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SUMMARY KEYWORDS

derivative, cubed, sine, dx, chain rule, du, integral, function, fourth, differentiating, equal, cancel, integrate, happy, steps, replace, terms, cosine, respect, check

SPEAKERS

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Okay, so greetings, welcome. We just went through one example of substitution where I kind of went through the steps as I did them. I summarized the steps here since I erased the last example. And now we want to new, do a new example here. So we are going to just start going through all the steps. The first one to go through is to actually identify right or outside and inside function, so I can kind of see this here, right. I can see my inside function. Usually it's easiest to do your inside first. Okay, so here we go. So this is going to be our U of X equals X to the fourth minus 3, and we check that we do actually see the derivative or something that's pretty close to the derivative over here. Okay, so that really is very close. It's not exactly it, but it's very close to the derivative. So if I look at this, so I have the U prime of X is actually equal to $4X$ cubed, which is pretty close to that, okay, so I'm happy. And then if you look at kind of my outside function, here's kind of my outside. So this is going to be my sine of U is going to be, right, so sine of, that probably should have been blue and red, but you can see, okay, so that's kind of my outside function. So I know my outside and my inside, so let's just write those over here for us. So we have that F of U is going to equal sine of U .

Right, where I have that U is going to, U of X is going to equal X to the fourth minus 3. Okay, and now I have my DU . Okay, so I need to take the derivative, so there's my U prime of X , and I just need to multiply that by DX . So that's not bad. So I'm going to take 2. So I have that my DU is going to equal, and I'm going to take that $4X$ cubed that I wrote up there, and then multiply that by DX , that's just the derivative of this function here.

Okay, now I want to solve for this DX . So I just bring it to the other side. So I just have dx is going to equal DU divided by $4X$ cubed. Now I'm ready to take these and substitute them in. Okay, so when I substitute them in, so this has to replace my DX . And this has to replace, a U has to replace that. So I'm going to get the integral of my X cubed is still there. I'm going to have to do some cancellation. Hopefully, if I chose my F and my U correctly, that'll just cancel there. So I have X cubed, and then I have my sine of U , right? You can see that there. So this is actually, just ends up being my sine of U , right? Once I replace that U , which is my X to the fourth minus 3. And now I have to replace my DX , right? Right here, this has to replace that. So I get DU over $4X$ cubed. And now I know that I did a good job because the X cubed, which is a significant part, that $1/4$ isn't really going to matter, because we can just pull that out. It doesn't, it's easy to work with, but all the X terms cancel. So let's

go ahead and do that. And we're going to get, so from this 4 here, I'm going to get a $\frac{1}{4}$ on the outside. So integral of sine of U DU. Ah, now I have an integral that's just in terms of U. So I'm happy so I can keep going.

And I'm going to integrate with respect to U. So I just get it. What's the integral of this? This is, I've got to take the antiderivative, I'm going to get $-\frac{1}{4} \cos U + C$. And now sometimes you might think you're done, but then you have to remember that I'm taking the integral with respect to X. So I need to get a function that's in terms of X right, all I need to do is put back in my U here, I just see my U a single time there. So I can just put my U into there. And I'm going to get $-\frac{1}{4} \cos U$, and then I just put in my U. So now I look back up and that's $X^4 - 3$.

Okay, and now I have no U's left or anything. So I'm happy. So now I just need to check it. So I'm going to just check and take the derivative. And always, when you're checking these, because of the nature of a substitution, you're going to be doing the chain rule. So make sure if you're not doing the chain rule, then you're making a mistake, right? Okay, so I have to get, so I can pull out, so I have $-\frac{1}{4}$. And then I'm going to get $-\sin X$ to the fourth minus 3 times the derivative with respect to X of the inside, I've got to do my chain rule always properly. And then the plus C ends up just dropping out because I'm taking the derivative of a constant, so I get zero. Okay, so what I end up getting here is, so these are going to cancel, so I'm going to end up getting $\frac{1}{4}$. So $\sin X$ to the fourth minus 3 times $4X^3$, which is going to equal X^4 .

Like this, and then I have to look back up and is this the same thing as this? And it is, so I'm happy and I'm done. Okay, so then I just get to write my answer, which was the answer I got before I was doing my check. So here we go. So here's my answer. Which is that the integral of $X^4 - 3$ DX is going to equal $\frac{1}{4} \sin X^4 - 3X^3 + C$. Don't forget your C. Okay, great. So I just went through these steps one by one, I look for my outside function, right, which is my F of U, was the outside of the inside. Usually, it's easiest to figure out the inside first, because it's kind of like inside a function, and then you can look for its derivative next door. And if roughly its derivative is door, you're happy. So we get, here's my inside function, my outside would be if I replaced the inside function with U, what would I get? So that's how I got my F of U was $\sin U$, that was my inside. More or less my derivative is next door, so I'm happy. Then I need to take the differential here. So I just did that derivative of U, which I kind of had already seen there, multiplied by DX. I solved for DX, because now I want to sub in the DX in here, and my U in there, so that everything is going to be in terms of U once I do canceling and I can just integrate with respect to U, a much more simple function. So I integrate with respect to U. But I need an answer in terms of X, so I switch back in terms of X. And the last step is that I just had to take the derivative, right, to check that actually the derivative that is indeed the integrand, so that I have indeed found an antiderivative. So I just take the derivative, I need to remember to use the chain rule. Some things to notice here, in case you forget, when you use the chain rule, I didn't change. Right, so I, I changed the outside function here, I didn't change the inside, but I took the derivative of the inside. And that's all that's kind of a little bit tricky. Sometimes when you get to these methods and you're used to, it gets very confusing because you're integrating and differentiating in the same time. So you have to kind of keep it straight in your mind when you're, right, because it's going to happen so many times probably, that you're going to forget that oh wait, I'm differentiating this so the power goes down. Instead of up, it's just something that you'll have to kind of, you know, kind of work to keep straight in your mind. But it's actually not that bad once you get the hang of it. You just

have to always kind of be alert. A lot of these things you just kind of have to be alert. Okay, well, great. So I hope this makes sense. And that, you know, you get some practice with this and find it's actually not so bad. I'll see you in the next lecture.