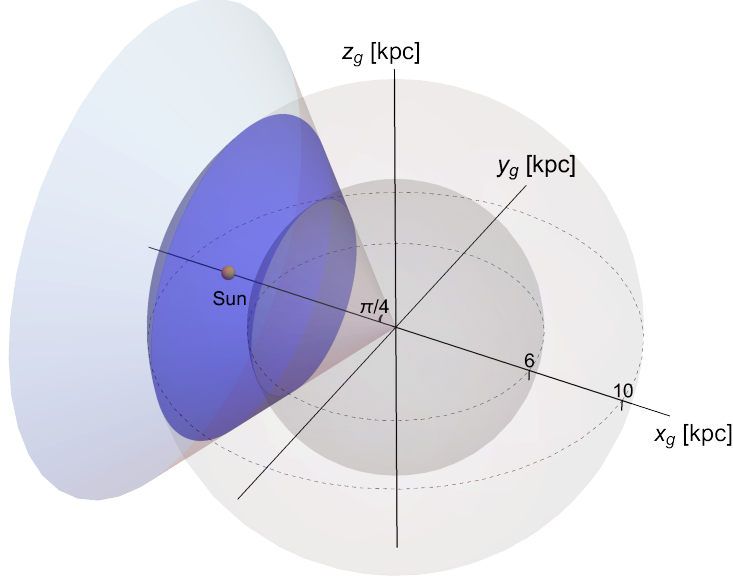


Figure 11: Solar region volume (blue shaded volume) define as the intersection of two concentric shells (grey shaded shells) of radii 6 to 10 kpc centered on the MW Galactic Center, and a cone (grey shaded cone) with its vertex at the Galactic origin, opening angle of $\pi/4$, and its axis aligned with the Sun's position. The yellow orb represents the hypothetical position of the Sun placed on the $x_g - y_g$ plane at a Galactocentric distance of 8 kpc. From Ref. [112].

DM particles in the Solar region, N_{MW} , the of DM particles originating from the LMC in the Solar region, N_{LMC} , the DM density, ρ_χ , the percentage of DM particles originating from the LMC, κ_{LMC} , and the κ_{LMC} range between all allowed Sun positions. The results are recorded in Tab. 6.

The DM density in the best-fit Solar region for all 15 MW-LMC analogs is in the range $\rho_\chi = [0.21 - 0.60] \text{ GeV/cm}^3$ which is in agreement with the Eagle [29] and Auriga [27] simulations, as well as local [85, 89–92, 97] and global [84, 86, 88, 93, 98, 99, 115, 116] estimates from observations.

11.1 Dark matter velocity distributions

The DM speed distributions for the MW-LMC analog systems are computed in the Galactic reference frame. The normalized DM speed distribution, $f(v)$, is,

$$f(v) = v^2 \int d\Omega_{\mathbf{v}} \tilde{f}(\mathbf{v}) , \quad (61)$$

Halo ID	N_{MW}	N_{LMC}	ρ_χ [GeV/cm ³]	κ_{LMC} [%]	κ_{LMC} Range [%]
1	7,760	11	0.21	0.14	[0.14 – 0.21]
2	8,581	55	0.23	0.64	[0.53 – 0.65]
3	11,621	3	0.35	0.026	[0.025 – 0.028]
4	12,483	12	0.34	0.096	[0.088 – 0.12]
5	8,669	136	0.24	1.5	[1.4 – 1.6]
6	13,290	5	0.38	0.038	[0.029 – 0.046]
7	18,467	6	0.53	0.032	[0.032 – 0.034]
8	12,949	1	0.38	0.0077	[0.0077 – 0.0082]
9	11,892	12	0.36	0.10	[0.069 – 0.13]
10	12,405	361	0.39	2.8	[2.8 – 3.1]
11	14,132	4	0.43	0.028	[0.021 – 0.039]
12	16,427	28	0.53	0.17	[0.17 – 0.21]
13	10,814	254	0.34	2.3	[2.3 – 3.0]
14	20,001	52	0.60	0.26	[0.26 – 0.31]
15	10,641	128	0.32	1.2	[0.81 – 1.3]

Table 6: Local DM properties for the 15 MW-LMC analogs at Pericenter in the Solar region. From left to right the columns are: halo ID, corresponding to the Auriga halo ID; N_{MW} the number of native DM particles in the best-fit Solar region; N_{LMC} , the number of LMC originating DM particles in the best-fit Solar region; ρ_χ , the DM density in the best-fit Solar region; κ_{LMC} , the percent of total DM particles originating from the LMC in the best-fit Solar region; and κ_{LMC} Range, the range of κ_{LMC} between all allowed Sun positions.

where $d\Omega_{\mathbf{v}}$ is the infinitesimal solid angle in the direction of $\mathbf{v}$, and the DM velocity distribution, $\tilde{f}(\mathbf{v})$, is normalized such that $\int dv f(v) = \int d^3v \tilde{f}(\mathbf{v}) = 1$.

Since the local circular speed for the MW is 220 km/s (Sec. 9.1) we scale the local DM speed

distributions of the 15 MW-LMC analog halo by a factor of $(220 \text{ km/s})/v_c$, where v_c is the local circular speed for each halo, to compare with the SHM and across halos more effectively. The bin size is set to a width of 25 km/s to limit statistical noise and optimize resolution.

Fig. 12 shows the DM speed distributions in the Galactic reference frame for the best-fit Sun position (defined in Sec. 10.2) for four of the 15 MW-LMC analogs, at the simulation equivalent of the first Pericenter approach. The black-shaded band represents the total DM in the Solar region, including DM native to the MW and DM originating from the LMC. The native MW DM particles are shown in the red shaded band, and the DM originating from the LMC is shown in blue (scaled down by a factor of 10 for illustration). The vertical extent of the bands specifies the $\pm 1\sigma$ Poisson error bar from the median value. The percent of LMC DM to total DM, κ_{LMC} is shown in the upper left corner of each panel, the Halo ID in the center top, and the ratio of MW+LMC to MW is shown in a residual plot beneath each panel.

The four MW-LMC analogies shown in Fig. 12 illustrate that the DM particles originating from the LMC that end up in the Solar region have a speed distribution that peaks in the high-speed tail of the MW speed distribution. For all 15 halos the LMC speed distribution peaks in the high-speed tail ($\gtrsim 500 \text{ km/s}$) of the MW distribution in the Galactic rest frame. This result is expected and confirms the findings in Ref. [40], but it is interesting to note in the cosmological setting the halo-to-halo scatter in results. In Fig. 12 Halo 13 (lower left) shows the largest κ_{LMC} of 2.3% and yet, due to the LMC speed distribution being broad across bins in v the ratio of MW+LMC to MW remains less than 1 across the high-speed tail. In contrast, Halo 2 (upper left) shows less than a third of the κ_{LMC} of Halo 13, and yet, due to the sharply peaked LMC speed distribution the ratio of MW+LMC to MW is near to or greater than 2 in three bins at the high-speed tail of the MW distribution.

Halo 6 (upper right) illustrates a halo for which the κ_{LMC} in the Solar region for the best-fit Sun position is ~ 60 times less than Halo 13 but still shows visible differences in the high-speed tails. Halo 15 (lower right) is an example of a halo where κ_{LMC} is relatively large and the LMC speed distribution is sharply peaked, leading to large differences in the high-speed tail of the MW

distributions (MW+LMC to MW ratio peaking near 4).

Therefore, the LMC speed distribution for the MW-LMC analog peaks in the high-speed tail of the MW speed distribution which leads to variations in the tails of the MW+LMC speed distribution, although the extent to which the distributions vary in the tails depends on the exact shape and width of the LMC speed distribution in the Solar region. We observe a large halo-to-halo scatter in the impact of the LMC on the tails of the DM speed distributions that are not strictly dependant on the number of LMC DM particles in the Solar region, i.e. κ_{LMC} .

To study the impact of the LMC on the MW at different points in its orbit of the MW we resimulate Halo 13 in finer resolution time steps³² (snapshots) and identify four snapshots of interest: isolated MW analog, *Iso.*, the LMC at Pericenter, *Peri.*, the MW-LMC system at the present-day equivalent *Pres.*, and the MW-LMC system at a future date, *Fut.* (see Tab. 5). In Fig. 13 the DM speed distributions are plotted for the four snapshots of Halo 13 in the Galactic reference frame in the Solar region for the best-fit Sun position.³³ The black shaded band is the MW+LMC DM, the red the MW DM, and the blue the LMC DM. The band height represents the $\pm 1\sigma$ Poisson errors on the median $f(v)$. Beneath each panel with an LMC is plotted a residual showing the ratio of the MW+LMC distribution to the MW-only distribution.

For all panels of Fig. 13 (except *Iso.* with $\kappa_{\text{LMC}} = 0\%$) the LMC speed distribution peaks in the high-speed tail of the MW speed distribution. The variation in the high-speed tail is most prominent (ratios of greater than 3) for *Peri.* (upper right) where κ_{LMC} has its highest value between the four snapshots of 0.85%. The *Pres.* snapshot (lower left) shows a sharp decrease in κ_{LMC} to 0.26% while the variation in the tail remains significant with ratios reaching 3. The *Fut.* snapshot (lower right) displays a slight drop in κ_{LMC} to 0.22% with a significant drop in the variation in the tails with ratios all below 2.

³²It is important to note that resimulating Halo 13 leads to stochastic changes in the phase space of the simulations that preclude a direct comparison with the original Halo 13 and the resimulated Halo 13.

³³Note that the isolated snapshot does not have an LMC analog with which to match the Sun-LMC geometry and therefore is presented as the DM speed distribution within the entire 6 – 10 kpc shell. The Poisson error bars appear smaller due to greater statistics—particle number—present in each bin.

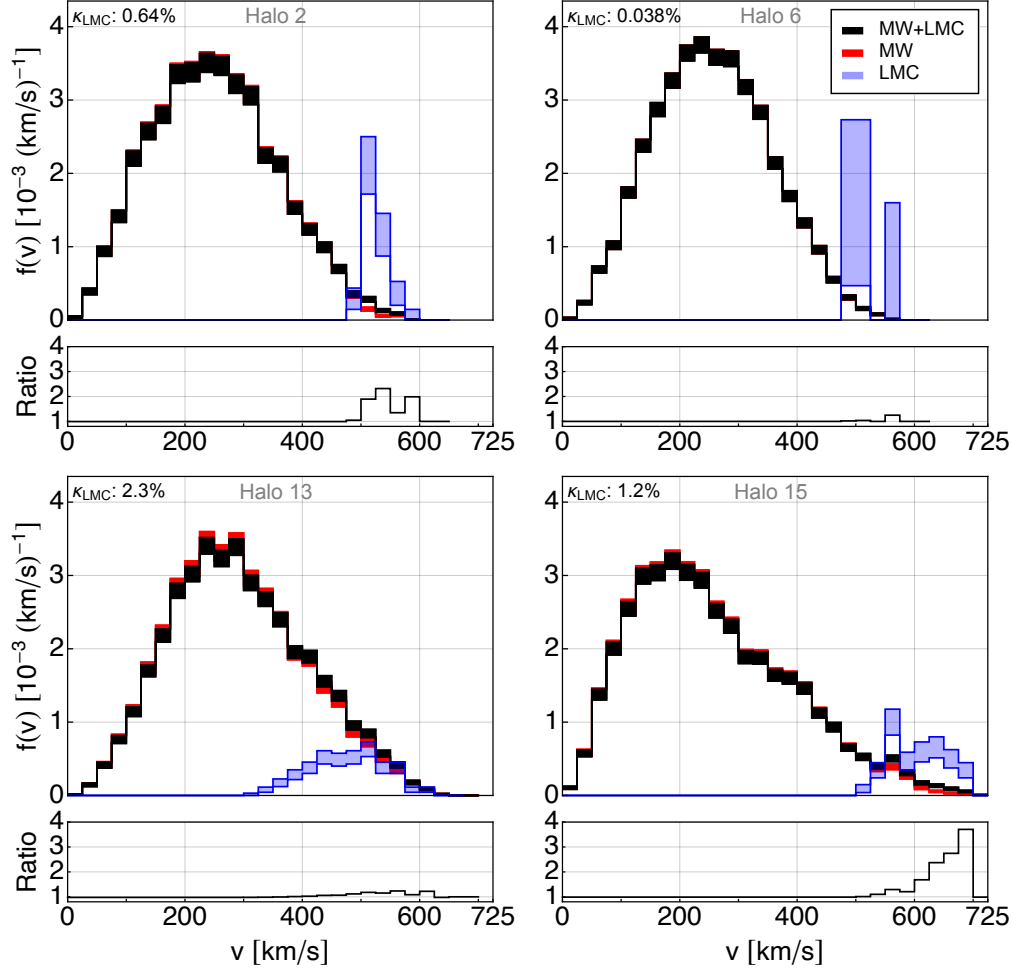


Figure 12: DM speed distributions for four MW-LMC analog halos at first Pericenter in the Galactic rest frame for the Solar region of the best-fit Sun position. DM from the MW+LMC is shown in black bands, MW-only in red, and LMC-only in blue (scaled down by a factor of 10 for visualization). The band height represents the $\pm 1\sigma$ Poisson error calculated using the number of particles in a bin of width 25 km/s. κ_{LMC} , the percent of LMC DM to the total DM is shown in the upper left corner of each panel, and a residual plot showing the ratio of MW+LMC to MW is shown underneath each panel. From Ref. [112].

