

PfaffModule8L14

📅 Thu, 2/10 2:08PM ⌚ 18:09

SUMMARY KEYWORDS

equals, bounds, dx, du, integrate, substitute, substitution, derivative, ln, function, indefinite integral, integral, write, step, g prime, terms, definite integrals, natural log, cancel, rule

SPEAKERS

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Welcome. We went through the substitution rule for indefinite integrals. And you could actually go through kind of like with integration by parts, you could go through the entire process, integrate, and then put your bounds in at the end if you want to do this for definite integrals. But also, so and this ends up getting a little confusing to students when people kind of learn this, and then they also learn the method that I'm going to show. And so we just keep very close track of which method you're using, and what variables are what. And so you're going to see certain things I'm going to do like, I'm going to label my bounds as to whether I'm talking about X or U, so that you're putting the correct bounds into the correct antiderivative, which is really kind of the key point. Okay? So let's go to the substitution rule for definite integrals. So this is the substitution rule for definite integrals.

Okay, so what we want to have, so we're going to suppose that we have, so we're going to have G' is going to be continuous on this interval, on the particular interval, so G' is continuous on this interval A, B . And we're going to have that F is going to be continuous on the range of, so F is continuous on the range of, I know that has a G' , but all G things I'm going to be red. So this is going to be on U equal G of X . So there's not a prime here. Okay, so this is our setup, if we need to be in the setup, in order to be able to do what we want to do, then we're going to have that the integral from, and see, I'm going to be very clear here, and we're talking about my bounds being X equals A to X equals B . Okay, so those are X bounds. Okay? And then I'm going to have F . So this is, you know, this should look familiar from when we learned about substitution before. So this is, you have your outside function and your inside function, and the derivative of your inside function should be next door like this, right?

Okay, so this is going to equal the integral, and now we're actually changing our bounds. Okay? So our bounds are U bounds now, how do you get your bounds? You plug in your old bounds into your U function, okay. So these are going to be our A and our B . And then we have F of, F of U DU , so we have F of U , DU . So now we're integrating with respect to U , instead of with respect to X . Okay, so this is the, this is the rule and then we're going to go through an example so that you can actually see how it's applied.

So here's our example. We're going to want to integrate, so from we want to integrate E , E to the fourth of DX over and I have X times the square root, yes, times the square root of $\ln$ of X . And the first thing I want to do is to write this in a format where it's going to be easier to work with. So I'm going to say that this is the integral. I'm going to note that these are my X bounds. So I'm going to kind of keep track of that for myself. I recommend keeping track of it by writing X equals this. X equals this, these are X bounds. I'm only going to use them if everything's in terms of X . And then I have, so how could, this is on the board. So I can write, it's a square root so that I can write as a half power, it's on the bottom. So that makes it a minus half power. So I get $\ln$ of X to the minus $1/2$. And then I want to multiply that by 1 over X , right? That's coming from there. And I'm just going to pull my DX to the side, this makes it much easier to see what I'm going to do. Okay, so let's just start going through this step. So the first step is that I want to identify my F of U .

And my U of X , right? So I need to identify my outside inside functions. This is very similar to what we were doing before. Okay, so looking at this, so my inside function, I usually think it's easier to do my inside first, okay, so this is my inside, like my U equals G of X , right? And then this over here, right there, this should be my G prime of X . So this is a good substitution problem, because I have my derivative next door. So that's all good. And then I can actually see that this gives me the U to the to the minus $1/2$ power, okay? So this kind of outside part here is going to give me that I've got U to the minus $1/2$ there. Okay. So having done all that, so my outside function, so I have my F of U is going to equal, so I, taking this to the minus $1/2$ just right, because this is the minus $1/2$ of my inside, my inside there. So this is going to be my U . And then I have, right, because my inside function is actually going to be that's this function here. Okay, so my U of X is the natural log of X . Great. Now what I want to do like before is I want to find my, right, because I need to do I need my DU here, so I have to figure out what my DU is. Okay, so I need to find step two, is that my DU is going to equal, so I have to put in my U prime or my G prime. So this is going to be my U prime of X DX . Okay, here's my U . So I have to find my U prime, which is just 1 over X . Okay, so I'm going to get that here. So this is just going to be DU is going to equal, so I'm taking the derivative of U of X . So I just get 1 over X DX . Okay, and then the last step that I do here is I want to solve for, right, because I'm going to want to replace in here everything and to make it in terms of U just like we did without bounds, okay? So in order to do that, I'm going to need to get rid of the DX . And so I'm going to want to solve for DX , okay, so step three is solve for DX .