Across all MW-LMC analog halos and snapshots the LMC DM is shown to affect the tails of the MW+LMC DM speed distributions in the Galactic reference frame. To analyze further the variation of the tails with respect to κ_{LMC} and the MW-LMC snapshot, and how these influence the direct detect parameter space, we translate the speed distributions into halo integrals—the input for

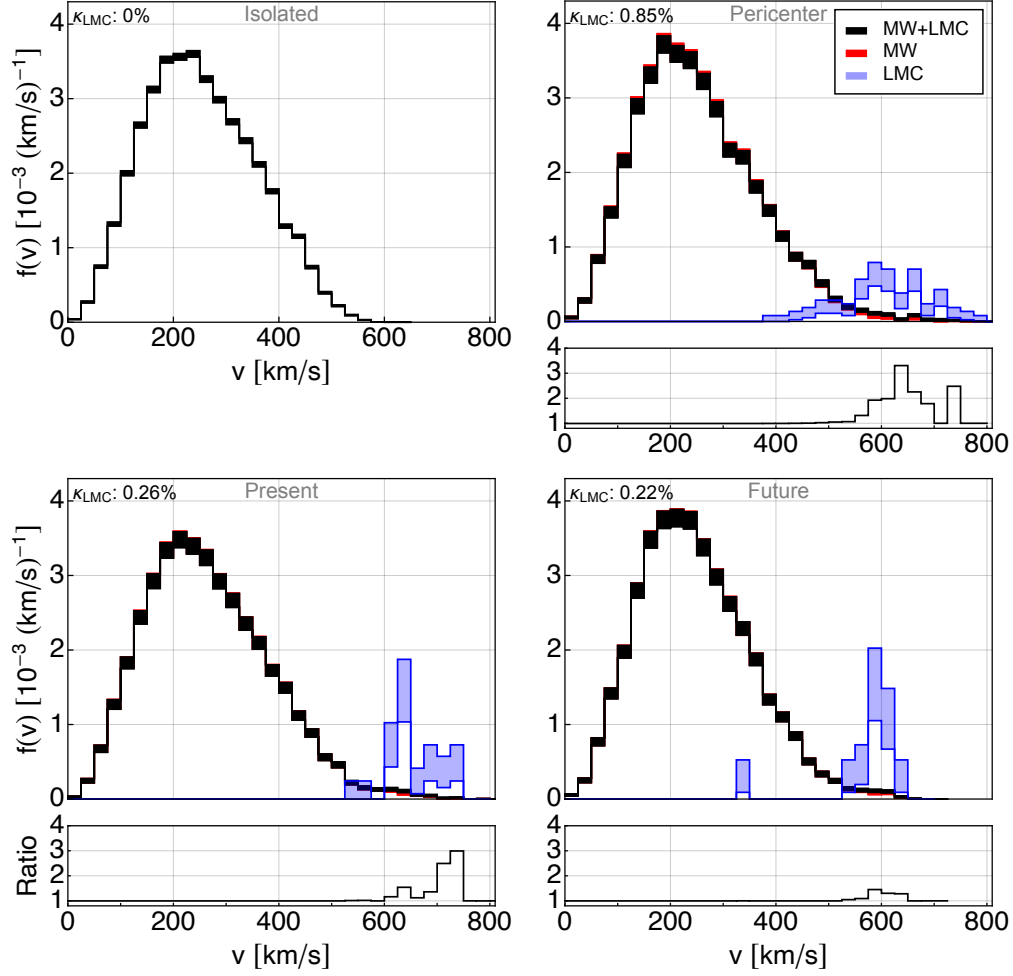


Figure 13: DM speed distributions for four snapshots of Halo 13 in the Galactic rest frame for the Solar region of the best-fit Sun position: isolated (upper left), Pericenter (upper right), present-day (lower left), and future (lower right). DM from the MW+LMC is shown in black bands, MW-only in red, and LMC-only in blue (scaled down by a factor of 10 for visualization). The band height represents the $\pm 1\sigma$ Poisson error calculated using the number of particles in a bin of width 25 km/s. κ_{LMC} , the percent of LMC DM to the total DM is shown in the upper left corner of each panel, and a residual plot showing the ratio of MW+LMC to MW is shown underneath each panel. From Ref. [112].

the event rate calculation. The following section presents the halo integrals and how we use them to quantify the LMC’s impact on the tails of the speed distributions.

11.2 Halo integrals

The astrophysical dependence on the DM velocity distribution in the event rate calculation enters via the halo integral, eq. (53). To get $\tilde{f}_{\text{det}}(\mathbf{v}, t)$ from the DM speed distributions in the Galactic frame we use eq. (56) to boost to the detector reference frame.³⁴ Eq. 56 depends on time, t , through the velocity of Earth in its annual orbit around the Sun.³⁵ Since we are concerned with direct detection limits and not the annual modulation signal arising from the LMC DM we present the time-averaged halo integrals where we average over a year.

Variation in the tails of the halo integral translates to changes in the exclusion limits of direct detection experiments. In order to quantify the differences between the halo integral for the total DM, MW+LMC, and the MW-only distributions we define a dimensionless metric,

$$\Delta\eta = \sum_{v_{\min}^i \geq 0.7v_{\text{esc}}^{\text{det}}} [\eta_{\text{MW+LMC}}(v_{\min}^i) - \eta_{\text{MW}}(v_{\min}^i)] \Delta v_{\min}, \quad (62)$$

where $\eta_{\text{MW+LMC}}(v_{\min}^i)$ is the midpoint value of the halo integral for the MW+LMC distribution at a bin i in $v_{\min}$, $\eta_{\text{MW}}(v_{\min}^i)$ is the MW-only distribution at the same i in $v_{\min}$, and $\Delta v_{\min}$ is the binsize. The sum is taken over all bins greater than 70% of the escape velocity in the detector frame, $v_{\text{esc}}^{\text{det}}$ ³⁶ to numerically reflect the importance of deviations in the tails of the halo integrals. Recall from Sec. 9 that greater $v_{\min}$ corresponds to a greater impact at lower DM masses on the direct detection exclusion limits.

Fig. 14 presents the time-averaged halo integrals for the four MW-LMC analog systems in Fig. 12 in the Solar region for the best-fit Sun position in the detector frame. The black and red solid line represents the linearly interpolated mean values for the halo integral calculated from the MW+LMC and MW-only DM velocity distribution in the detector frame, respectively. The red and black bands

³⁴We use $v_c = 220$ km/s when boosting to the detector frame since we are scaling all DM speed distributions to have a peak speed of 220 km/s

³⁵We neglect the small eccentricity of Earth's orbit.

³⁶Other metrics including a relative difference, the area between the halo integral curves, different fractions of $v_{\text{esc}}^{\text{det}}$, etc., with the current definition arrived at due to its correspondence with results in Sec. 12 as well as preserving the general trends with other metrics.

represent the $\pm 1\sigma$ uncertainties. Each panel in Fig. 14 contains a residual plot that shows the ratio of the MW+LMC to the MW-only halo integral at $v_{\min}^i$. Note that the ratio is undefined if the MW-only halo integral is zero for some $v_{\min}^i$.

As seen in Fig. 14 Halo 6 and Halo 15 show variation in the high $v_{\min}$ tails, with their ratios reaching ~ 7 for Halo 6 and ~ 1.5 for Halo 15. In contrast, Halos 2 and 13 show little to no variation in their halo integrals, with their ratios remaining smaller than 1.1 for all $v_{\min}$. It is interesting to note that the halo integral that shows the largest variation in the tail, Halo 6, is also the halo with the smallest κ_{LMC} , while the halo with the largest κ_{LMC} , Halo 13, shows the least variation. This fact corroborates the conclusion from Sec. 11.1 that the shape and width of the LMC DM velocity distribution determines the variation in the tails of the distribution, despite having a large κ_{LMC} . In our analysis, we find that between halos greater κ_{LMC} correlates with larger $\Delta\eta$ (variation in the tail), but that the Sun position, parameterized by the cosine angles (eq. (59)), has a pronounced effect on $\Delta\eta$ intra-halo.

We identify three factors that impact the tails of the halo integrals: the percent of DM particles in the Solar region originating from the LMC, or, κ_{LMC} ; the Sun’s position, i.e. particular cosine angle used to parameterize the Solar region; and the MW DM response to the Pericenter approach of the LMC. In the following section, we discuss how the halo integrals depend on each of the above phenomena in turn.

11.3 Halo integrals and percent LMC

DM particles from the LMC become gravitationally unbound from the LMC at infall. At any given time, some number of the total DM particles in the Solar region are DM particles that originated from the LMC. These particles have on average higher speeds than the native MW particles, leading to changes in the high-speed tails of the local DM velocity distribution, as shown in Sec. 11.1. It is expected that the greater the number of DM particles originating from the LMC in the Solar region the more pronounced will be the effect on the DM speed distribution, and therefore, on the halo integrals and direct detection limits inferred therefrom.

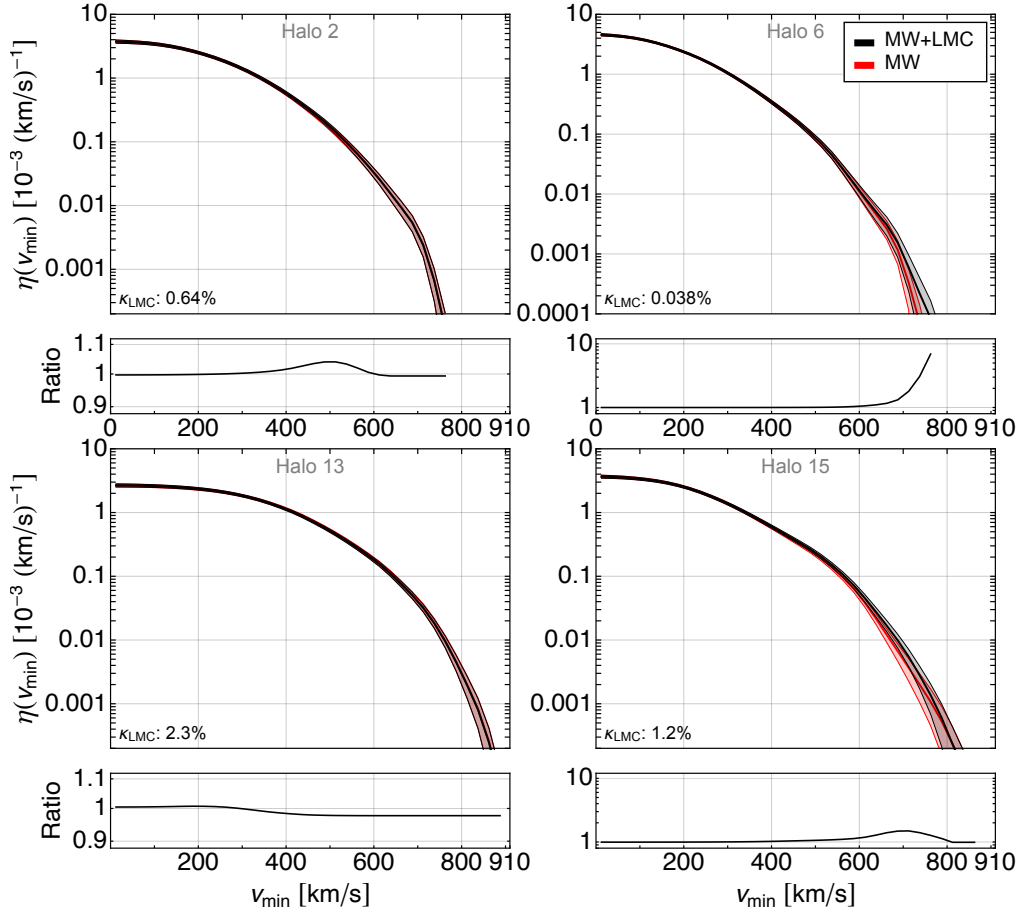


Figure 14: Time-averaged halo integrals in the Solar region for the best-fit Sun position in the detector frame for Halo 2 (upper left), Halo 6 (upper right), Halo 13 (lower left), and Halo 15 (lower right). The solid lines, black and red, represent the mean value of the halo integral for the MW+LMC and MW-only DM velocity distributions, respectively. The shaded bands are the $\pm 1\sigma$ deviation from the mean. κ_{LMC} for the best-fit Sun position is specified in the lower left corner of each panel. Below each panel is a residual plot representing the ratio of the MW+LMC to MW-only halo integrals. All plots are on a log-linear scale. From Ref. [112].

Fig. 15 shows the correlation between the metric for the variation in the tails of the halo integrals (eq 62), $\Delta\eta$, and κ_{LMC} for two populations—all 15 MW-analogs at Pericenter (left) and snapshots

from Halo 13 ranging from ~ 314 Myr prior to the present day to ~ 175 Myr (*Fut.*) after. In each panel $\Delta\eta$ is plotted for three different Solar regions: the Solar region for the best-fit Sun position (black squares), the Solar region that maximizes $\Delta\eta$ (yellow dots), and the Solar region that minimizes $\Delta\eta$ (blue dots). The Solar regions will be expounded on further in Sec. 11.4.

In general, both panels of Fig. 15 show that $\Delta\eta$ increases with increasing κ_{LMC} , with the largest $\Delta\eta$ in the upper right corner of both panels.

Owing to the general trend between $\Delta\eta$ and κ_{LMC} , we also expect that the variation in the halo integrals, and therefore the effect on the direct detection limits, will be at its highest in the snapshot where κ_{LMC} is a maximum. To investigate this, we plot in Fig. 16 the snapshots starting with ~ 314 Myr prior and up to ~ 175 Myr after *Pres.*. Each point represents a snapshot of Halo 13 in the Solar region for the best-fit Sun position. The y-axis represents the variation in the halo integral tails, $\Delta\eta$, and the color bar indicates κ_{LMC} . The vertical dashed lines represent the snapshots in Tab. 5, excluding *Iso.* since there is no LMC with which to match the Sun-LMC geometry. Fig. 16 shows that Halo κ_{LMC} in the snapshots just before *Peri.*,³⁷ with a consistent decline thereafter. As expected, the snapshots with the highest $\Delta\eta$ are also those among the highest κ_{LMC} , occurring just before *Peri.*.

In Fig. 15 there exist exceptions to the rule, i.e. halos or snapshots with a large κ_{LMC} that still exhibit low, and sometimes zero, $\Delta\eta$. One such outlier is the original Halo 13 at Pericenter. In Fig. 14 the lower left panel shows Halo 13, which has a κ_{LMC} of 2.3% and almost no variation in the tail of the halo integral between MW+LMC and MW-only. Furthermore, in Fig. 15 Halo 13 can be seen on the left panel as the black square at $\kappa_{\text{LMC}} = 2.3\%$ and $\Delta\eta = 0$. Further examination shows that the Sun position for the original Halo 13 does not have an allowed Sun position that provides a good fit to the observed Sun-LMC geometry, namely, the best-fit Sun position cosine angles for Halo 13 at Pericenter are $\cos\alpha = -0.796$ and $\cos\beta = -0.090$, compared with the observed cosine angle (eq. (60)) we note that $\cos\beta$ is not in good agreement.

That the particular Sun position impacts the degree to which the halo integral varies in its tail,

³⁷Two snapshots, ~ 174 Myr and ~ 191 Myr, prior to $t_{\text{Pres.}}$ are tied for the largest κ_{LMC} with a value of 1.0% each.

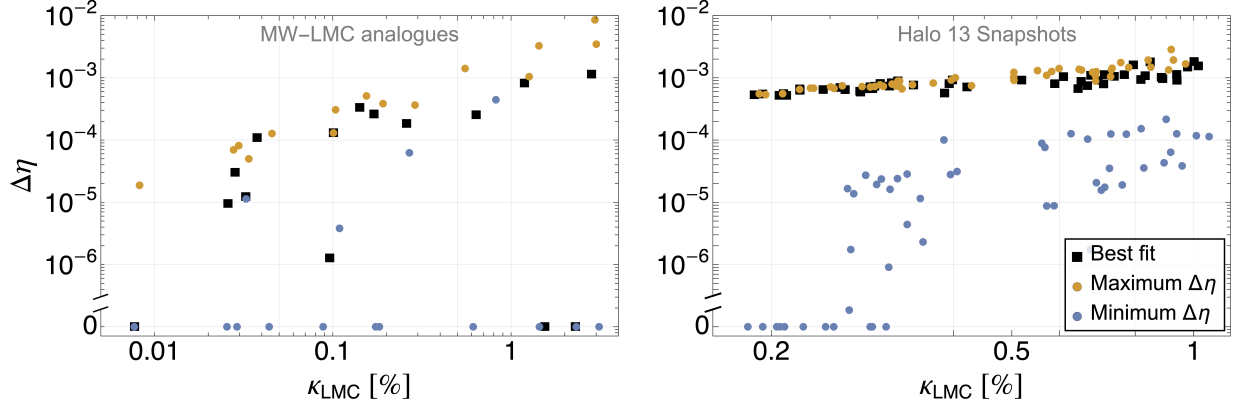


Figure 15: $\Delta\eta$ and κ_{LMC} plotted on a log-log scale for 15 MW-LMC analog systems at Pericenter (left) and resimulated Halo 13 snapshots from ~ 314 Myr before the present day to ~ 175 Myr after. Three populations are plotted for each Halo/snapshot: the Solar region for the best-fit Sun position (black square), the Solar region that maximizes $\Delta\eta$ (yellow dot), and the Solar region that minimizes $\Delta\eta$ (blue dot). From Ref. [112].

notwithstanding κ_{LMC} , can also be seen in Fig. 15 where the allowed Sun positions that maximize and minimize $\Delta\eta$ are plotted as blue and yellow dots respectively. The importance of the choice of Sun position, and whether the best-fit Sun position occupies a privileged geometry with respect to observing variations in the halo integral are the subject of the following section.

11.4 Variation due to the choice of Sun’s position

In Sec. 11.3 the variation in the tails of the halo integrals was shown to be linearly correlated with the percent LMC in the Solar region, κ_{LMC} . However, Fig. 15 made evident that among the allowed Sun positions for each Halo or snapshot there is a large deviation in $\Delta\eta$ —the allowed Sun positions that minimize $\Delta\eta$ (blue dots) are orders of magnitude lower than the Sun position that maximizes $\Delta\eta$ (yellow dots). This result is related to the discussions in Secs. 11.1 and 11.2 where the variations in the tails were seen to depend on the shape, width, and position of the LMC speed distribution with respect to the MW-only speed distribution. The choice of Sun position influences the relative position and velocity of the LMC DM with respect to the MW-only DM. In particular, recall that

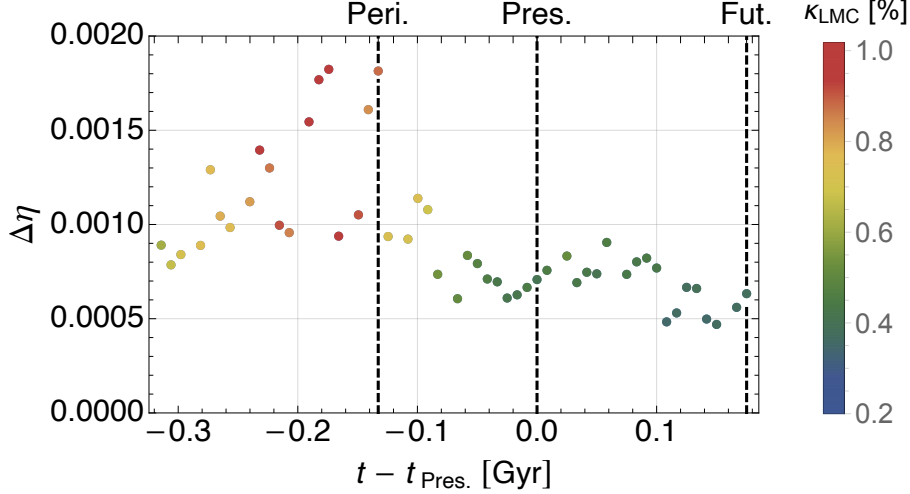


Figure 16: Halo 13 snapshots from ~ 314 Myr before the present day to ~ 175 Myr after in the Solar region for the best-fit Sun position. Snapshots are specified by their distance from the present-day snapshot, $t - t_{\text{Pres.}}$. The y-axis shows the range in $\Delta\eta$ for all snapshots, while the color bar specifies κ_{LMC} . The vertical dashed black lines represent three snapshots of interest: Pericenter (*Peri.*), present-day (*Pres.*), and future (*Fut.*). From Ref. [112].

$\cos \beta$ (eq. (59)) represents the angle between the velocity vector of the Sun with the velocity vector of the LMC. A negative $\cos \beta$ implies that the relative speeds of the LMC DM and MW DM are highest,³⁸ and therefore the variation in the tail greater.

To illustrate this point more clearly we have plotted in Fig. 17 the time-averaged halo integrals for the resimulated Halo 13 at the present-day snapshot for two Solar regions—the Solar region for the best-fit Sun position (left) and the Solar region that minimizes $\Delta\eta$ (right). The black and red solid lines are the mean values of the halo integral for the MW+LMC and MW-only DM respectively. The bands represent $\pm 1\sigma$ deviation from the mean. In the lower-left corner of each plot is the percent LMC, κ_{LMC} , and the cosine angles, $\cos \alpha$ and $\cos \beta$. Beneath each panel is a residual plot showing the ratio of MW+LMC to MW-only halo integrals. Despite κ_{LMC} being similar for both Sun positions, 0.26% compared to 0.30%, $\Delta\eta$ is considerably larger for the Solar region for the

³⁸A physical interpretation for $\cos \alpha$ is not as obvious, although analysis has shown that in terms of $\Delta\eta$ it is as important $\cos \beta$, justifying the equal treatment in using the least squares method to fit the best-fit Sun position.