And when I do that here, I just get DX equals XDU . Okay, great. Now, I can start substituting everything in. So I'm going to go ahead and do that. So I want to substitute. I'm going to, actually right over there. I think I'm going to want more vertical room. So I'm going, so now I'm in step four. So I'm going to want to substitute in, right, so I have two things to substitute in. The first one is my U of X , right, which is equal to the natural log of X . Right? And I also have the my DX equals XDU . So I want to substitute these two things into the integral and change the bounds.

Okay, so I'm going to do two things in here so that everything becomes in terms of U . Okay? So you mean. So here we go. So I'm going to get the integral of, and now I want to take, right, I want to take U of my bounds. So I want to take U of each of my bounds. So my bounds here were E , and E to the fourth. So I want to plug those into U . Okay? So I've got E and E to the fourth of, and now I have, right, so I have U to the minus $1/2$, and then I have, right, and then I still have my 1 over X from before. And then I have, and then the last part, so I want to go XDU , this came from my replacing my DX , okay? And then, lucky for us, these are going to cancel. So I'm going to get, so this is going to, oh, but I

don't want to cancel it yet. But I want to point out, so what are these here? So this is going to equal, so I'm going, we are in fact taking the natural log. So this is the natural log of E to the fourth, which is going to be 4. And this is the natural log of E , which is just going to equal 1. Okay, so those are my new bounds. And I want to cancel until everything's in terms of U . So that's the next step. So step five.

Cancel until everything is in terms of U . Okay, so I'm going to get the integral. And so I can just kind of write these in now. So these are, but it helps me to write U equals 1, and U equals 4, so that I don't forget that I'm in a U world now. Okay, and then these cancel out very nicely. So I'm just going to get, now I have U to the minus $1/2$ DU . Okay, great. Now, the last thing I need to do is just to integrate. So step six, integrate with respect to U . So don't get confused by that. In fact, let's kind of make this clear that this is a U here, we're integrating with respect to U . Okay, so all this stuff is U . Okay? So I have the integral from U equals 1, to U equals 4 of, and then we're going to have a U in here and DU , so then we want to put this, so this is to the minus $1/2$ is going to equal, and this is just a U integral, everything's in terms of U . So I can integrate it just like normal. So I'm going to get, right, so I want to have 1 over so that power minus $1/2$ plus 1 times, and then I have U to the minus $1/2$ plus 1 . And then I want to evaluate at my two bounds, which are U bounds. So this is good, everything staying in terms of U , so that makes sense. U equals 1 to U equals 4. And so then I can just kind of write this a little nicer now. So this is $1/2$, it's on the bottom, so that becomes 2. So this is 2 and I have U to the half.

And I'm going to go from U equals 1 to 4 right? And then we know how to do this super duper carefully, oops that's a U equals 4. Right where I just first replaced, so I've got, I've got this is going to equal so I do my something minus my something ever so carefully. I'm being so so so so careful. Okay, where I write, so I write 2, and then this goes to the $1/2$, and 2 and this goes to the $1/2$. Okay? And first I put in my 4, and then my 1 right, so this ends up square rooted, so it becomes 2, so that becomes 4. Right? So this is a 4 and then I'm going to just minus a 2, and so I should get 2.

And that's my final answer. Okay, so what happened here? And one thing I'm going to say before anything else is a very important thing here, which is that X never returns. And so maybe we can even emphasize that here. This is the last time we ever see X . We don't substitute back in for X later. Right, it does not get substituted back in. Okay, that's a good talk mistake that students make. Because you do it when you do for an indefinite integral, you do substitute it back in for U . Okay, so this is just kind of like normal substitution in the beginning, where I need to find my outside function, my inside function, the derivative of my inside function should be next to it. And at the start my bounds are in terms of X . It can really help to write that with an X there so that you keep that straight. But then we want to try to transform everything to be in terms of U , with the key point being here it's never coming back. Okay, X never comes back. Okay, so my outside function, my inside function, I have to take the derivative of my inside function to figure out what DU . So DU is going to be the derivative of the inside function DX , the derivative of $\ln$ of X is 1 over X , so I have DU equals this. My favorite way to do this is just to solve for DX , and then put that in. Okay, so that's what we did here. And now once we've got kind of, we put in things in terms of U , you can also kind of, you need to put your bounds in, as we switch everything to be in terms of U , your bounds need to be in terms of U , so you need to plug them into the function. So that's what we're going to do there. And so now I have my bounds are in terms of U , my functions in terms of U , I can integrate with respect to U . So that all becomes very clear and easy. So here we go, we just integrate with respect to U . This is a minus $1/2$

power, so I can just add 1, add 1, and then I have to put in my bounds. And then I just work this through where I kind of do this carefully with parentheses, so I don't make any mistakes. Okay, so key point, once X disappears, it never comes back. So it's a good thing to kind of keep in mind to the whole thing. Great. So, I hope this was helpful and I will see you in the next lecture.