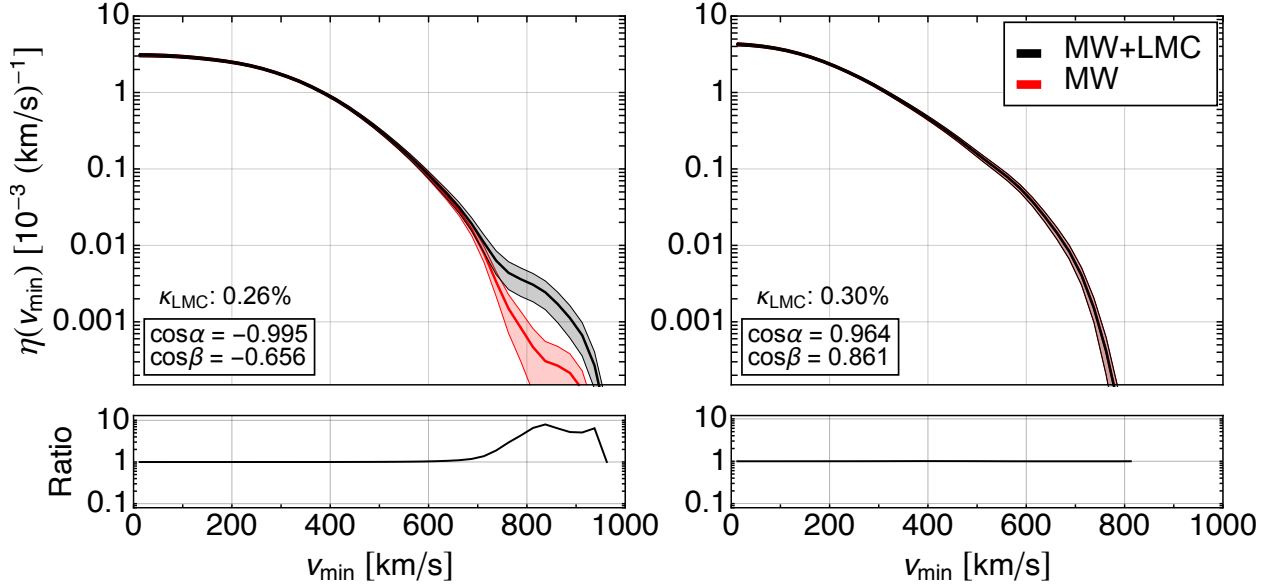


Figure 17: Halo 13 time-averaged halo integrals at the present-day snapshot in the Solar region for the best-fit Sun position (left) and the Solar region that minimizes $\Delta\eta$ (right). Black and red solid lines are the mean value of the halo integral for the MW+LMC and MW-only DM distributions respectively. The shaded bands represent $\pm 1\sigma$ standard deviation from the mean. In each plot κ_{LMC} and the cosine angles are specified in the lower left corner. Underneath each panel is a residual plot representing the ratio of the MW+LMC to the MW-only halo integral. All plots are log-linear scale. From Ref. [112].

best-fit Sun position (left), where the ratio reaches ~ 10 in the tail. Notice that the cosine angles for the Solar region that minimizes $\Delta\eta$ (right) are in the opposite direction to the best-fit cosine angles (left).

To better visualize the impact of particular cosine angles on $\Delta\eta$, Fig. 18 shows all allowed Sun positions parameterized by the cosine angles, $\cos\alpha$ and $\cos\beta$. The left plot shows the cosine angles colored by κ_{LMC} ; the right plot shows the cosine angles colored by $\Delta\eta$. In each plot, the black square represents the observed cosine angles to which the best-fit Sun position is a match. Comparing between left and right plots, we see that κ_{LMC} is at a minimum in the negative $\cos\alpha$ and $\cos\beta$ quadrant and greatest in the positive quadrant, while $\Delta\eta$ has its maximum in the negative

quadrant and minimum in the positive quadrant. This implies that, at a particular snapshot, κ_{LMC} has a sub-dominant impact on the variation in the halo integral compared to the position of the Sun. That is to say, although $\Delta\eta$ is broadly correlated with κ_{LMC} , within a snapshot³⁹ the Sun position determines the variation of the halo integral.

Fig. 18 shows the observed value of the cosine angle (eq. (60)) as a black square in each plot. For this particular snapshot, the best-fit Sun position cosine angles provide a close match to the observed values. As seen in the right plot of Fig. 18 $\Delta\eta$ is at its maximum in the negative $\cos\alpha$ and $\cos\beta$ quadrant, where the observed cosine angle lies. That $\Delta\eta$ is at a maximum in the negative $\cos\alpha$ and $\cos\beta$ quadrant is true across snapshots and for the 15 MW-LMC analog halos. Fig. 15 shows this fact well, where the best-fit Sun positions (black squares) track well with the Sun positions that maximize $\Delta\eta$ (except for a few outliers which are described above). Therefore, by virtue of the observed cosine angles being in the quadrant where $\Delta\eta$ is a maximum, we claim that the Solar region for the best-fit Sun position exists in a privileged position with respect to the LMC DM having an impact on the variation in the halo integral.

In the next section we investigate the gravitational response of the native MW DM particles to the near-orbit passage of the LMC, in the absence of the LMC DM in the Solar regions for the best-fit Sun position.

11.5 MW response to the LMC

In addition to the number of high-speed LMC DM particles in the Solar region, and the particular Sun position with respect to the LMC parameterized by the cosine angles, the MW DM velocity distribution is also impacted by a gravitational wake induced in the MW due to the passage of the LMC [71, 117]. The response of the MW to the passage of the LMC has been studied in idealized simulations [39, 40] but not in the context of fully cosmological simulations. This section presents the evidence for a gravitational wake induced by the LMC whose effect is to boost the halo integral, both of the MW-only and the MW+LMC distributions, to higher v_{min} up to the present day.

³⁹ κ_{LMC} within a snapshot varies on average 0.15% between Sun positions.

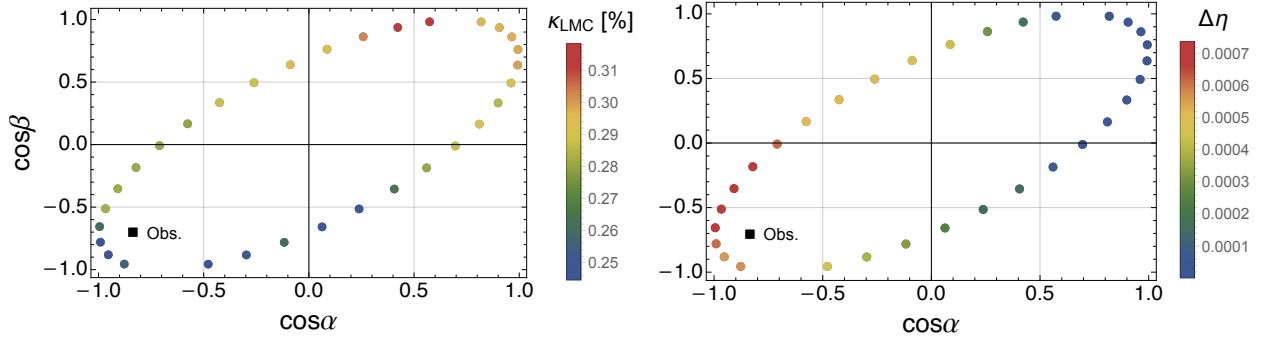


Figure 18: All allowed Sun positions parameterized by cosine angles for Halo 13 at the present-day snapshot. Colour bars represent κ_{LMC} (left) and $\Delta\eta$ (right). The black square in each plot signifies the observed values for the cosine angles (eq. (60)) to which the best-fit Sun position cosine angles are fit. From Ref. [112].

Fig. 19 displays the time-averaged halo integrals for the four snapshots of the resimulated Halo 13 in Tab. 5. For each snapshot, the MW+LMC and MW-only DM distributions (solid and dashed curves, respectively) are plotted, except *Iso.* which does not have an LMC analog. All snapshots are in the Solar region for the best-fit Sun position, except for *Iso.* which does not have an LMC with which to match Sun-LMC geometry and therefore is presented in a 6 – 10 kpc shell centered on the Galactic Center. Furthermore, the halo integral for the MW SHM defined by eq. (58) is plotted for comparison using $v_c = 220$ km/s and $v_{\text{esc}} = 544$ km/s and is shown as the blue solid curve in Fig. 19.

Comparing first the SHM with *Iso.* snapshot shows that the proxy for the MW analog in the absence of the LMC is in near agreement with the SHM for the MW halo integral. Both tails for the SHM and *Iso.* extend to ~ 800 km/s in v_{min} . However, the near passage of the LMC induces a large boost in v_{min} to the MW-only distribution at Pericenter (green dashed curve) up to 900 km/s in v_{min} , ~ 100 km/s greater than the *Iso.* snapshot. The boost in v_{min} of the MW-only distribution is sustained up to the *Pres.* snapshot (dashed orange curve), and relaxes back to the *Iso.* level by the *Fut.* snapshot (magenta dashed curve).

The addition of the high-speed LMC DM to the Solar region serves to boost the halo integrals

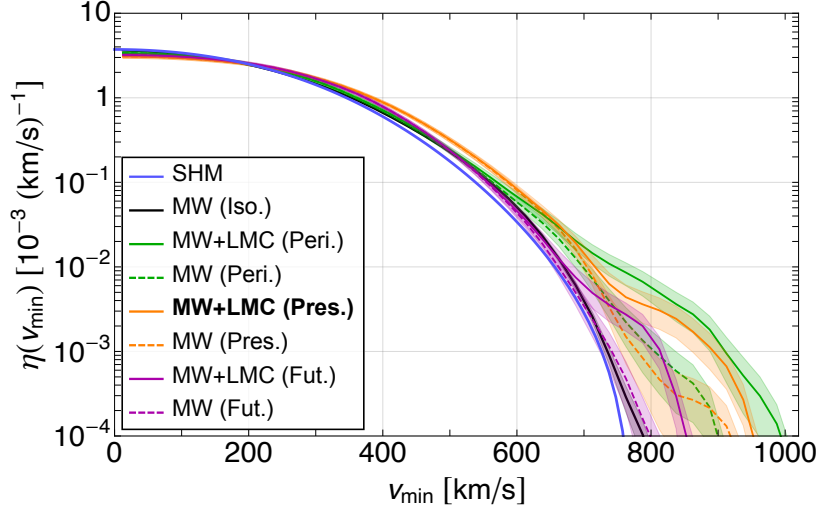


Figure 19: Halo 13 time-averaged halo integrals for the four snapshots of interest: *Iso.*, isolated MW analog (black solid line); *Peri.*, the Pericenter approach (green lines); *Pres.*, the present-day simulation equivalent (orange lines); and *Fut.*, the future snapshot (magenta lines). Each snapshot, excluding *Iso.*, shows the halo integral for the MW+LMC distribution (solid lines) and the MW-only distribution (dashed). The bands represent $\pm 1\sigma$ standard deviation from the mean. All snapshots are in the Solar region for the best-fit Sun position except *Iso.* which is in a $6 - 10$ kpc shell from the Galactic Center. The analytic SHM for the MW with $v_c = 220$ km/s and $v_{\text{esc}} = 544$ km/s is shown as the blue solid curve. From Ref. [112].

still further, with the largest effect occurring for the *Peri.* snapshot where the MW+LMC curve is boosted to ~ 1000 km/s in v_{min} , ~ 100 km/s greater than its MW-only counterpart. While smaller but still significant boosts in v_{min} are observed for *Pres.* and *Fut.*

Fig. 19 demonstrates that in the full cosmological simulations the response of the MW due to the passage of the LMC at Pericenter and extending to present-day is as, or nearly as, significant as the high-speed LMC DM in the Solar region in influencing the tails of the halo integrals. The combined effect of the LMC on the DM in the Solar region of the MW is to boost its halo integral up to ~ 200 km/s in v_{min} , in the *Peri.* snapshot, and more than 150 km/s in v_{min} by the *Pres.* snapshot.

12 Implications for dark matter direct detection

The previous section showed that the halo integrals for MW-LMC analog systems can be boosted to higher $v_{\min}$ due to both the presence of high-speed DM particles originating from the LMC at infall in the Solar neighborhood and to the gravitational wake induced by the Pericenter approach of the LMC. Fig. 19 showed that the combined boost in $v_{\min}$ of the halo integral can be up to ~ 150 km/s at present-day in the Solar region for the best-fit Sun position with respect to the isolated MW and the SHM. The SHM adopted in direct detection experiments assumes a MW DM halo in absence of the LMC, which leads to underestimations of the halo integrals at large $v_{\min}$, and therefore lower sensitivities to low DM masses, as discussed in Sec. 9, in the scattering cross-section and DM mass parameter space.

In this section the halo integrals studied in Sec. 11.2 are used in computing the signals from three idealized direct detection experiments based on near-future detectors. To simulate the constraints in the DM-SM scattering cross-section and DM mass plane for DM-nucleus and DM-electron recoils we employ the Poisson likelihood method used in the DDCalc [118] and QEDark [119, 120] packages respectively. To determine the direct detection exclusion limits at a particular confidence level we use the properties of existing or future experiments and the local DM properties extracted from the Auriga simulation described in Sec. 11.

12.1 DM-nucleus exclusion limits

The differential event rate for DM-nucleus scattering in terms of the halo integral is given in eq. (52). To analyze LMC's impact on the direct detection exclusion limits we consider xenon and germanium-based experiments, based on the near-future sensitivity of LUX-ZEPLIN (LZ) [121] and SuperCDMS [122]. These two experiments are chosen because LZ is designed to have large exposures and is sensitive to large WIMP DM masses and low cross-section, while SuperCDMS is sensitive to sub-GeV DM, together probing a large range of DM masses.

For the xenon-based experiment, we take the following specifications: an energy range of $[2 -$

50]keV, an energy resolution of $\sigma_E = 0.065E_R + 0.24 \text{ keV} \sqrt{E_R/\text{keV}}$ [123], with an exposure of $5.6 \times 10^6 \text{ kg days}$ at 50% efficiency [121]. This exposure is expected to be achieved by LZ in 5 years of operation.

For the germanium-based experiment, we consider a low-energy and high-energy threshold crystal target separately. For the low energy threshold target we choose an energy range informed by the projected high-voltage (HV) detector from SuperCDMS [122] in the energy range $[40 - 300] \text{ eV}$ with a constant signal efficiency of 85%, an exposure of $1.6 \times 10^4 \text{ kg days}$ with a flat background of $10 \text{ keV}^{-1} \text{ kg}^{-1} \text{ days}^{-1}$ [122, 124]. For the high energy threshold target we choose the energy range $[3 - 30] \text{ keV}$, an exposure of $2.04 \times 10^4 \text{ kg days}$, a flat efficiency of 75%, and 1 expected background event. This exposure is expected to be achieved by SuperCDMS after 5 years of operation [122].

Figs. 20 and 21 show the projected exclusion limits at the 90% confidence level in the DM mass and spin-independent DM-nucleon cross-section parameter space for the xenon and germanium-based experiments respectively. The left panel shows the full parameter space in DM mass and cross-section and the right panel shows only the lower DM mass range. Exclusion curves are plotted for the SHM (blue solid) and the four representative snapshots from the resimulated Halo 13 defined in Tab. 5. Each exclusion curve is computed using the halo integrals in Fig. 19, with the solid and dashed curves representing the mean values and the shaded region representing $\pm 1\sigma$ standard deviations from the mean. All snapshots are in the Solar region for the best-fit Sun position, with the exception of *Iso.* which is in a Solar region defined by a $6 - 10 \text{ kpc}$ shell centered on the Galactic Center. In order to compare exclusion curves with equal normalization the local DM density, ρ_χ , is set to 0.3 GeV/cm^3 as in the SHM.⁴⁰ Beneath each panel the ratio for the MW+LMC and MW-only DM distributions are plotted for snapshots that have an LMC component.

By virtue of $v_{\min}$ increasing for low DM masses, the event rate becomes sensitive to small changes in the tails of halo integrals. The variations in the halo integrals seen in Fig. 19 are reflected in the shifts of the concomitant exclusion curves in Figs. 20 and 21, with a pronounced effect on the low

⁴⁰Recall that ρ_χ is a normalization constant in the rate equation and results in shifting the exclusion curves in cross-section.

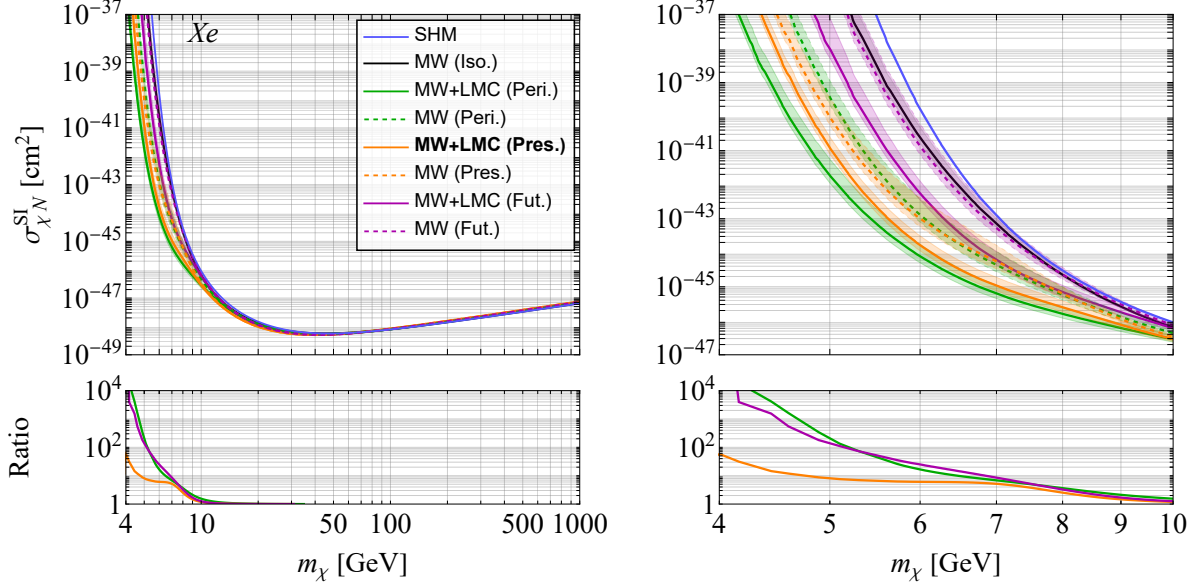


Figure 20: Projected exclusion limit curves at 90% confidence in the spin-independent DM-nucleon cross-section and DM mass parameter space for four representative snapshots in time of the MW-LMC analog Halo 13: *Iso.*, the MW analog in isolation from the influence of the LMC analog; *Peri.*, the LMC at its Pericenter approach to the MW analog; *Pres.* the MW-LMC analog system at its present-day equivalent to the observed MW-LMC system; and *Fut.*, the MW-LMC analog system at a future point in time. For each snapshot excluding *Iso.* the solid line is the total DM, i.e. MW+LMC, in the Solar region for the best-fit Sun position and the dashed line is the MW-only distribution in the same Solar region. The *Iso.* snapshot (solid black) shows the MW DM in a 6 – 10 kpc shell centered on the Galactic Center. The SHM exclusion curve is also plotted (blue solid) for reference. The right panel is the same as the left but zoomed into the low DM mass range. Beneath each panel is a residual plot showing the ratio of the MW+LMC to the MW-only distributions, for the snapshots that have both. From Ref. [112].

DM mass region shown in the right panels. For both xenon and germanium-based experiments the effect of the LMC is to shift the exclusion curves to lower cross-sections and hence higher sensitivities to low DM masses. For both experiments, the weakest exclusion limits come from the SHM and the isolated MW, and the strongest from the MW+LMC at Pericenter, whose halo integral has the

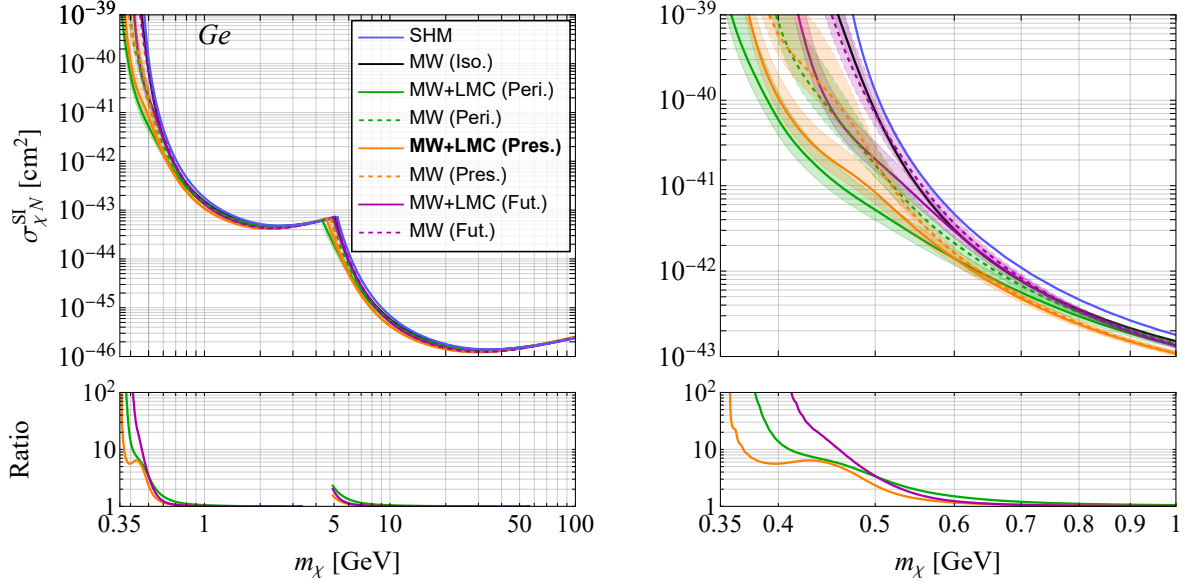


Figure 21: Same as Fig. 20, but for a future germanium based experiment. From Ref. [112].

largest boost in $v_{\min}$.

Exclusion curves for the xenon-based experiment in Fig. 20 show that at present-day the MW+LMC (orange solid) distribution can be up to three orders of magnitude lower than the isolated MW (black solid), and the SHM (blue solid), at $m_\chi \sim 6$ GeV, and one order of magnitude at $m_\chi \sim 8$ GeV. Furthermore, holding the cross-section fixed, the impact of the LMC is to shift all snapshots towards lower DM masses, extending the detector sensitivity for low DM masses.

A similar conclusion for the germanium-based experiment can be drawn from Fig. 21 where the mass scale probes down to ~ 0.35 GeV. The exclusion curves for the present-day MW+LMC distribution (orange solid) as compared to the isolated MW (black solid) shift toward lower cross-sections beginning at $m_\chi \sim 1$ GeV, extending to an order of magnitude at $m_\chi \sim 0.5$ GeV, and reaching up to three orders of magnitude at $m_\chi \sim 0.4$ GeV. Additionally, the impact of the LMC is seen to, holding the cross-section fixed, shift the sensitivity toward lower DM masses as was the case with the xenon experiment.

These results cohere with those found in idealized simulations [40], confirming that the LMC impacts the sensitivity of low DM mass models in the context of full cosmological simulations.

12.2 DM-electron exclusion limits

We consider a silicon-based experiment wherein the DM scatters with an electron as opposed to a nucleus. The sensitivities considered are based on the next generation DAMIC-M experiment [125, 126], and probe the DM mass in the MeV range. We choose an exposure of 1 kg year assuming no background events and an ionization threshold of 1 electron-hole pair.

Fig. 22 shows the projected exclusion limits at the 95% confidence level in the DM mass and DM-electron cross-section parameter space for the three functional forms for the form factors, F_{DM} : $F_{\text{DM}} = 1$ (left), $F_{\text{DM}} \propto q^{-1}$ (middle), and $F_{\text{DM}} \propto q^{-2}$ (right). Each panel contains the SHM reference curve (blue solid) and the four representative MW-LMC analog snapshots from the resimulated Halo 13 defined in Tab. 5. Excluding *Iso.* the solid curves represent the MW+LMC DM distribution while the dashed represent the MW-only distribution, both in the Solar region for the best-fit Sun position. The *Iso.* snapshot is in a Solar region defined as a 6 – 10 kpc shell centered on the Galactic Center. The bands represent $\pm 1\sigma$ standard deviation from the mean. In order to compare all snapshots with equal normalization the DM density, ρ_χ , is taken to be a constant $0.3 \text{ GeV}/\text{cm}^3$, as in the SHM. Beneath each panel is a ratio plot for the MW+LMC and MW-only DM distributions for snapshots that have an LMC component

Fig. 22 shows the same trends as in the xenon and germanium detectors, i.e. the exclusion curve shifts to lower cross-sections and smaller DM masses. For all three form factors, the exclusion curves for the present-day snapshot begin to diverge from the *Iso.* and SHM curves at a DM mass of $\sim 5 \text{ MeV}$ and below. The effect is not as pronounced as in Figs. 20 and 21 but can still be up to a factor of ~ 4 for a $m_\chi \sim 1 \text{ MeV}$, and up to ~ 50 for $m_\chi \sim 0.6 \text{ MeV}$. For all choices of DM form factor, the presence of the LMC at fixed cross-sections shifts the exclusion limits toward smaller DM masses, extending the detector sensitivity by fractions of MeV for low DM masses.

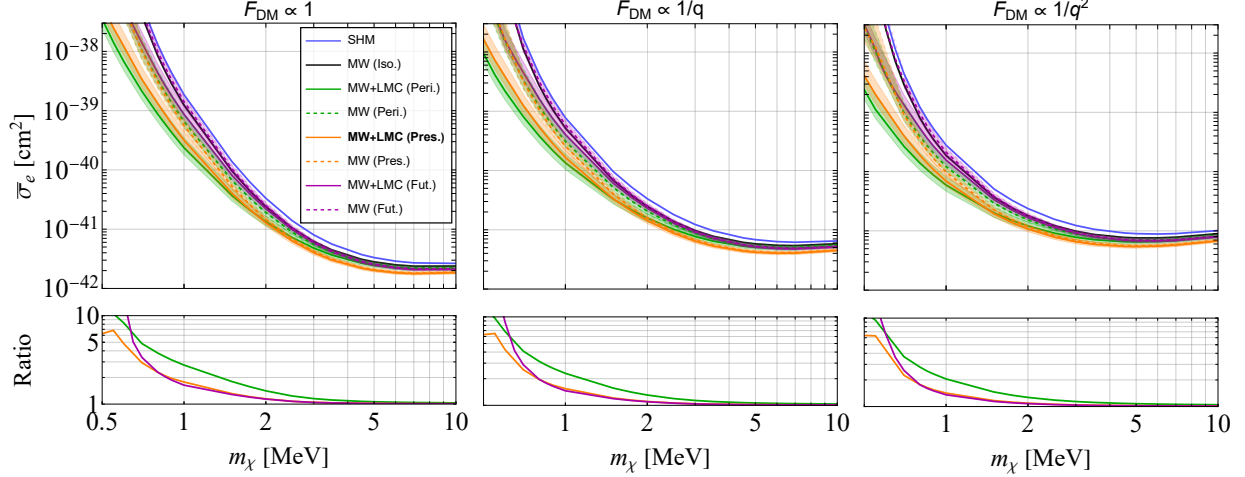


Figure 22: Projected exclusion limit curves at 95% confidence for a silicon-based experiment in the DM-electron cross-section and DM mass parameter space for four representative snapshots in time of the MW-LMC analog Halo 13: *Iso.*, the MW analog in isolation from the influence of the LMC analog; *Peri.*, the LMC at its Pericenter approach to the MW analog; *Pres.* the MW-LMC analog system at its present-day equivalent to the observed MW-LMC system; and *Fut.*, the MW-LMC analog system at a future point in time. For each snapshot excluding *Iso.* the solid line is the total DM, i.e. MW+LMC, in the Solar region for the best-fit Sun position and the dashed line is the MW-only distribution in the same Solar region. The *Iso.* snapshot (solid black) shows the MW DM in a 6 – 10 kpc shell centered on the Galactic Center. The SHM exclusion curve is also plotted (blue solid) for reference. The three panels represent different choices for the form factor, F_{DM} : $F_{DM} = 1$ (left), $F_{DM} \propto q^{-1}$ (middle), and $F_{DM} \propto q^{-2}$ (right). Beneath each panel is a residual plot showing the ratio of the MW+LMC to the MW-only distributions, for the snapshots that have both. From Ref. [112].

13 Conclusion

In Part 2 of this thesis we analyzed the Aurgia suite of magneto-hydrodynamical simulations with a CDM cosmology and realistic baryonic physics to determine the effects of satellites like the LMC on the local properties of host halos like the MW. We identified 15 MW-LMC analog systems by matching the satellite stellar mass and distance at Pericenter to the observed values for the LMC. We then matched the Sun-LMC geometry and identified allowed Sun positions in the simulations, with the best-fit Sun position being the one that matched with the observed values for the cosine angles between the Sun’s velocity vector and the position and velocity of the LMC with respect to the MW.

We then extracted the local DM velocity distributions for the 15 MW-LMC candidate analogs and for one halo (Halo 13), resimulated for finer snapshots near Pericenter. Using the velocity distributions we computed the halo integrals for the candidate halos and four snapshots from Halo 13. Using the halo integrals as a proxy for the effect of the direct detection signals, we then analyzed the impact on the tails of the halo integral due to the Pericenter approach of the LMC. We found that three factors determined the degree to which the tail of the halo integral varied: the percent of LMC DM particles in the Solar region, the particular choice of Sun position, with those close to the best-fit (observed) values having the most pronounced effect, and the gravitational boost to the native MW particles due to the wake induced by the LMC.

These factors were then seen to have the combined effect of shifting the expected exclusion curve of near-future direct detection experiments searching for nuclear or electron recoils due to scattering with DM toward lower cross-sections and smaller DM matter masses, increasing their sensitivities.

Our results are largely consistent with the findings of Ref. [40] and underscore the importance of the LMC’s impact on the local DM distribution in a fully cosmological context for direct detection experiments of near-future detectors.

Appendix

A General relaxation method for solving ODEs

Consider a system of coupled first-order differential equations,

$$\mathbf{y}'(x) + \mathbf{F}(\mathbf{y}, x, \mathbf{p}) = 0, \quad (\text{A.1})$$

where $\mathbf{y}'(x)$ denotes the first derivative with respect to x , $\mathbf{y}$ and $\mathbf{F}$ are vectors of n functions, and $\mathbf{p}$ is a vector of m constant parameters. To solve eq. (A.1) for $\mathbf{y}$ and $\mathbf{p}$ simultaneously we need to satisfy $n + m$ boundary conditions.

We start by discretizing our solution space into N points, $x_1, x_2, \dots, x_N$. The functions to be solved for are then,

$$\mathbf{y}_i = \mathbf{y}(\mathbf{x}_i), \quad (\text{A.2})$$

where i corresponds to a particular x . We define a new vector, $\mathbf{E}_i$, using the finite difference method we write eq. A.1 in the discretized form,

$$\mathbf{E}_i(\mathbf{y}_{i+1}, \mathbf{y}_i, \mathbf{p}) = \mathbf{y}_{i+1} - \mathbf{y}_i + (x_{i+1} - x_i) \mathbf{F}(\bar{\mathbf{y}}_i, \bar{x}_i, \mathbf{p}), \quad (\text{A.3})$$

where $\bar{\mathbf{y}}_i$ and $\bar{x}_i$ denote the midpoint value between grid points $i + 1$ and i .

After discretization of eq. (A.1) we are left with $N \times n + m$ unknowns to be solved. By setting $\mathbf{E}_1 = \dots = \mathbf{E}_{N-1} = 0$ we have $N - 1 \times n$ equations and $n + m$ boundary conditions, enabling a solution to the system of coupled equations. If the ODEs were linear in $\mathbf{y}$ we could solve by matrix inversion. However, the relaxation method accommodates a nonlinear function $\mathbf{F}$ in $\mathbf{y}$ and $\mathbf{p}$.

The relaxation method proceeds by starting with an initial guess for the unknowns $\mathbf{y}$ and $\mathbf{p}$ and iterating toward a better solution by calculation shifts in the solution space to arrive at a new guess, i.e.,

$$\mathbf{y}_i \longrightarrow \mathbf{y}_i^{\text{new}} = \mathbf{y}_i + \Delta \mathbf{y}_i, \quad \mathbf{p} \longrightarrow \mathbf{p}^{\text{new}} = \mathbf{p} + \Delta \mathbf{p}. \quad (\text{A.4})$$

This process is repeated until $\mathbf{y}$ and $\mathbf{p}$ provide a converged solution.

In order to calculate the appropriate shifts, $\Delta \mathbf{y}_i$ and $\Delta \mathbf{p}$, for each iteration such that we approach a solution, that is, $\mathbf{E}_i(\mathbf{y}_{i+1}^{\text{new}}, \mathbf{y}_i^{\text{new}}, \mathbf{p}^{\text{new}}) = 0$, we approximate the non-linear functions in $\mathbf{E}_i$ by linearizing in the shifts. This gives,

$$\begin{aligned} E_{ai}(\mathbf{y}_{i+1}^{\text{new}}, \mathbf{y}_i^{\text{new}}, \mathbf{p}^{\text{new}}) &\approx E_{ai}(\mathbf{y}_{i+1}, \mathbf{y}_i, \mathbf{p}) + \sum_{b,j} \frac{\partial E_{ai}}{\partial y_{bj}} \Delta y_{bj} + \sum_q \frac{\partial E_{ai}}{\partial p_q} \Delta p_q \\ &\approx E_{ai}(\mathbf{y}_{i+1}, \mathbf{y}_i, \mathbf{p}) + \Delta y_{a,i+1} - \Delta y_{ai} + (x_{i+1} - x_i) \\ &\quad \left(\sum_b \frac{\partial F_a}{\partial y_b} \frac{\Delta y_{b,i+1} + \Delta y_{bi}}{2} + \sum_q \frac{\partial F_a}{\partial p_q} \Delta p_q \right), \end{aligned} \quad (\text{A.5})$$

where a and b label the components of $\Delta \mathbf{y}_i$ and $\mathbf{F}$, and q labels the component of $\mathbf{p}$.

To solve we cast eq. (A.5) into a matrix equation,

$$\mathbf{M} \Delta \mathbf{Y} = -\mathbf{E} \quad (\text{A.6})$$

where $\Delta \mathbf{Y}$ is the $N \times n + p$ component vector,

$$\Delta \mathbf{Y} = \begin{pmatrix} \Delta \mathbf{y}_1 \\ \vdots \\ \Delta \mathbf{y}_N \\ \Delta \mathbf{p} \end{pmatrix}. \quad (\text{A.7})$$

Therefore, to solve eq. (A.6) by matrix inversion we require that $\mathbf{M}$ be a square matrix of dimension $(N \times n + m) \times (N \times n + m)$, and $\mathbf{E}$ a vector with dimension equal to $\Delta \mathbf{y}$. To get the correct dimensions we define the boundary conditions as,

$$\mathbf{H}(\mathbf{y}_1, \mathbf{y}_N, \mathbf{p}) = 0, \quad (\text{A.8})$$

where $\mathbf{H}$ is an $(n + m)$ vector. Approximating $\mathbf{H}$ by linearizing as before,

$$H_\alpha(\mathbf{y}_1^{\text{new}}, \mathbf{y}_N^{\text{new}}, \mathbf{p}^{\text{new}}) \approx H_\alpha(\mathbf{y}_1, \mathbf{y}_N, \mathbf{p}) + \sum_a \left(\frac{\partial H_\alpha}{\partial y_{a1}} \Delta y_{a1} + \frac{\partial H_\alpha}{\partial y_{aN}} \Delta y_{aN} \right) + \sum_q \frac{\partial H_\alpha}{\partial p_q} \Delta p_q, \quad (\text{A.9})$$

where α label the components of $\mathbf{H}$.

We can now define our invertible square matrix, $\mathbf{M}$, by

$$\mathbf{M} = \begin{pmatrix} \underbrace{-\mathbb{1}_n + \mathbb{A}_1}_{n \text{ cols}} & \underbrace{\mathbb{1}_n + \mathbb{A}_1}_{n \text{ cols}} & \underbrace{\mathbb{O}_{n \times n}}_{n \text{ cols}} & \dots & \underbrace{\mathbb{O}_{n \times n}}_{n \text{ cols}} & \underbrace{\mathbb{O}_{n \times n}}_{n \text{ cols}} & \underbrace{\mathbb{B}_1}_{m \text{ cols}} \\ \mathbb{O}_{n \times n} & -\mathbb{1}_n + \mathbb{A}_2 & \mathbb{1}_n + \mathbb{A}_2 & \mathbb{O}_{n \times n} & \dots & \mathbb{O}_{n \times n} & \mathbb{B}_2 \\ & & \ddots & \ddots & & & \vdots \\ \mathbb{O}_{n \times n} & \mathbb{O}_n & \dots & \mathbb{O}_{n \times n} & -\mathbb{1}_n + \mathbb{A}_{N-1} & \mathbb{1}_n + \mathbb{A}_{N-1} & \mathbb{B}_{N-1} \\ \underbrace{\mathbb{C}_1}_{n \text{ cols}} & \underbrace{\mathbb{O}_{(n+m) \times n}}_{n \text{ cols}} & \dots & \dots & \underbrace{\mathbb{O}_{(n+m) \times n}}_{n \text{ cols}} & \underbrace{\mathbb{C}_N}_{n \text{ cols}} & \underbrace{\mathbb{D}}_{m \text{ cols}} \end{pmatrix} \begin{matrix} \} n \text{ rows} \\ \} n \text{ rows} \\ \vdots \\ \} n \text{ rows} \\ \} n + m \text{ rows} \end{matrix} \quad (\text{A.10})$$

where $\mathbb{1}_n$ are identity matrices, $\mathbb{O}_{n \times n}$ 0 matrices, $\mathbb{A}$ and $\mathbb{B}$ are matrix blocks defined,

$$(\mathbb{A}_i)_{ab} = \frac{1}{2}(x_{i+1} - x_i) \left. \frac{\partial F_a}{\partial y_b} \right|_{\substack{\mathbf{y}=\bar{\mathbf{y}}_i \\ x=\bar{x}_i}} \quad \text{and} \quad (\mathbb{B}_i)_{aq} = (x_{i+1} - x_i) \left. \frac{\partial F_a}{\partial p_q} \right|_{\substack{\mathbf{y}=\bar{\mathbf{y}}_i \\ x=\bar{x}_i}}, \quad (\text{A.11})$$

from $\mathbf{E}_{\mathbf{ai}}$ and $\mathbb{C}$ and $\mathbb{D}$ come from $\mathbf{H}$, defined,

$$(\mathbb{C}_1)_{\alpha a} = \frac{\partial H_\alpha}{\partial y_{a1}}, \quad (\mathbb{C}_N)_{\alpha a} = \frac{\partial H_\alpha}{\partial y_{aN}}, \quad (\mathbb{D})_{\alpha q} = \frac{\partial H_\alpha}{\partial p_q}. \quad (\text{A.12})$$

Adding $\mathbf{H}$ to the $\mathbf{E}$ vector we have, finally,

$$\mathbf{E} = \begin{pmatrix} \mathbf{E}_1 \\ \vdots \\ \mathbf{E}_{N-1} \\ \mathbf{H} \end{pmatrix}. \quad (\text{A.13})$$

Therefore, at each iteration the relaxation method takes an initial guess for $\mathbf{y}_i$ and $\mathbf{p}$ and computes the $\mathbf{M}$ matrix and $\mathbf{E}$ vector and solves for the shifts via,

$$\Delta \mathbf{Y} = -\mathbf{M}^{-1} \mathbf{E}. \quad (\text{A.14})$$

The solutions are updated through eq. (A.4) and checked for convergence. If the new solutions do not satisfy eq. (A.3) the process is repeated until the solutions converge.

B Spherical Jeans model without baryons

The case of the spherical Jeans model without baryons serves as an initial guess for the nonspherical Jeans model relaxation code. The solutions for the simplified case are presented here.

From eqs. (3) and (5), neglecting the terms due to baryons, we have two first-order equations for the DM density, $\rho_{\text{dm}}(r)$, and enclosed mass, $M_{\text{dm}}(r)$, i.e.,

$$\frac{\partial \rho_{\text{dm}}}{\partial r} = -\frac{G\rho_{\text{dm}}}{\sigma_0^2 r^2} M_{\text{dm}}, \quad \frac{\partial M_{\text{dm}}}{\partial r} = 4\pi r^2 \rho_{\text{dm}}. \quad (\text{B.1})$$

Eq. (B.1) can then be expressed as a system of first-order equations for two new functions $h(x)$, $j(x)$ by a change of variables,

$$\rho_{\text{dm}}(r) = \rho_0 e^{h(x)}, \quad M_{\text{dm}}(r) = \frac{4\pi}{3} \rho_0 r^3 e^{j(x)}, \quad (\text{B.2})$$

where $x = r/r_0$, with central density $\rho_0 = \rho_{\text{dm}}(0)$, and characteristic radius $r_0 = \sigma_0/\sqrt{4\pi G\rho_0}$.

Rewriting the differential equations in terms of these functions we have,

$$\frac{\partial h}{\partial x} + \frac{x}{3} e^{j(x)} = 0, \quad \frac{\partial j}{\partial x} + \frac{3}{x} (1 - e^{h(x)-j(x)}) = 0. \quad (\text{B.3})$$

The boundary conditions for these functions are such that $h(0) = j(0) = 0$. We then can solve for eq. (B.3), noting that the solutions for $h(x)$ and $j(x)$ do not depend on additional inputs for ρ_{dm} and M_{dm} and therefore need only be solved once.

Here, as in the nonspherical case, we follow the outside-in approach by fixing the boundary conditions at r_m ,

$$\rho_{\text{dm}}(r_m) = \rho_m, \quad M_{\text{dm}}(r_m) = M_m, \quad (\text{B.4})$$

where $\rho_m = \rho_{\text{NFW}}(r_m)$, $M_m = M_{\text{NFW}}(r_m)$ are determined by the NFW profile of the outer halo, and iteratively solving for r_0 and σ_0 in the inner halo.

Defining the ratio

$$\mathcal{R} = \frac{M_m}{4\pi r_m^3 \rho_m}, \quad (\text{B.5})$$

the matching condition at r_m can be expressed as a single condition

$$\frac{M_{\text{dm}}(r_m)}{4\pi r_m^3 \rho_{\text{dm}}(r_m)} = \frac{1}{3} e^{j(x_m)-h(x_m)} = \mathcal{R}, \quad (\text{B.6})$$

where $x_m = r_m/r_0$. Eq. (B.6) is solved numerically to determine x_m . The unknowns are solved via,

$$r_0 = r_m/x_m, \quad \rho_0 = \rho_m e^{-h(x_m)}, \quad \sigma_0 = \sqrt{4\pi G\rho_0 r_0^2}, \quad (\text{B.7})$$

and translated back into $\rho_{\text{dm}}(r)$, $M_{\text{dm}}(r)$ for the initial guess for the nonspherical Jeans model.

C Halo shape parameters

To calculate the halo shape from the numerical solutions to the nonspherical Jeans model, we start by noting that the equation of a spheroid in two dimensions is,

$$\frac{R^2}{a^2} + \frac{z^2}{c^2} = 1, \quad (\text{C.1})$$

where the ratio of the semi-minor axis, c , to the semi-major axis, a , gives a measure for the sphericity of the halo, where $c/a = 1$ implies a perfect sphere, $c/a < 1$ an oblate spheroid (flattened along z), and $c/a > 1$ a prolate spheroid (elongated along z).

Let $q = c/a$ and we can define an effective radius such that the volume of a spheroid is $\frac{4\pi}{3}a^2c = \frac{4\pi}{3}r_{\text{eff}}^3$, where r_{eff} is,

$$r_{\text{eff}} = \sqrt{R^2 q^{2/3} + z^2 q^{-4/3}}. \quad (\text{C.2})$$

To calculate q we begin by computing the moment-of-inertia tensor for a thin shell about a shell's principle axes,

$$I_{ij} = \int_{\text{shell}} d^3r r_i r_j \rho_{\text{dm}}(\mathbf{r}), \quad (\text{C.3})$$

where the indices i, j label three Euclidean axes (x, y, z) and the integral is taken over a thin shell of thickness Δr_{eff} for a given r_{eff} and q . For the case with azimuthal symmetry eq. (C.3) becomes diagonal with $I_{xx} = I_{yy} = I_{RR}$ and I_{zz} . On a spheroidal shell the radius is then,

$$r(\theta) = \frac{r_{\text{eff}}}{\sqrt{\sin^2 \theta q^{4/3} + \cos^2 \theta q^{-2/3}}}, \quad (\text{C.4})$$

with the thickness,

$$\Delta r(\theta) = \frac{\Delta r_{\text{eff}}}{\sqrt{\sin^2 \theta q^{4/3} + \cos^2 \theta q^{-2/3}}} = \frac{\Delta r_{\text{eff}}}{r_{\text{eff}}} \times r(\theta). \quad (\text{C.5})$$

The moment-of-inertia tensor is then,

$$I_{RR} = \int_{\text{shell}} dr d\theta r^4 \sin^3 \theta \rho_{\text{dm}}(r, \theta) = \frac{\pi \Delta r_{\text{eff}}}{r_{\text{eff}}} \int_0^\pi d\theta r(\theta)^5 \sin^3 \theta \rho(r(\theta), \theta) \quad (\text{C.6})$$

$$I_{zz} = \int_{\text{shell}} dr d\theta r^4 \sin \theta \cos^2 \theta \rho_{\text{dm}}(r, \theta) = \frac{2\pi \Delta r_{\text{eff}}}{r_{\text{eff}}} \int_0^\pi d\theta r(\theta)^5 \sin \theta \cos^2 \theta \rho(r(\theta), \theta). \quad (\text{C.7})$$

The shape q (or c/a) is then recovered in the usual way,

$$q = \sqrt{I_{zz}/I_{RR}} . \quad (\text{C.8})$$

For the numerical profiles, given r_{eff} , we start with an initial guess of $q = 1$ and compute the new q via eq. (C.8), using this q as a new initial guess and iterating the process until convergence is achieved.

D Tesseral spherical harmonics

The tesseral harmonics are a set of real-valued spherical harmonics defined as a linear combination of the usual spherical harmonics, Y_ℓ^m , i.e.,

$$Z_{\ell m} = \begin{cases} \frac{1}{\sqrt{2}} (Y_\ell^{-m} + (-1)^m Y_\ell^m) & m > 0 \\ Y_\ell^0 & m = 0 \\ \frac{i}{\sqrt{2}} (Y_\ell^m - (-1)^m Y_\ell^{-m}) & m < 0 \end{cases} , \quad (\text{D.1})$$

where $(-1)^m$ is the Condon-Shortley phase and is used for consistency with associated Legendre polynomials, i.e.,

$$P_\ell^m(x) = (-1)^m (1-x^2)^{m/2} \frac{d^m}{dx^m} P_\ell(x) , \quad (\text{D.2})$$

in terms of which the tesseral harmonics may also be defined,

$$Z_{\ell m}(\theta, \varphi) = \begin{cases} (-1)^m \sqrt{\frac{2(2\ell+1)}{4\pi}} \sqrt{\frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(\cos \theta) \cos(m\varphi) & m > 0 \\ \sqrt{\frac{2\ell+1}{4\pi}} P_\ell^0(\cos \theta) & m = 0 \\ (-1)^m \sqrt{\frac{2(2\ell+1)}{4\pi}} \sqrt{\frac{(\ell-|m|)!}{(\ell+|m|)!}} P_\ell^{|m|}(\cos \theta) \sin(|m|\varphi) & m < 0 \end{cases} , \quad (\text{D.3})$$

The tesseral harmonics form a complete orthonormal basis, i.e.,

$$\int d\Omega Z_{\ell m}(\theta, \varphi) Z_{\ell' m'}(\theta, \varphi) = \delta_{\ell\ell'} \delta_{mm'} . \quad (\text{D.4})$$

References

- [1] H. Andernach and F. Zwicky, English and spanish translation of zwicky's (1933) the redshift of extragalactic nebulae, 2017, 1711.01693.
- [2] B. Ryden, Introduction to cosmology (Cambridge University Press, 1970).
- [3] V.C. Rubin and W.K. Ford, Jr., *Astrophys. J.* 159 (1970) 379.
- [4] R. Massey, T. Kitching and D. Nagai, *Mon. Not. Roy. Astron. Soc.* 413 (2011) 1709, 1007.1924.
- [5] N.A. Bahcall et al., *Science* 284 (1999) 1481, astro-ph/9906463.
- [6] V. Springel, C.S. Frenk and S.D.M. White, *Nature* 440 (2006) 1137.
- [7] Planck, N. Aghanim et al., *Astron. Astrophys.* 641 (2020) A6, 1807.06209, [Erratum: *Astron. Astrophys.* 652, C4 (2021)].
- [8] J.F. Navarro, C.S. Frenk and S.D.M. White, *Astrophys. J.* 490 (1997) 493, astro-ph/9611107.
- [9] A. Burkert, *The Astrophysical Journal* 447 (1995).
- [10] M.G. Walker and J. Penarrubia, *Astrophys. J.* 742 (2011) 20, 1108.2404.
- [11] P. Salucci et al., *Mon. Not. Roy. Astron. Soc.* 378 (2007) 41, astro-ph/0703115.
- [12] G. Castignani et al., *Natural Science* 04 (2012) 265.
- [13] D.H. Weinberg et al., *Proc. Nat. Acad. Sci.* 112 (2014) 12249, 1306.0913.
- [14] J.S. Bullock and M. Boylan-Kolchin, *Annual Review of Astronomy and Astrophysics* 55 (2017) 343.
- [15] D.N. Spergel and P.J. Steinhardt, *Phys. Rev. Lett.* 84 (2000) 3760, astro-ph/9909386.
- [16] S. Tulin and H.B. Yu, *Phys. Rept.* 730 (2018) 1, 1705.02358.
- [17] M. Rocha et al., *Mon. Not. Roy. Astron. Soc.* 430 (2013) 81, 1208.3025.

- [18] M. Kaplinghat et al., Phys. Rev. Lett. 113 (2014) 021302, 1311.6524.
- [19] M. Kaplinghat, S. Tulin and H.B. Yu, Phys. Rev. Lett. 116 (2016) 041302, 1508.03339.
- [20] L. Sagunski et al., (2020), 2006.12515.
- [21] A. Robertson et al., Monthly Notices of the Royal Astronomical Society 501 (2020) 4610.
- [22] A. Kamada et al., Phys. Rev. Lett. 119 (2017) 111102, 1611.02716.
- [23] T. Ren et al., Phys. Rev. X9 (2019) 031020, 1808.05695.
- [24] D. Baxter et al., Eur. Phys. J. C 81 (2021) 907, 2105.00599.
- [25] A.K. Drukier, K. Freese and D.N. Spergel, Phys. Rev. D33 (1986) 3495.
- [26] R. Poole-McKenzie et al., JCAP 11 (2020) 016, 2006.15159.
- [27] N. Bozorgnia et al., JCAP 07 (2020) 036, 1910.07536.
- [28] N. Bozorgnia and G. Bertone, Int. J. Mod. Phys. A 32 (2017) 1730016, 1705.05853.
- [29] N. Bozorgnia et al., JCAP 05 (2016) 024, 1601.04707.
- [30] C. Kelso et al., JCAP 1608 (2016) 071, 1601.04725.
- [31] J.D. Sloane et al., Astrophys. J. 831 (2016) 93, 1601.05402.
- [32] Gaia, A.G.A. Brown et al., Astron. Astrophys. 616 (2018) A1, 1804.09365.
- [33] SDSS, D.G. York et al., Astron. J. 120 (2000) 1579, astro-ph/0006396.
- [34] N.W. Evans, Proceedings of the International Astronomical Union 14 (2019) 113.
- [35] A. Helmi et al., Nature 563 (2018) 85.
- [36] C.A.J. O'Hare et al., Phys. Rev. D 98 (2018) 103006, 1807.09004.
- [37] C.A.J. O'Hare et al., Phys. Rev. D 101 (2020) 023006, 1909.04684.

- [38] D.S. Aguado et al., *Mon. Not. Roy. Astron. Soc.* 500 (2020) 889, 2007.11003.
- [39] K. Donaldson, M.S. Petersen and J. Peñarrubia, *Mon. Not. Roy. Astron. Soc.* L513 (2022) 46, 2111.15440.
- [40] G. Besla, A. Peter and N. Garavito-Camargo, *JCAP* 11 (2019) 013, 1909.04140.
- [41] J.N. Garavito Camargo, *The Effect of the Large Magellanic Cloud on the Dark Matter Halo of the Milky Way*, PhD thesis, Arizona U., 2021.
- [42] M.S. Petersen and J. Peñarrubia, *Nature Astronomy* 5 (2020) 251.
- [43] C. Conroy et al., *Nature* 592 (2021) 534.
- [44] M.S. Pawłowski et al., (2021), 2111.05358.
- [45] P. Creasey et al., *Mon. Not. Roy. Astron. Soc.* 468 (2017) 2283, 1612.03903.
- [46] M. Kaplinghat et al., *Physical Review Letters* 113 (2014).
- [47] M. Kaplinghat, S. Tulin and H.B. Yu, *Physical Review Letters* 116 (2016).
- [48] Y.M. Zhong, D. Yang and H.B. Yu, (2023), 2306.08028.
- [49] A.A. Dutton and A.V. Macciò, *Mon. Not. Roy. Astron. Soc.* 441 (2014) 3359, 1402.7073.
- [50] M.D. Schneider, C.S. Frenk and S. Cole, *Journal of Cosmology and Astroparticle Physics* 2012 (2012) 030.
- [51] J. Binney and S. Tremaine, *Galactic Dynamics* (Princeton University Press, 2008).
- [52] M. Cautun et al., *Monthly Notices of the Royal Astronomical Society* 494 (2020) 4291.
- [53] M. Valli and H.B. Yu, *Nat. Astron.* 2 (2018) 907, 1711.03502.
- [54] M. Kaplinghat, T. Ren and H.B. Yu, (2019), 1911.00544.

- [55] B.P. Moster, T. Naab and S.D.M. White, *Monthly Notices of the Royal Astronomical Society* 428 (2012) 3121.
- [56] P.S. Behroozi, R.H. Wechsler and C. Conroy, *The Astrophysical Journal* 770 (2013) 57.
- [57] J. Schaye et al., 446 (2015) 521, 1407.7040.
- [58] T.C. Licquia and J.A. Newman, *The Astrophysical Journal* 806 (2015) 96.
- [59] A.H.G. Peter et al., *Mon. Not. Roy. Astron. Soc.* 430 (2013) 105, 1208.3026.
- [60] R.A. Crain et al., 450 (2015) 1937, 1501.01311.
- [61] A. Robertson, R. Massey and V. Eke, 465 (2017) 569, 1605.04307.
- [62] A. Robertson et al., 476 (2018) L20, 1711.09096.
- [63] K. Bondarenko et al., *Journal of Cosmology and Astroparticle Physics* 2021 (2021) 043.
- [64] G.E. Lawrence et al., (2022), 2207.07644.
- [65] M. Kuhlen et al., *Astrophys. J.* 784 (2014) 161, 1308.1703.
- [66] E. Rahimi et al., *Journal of Cosmology and Astroparticle Physics* 2023 (2023) 040.
- [67] G.C. Myeong et al., *Mon. Not. Roy. Astron. Soc.* 488 (2019) 1235, 1904.03185.
- [68] L. Necib, M. Lisanti and V. Belokurov, (2018), 1807.02519.
- [69] L. Necib et al., *Nature Astron.* 4 (2020) 1078, 1907.07190.
- [70] N. Bozorgnia et al., *Journal of Cosmology and Astroparticle Physics* 2020 (2020) 036–036.
- [71] N. Garavito-Camargo et al., (2019), 1902.05089.
- [72] G. Besla et al., *The Astrophysical Journal* 668 (2007) 949.
- [73] E. Vasiliev, *Dear magellanic clouds, welcome back!*, 2023, 2306.04837.

- [74] F.A. Gómez et al., *Astrophys. J.* 802 (2015) 128, 1408.4128.
- [75] E. Vasiliev, *Galaxies* 11 (2023) 59.
- [76] J. Billard et al., *Reports on Progress in Physics* 85 (2022) 056201.
- [77] J. Aalbers et al., *Journal of Physics G: Nuclear and Particle Physics* 50 (2022) 013001.
- [78] R.H. Helm, *Phys. Rev.* 104 (1956) 1466.
- [79] R. Essig et al., *JHEP* 05 (2016) 046, 1509.01598.
- [80] R. Schönrich, J. Binney and W. Dehnen, *Monthly Notices of the Royal Astronomical Society* 403 (2010) 1829.
- [81] C. Savage, K. Freese and P. Gondolo, *Physical Review D* 74 (2006).
- [82] J.I. Read, *Journal of Physics G: Nuclear and Particle Physics* 41 (2014) 063101.
- [83] J. Richard et al., (2009), 0911.3302.
- [84] M. Weber and W. de Boer, *Astronomy and Astrophysics* 509 (2010) A25.
- [85] P. Salucci et al., *Astronomy & Astrophysics* 523 (2010) A83.
- [86] P.J. McMillan, *Monthly Notices of the Royal Astronomical Society* 414 (2011) 2446.
- [87] S. Garbari, J.I. Read and G. Lake, *Monthly Notices of the Royal Astronomical Society* 416 (2011) 2318.
- [88] F. Iocco et al., *Journal of Cosmology and Astroparticle Physics* 2011 (2011) 029.
- [89] S. Garbari, J. Read and G. Lake, *EPJ Web of Conferences* 19 (2012) 01008.
- [90] J. Bovy and S. Tremaine, *The Astrophysical Journal* 756 (2012) 89.
- [91] L. Zhang et al., *The Astrophysical Journal* 772 (2013) 108.

- [92] J. Bovy and H.W. Rix, *The Astrophysical Journal* 779 (2013) 115.
- [93] M. Pato, F. Iocco and G. Bertone, *Journal of Cosmology and Astroparticle Physics* 2015 (2015) 001.
- [94] H. Silverwood et al., *Monthly Notices of the Royal Astronomical Society* 459 (2016) 4191.
- [95] Y. Huang et al., *Monthly Notices of the Royal Astronomical Society* 463 (2016) 2623.
- [96] P.J. McMillan, *Monthly Notices of the Royal Astronomical Society* 465 (2016) 76.
- [97] S.H. Lim et al., *Mapping dark matter in the milky way using normalizing flows and gaia dr3*, 2023, 2305.13358.
- [98] X. Ou et al., *The dark matter profile of the milky way inferred from its circular velocity curve*, 2023, 2303.12838.
- [99] Y. Zhou et al., *The Astrophysical Journal* 946 (2023) 73.
- [100] A.M. GREEN, *Modern Physics Letters A* 27 (2012) 1230004.
- [101] R.J.J. Grand et al., *Mon. Not. Roy. Astron. Soc.* 467 (2017) 179, 1610.01159.
- [102] V. Springel, *Monthly Notices of the Royal Astronomical Society* 401 (2010) 791.
- [103] J. Schaye et al., *MNRAS* 446 (2015) 521, 1407.7040.
- [104] R.A. Crain et al., *MNRAS* 450 (2015) 1937, 1501.01311.
- [105] Planck, P.A.R. Ade et al., *Astron. Astrophys.* 594 (2016) A13, 1502.01589.
- [106] C. Power et al., *Mon. Not. Roy. Astron. Soc.* 338 (2003) 14, astro-ph/0201544.
- [107] A. Jenkins, *MNRAS* 434 (2013) 2094, 1306.5968.
- [108] M. Salem et al., 815 (2015) 77, 1507.07935.
- [109] G. Pietrzyński et al., *Nature* 567 (2019) 200.

- [110] N. Kallivayalil et al., *Astrophys. J.* 764 (2013) 161, 1301.0832.
- [111] R.P. van der Marel et al., *Astron. J.* 124 (2002) 2639, astro-ph/0205161.
- [112] A. Smith-Orlik et al., The impact of the large magellanic cloud on dark matter direct detection signals, 2023, 2302.04281.
- [113] V. Springel et al., *Mon. Not. Roy. Astron. Soc.* 328 (2001) 726, astro-ph/0012055.
- [114] M. Schaller et al., *Mon. Not. Roy. Astron. Soc.* 461 (2016) L56, 1605.02770.
- [115] R. Catena and P. Ullio, *Journal of Cosmology and Astroparticle Physics* 2010 (2010) 004.
- [116] P.F. de Salas et al., *JCAP* 10 (2019) 037, 1906.06133.
- [117] N. Garavito-Camargo et al., *Astrophys. J.* 919 (2021) 109, 2010.00816.
- [118] T. Bringmann et al., *European Physical Journal C, Particles and Fields (Online)* 77 (2017) 1.
- [119] P.G. et al., *Journal of Physics: Condensed Matter* 21 (2009) 395502.
- [120] R. Essig, J. Mardon and T. Volansky, *Phys. Rev. D* 85 (2012) 076007, 1108.5383.
- [121] LZ, D.S. Akerib et al., (2015), 1509.02910.
- [122] SuperCDMS, R. Agnese et al., *Phys. Rev. D* 95 (2017) 082002, 1610.00006.
- [123] G. Bertone et al., *JCAP* 03 (2018) 026, 1712.04793.
- [124] F. Kahlhoefer, S. Kulkarni and S. Wild, *JCAP* 1711 (2017) 016, 1707.08571.
- [125] DAMIC, DAMIC-M, S.J. Lee, B. Kilminster and A. Macchiolo, *JINST* 15 (2020) C02050, 2001.01209.
- [126] DAMIC-M, N. Castelló-Mor, *Nucl. Instrum. Meth. A* 958 (2020) 162933, 2001.01476.