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SUMMARY KEYWORDS

derivative, integral, du, dx, $3x$, square root, halves, squared, equal, cubed, replaced, integrand, step, substitution, function, thirds, circumstance, integrate, respect, du dx

SPEAKERS

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Okay welcome. So I'm going to go through, so this will be one of two lectures where I'm going to go through examples of substitution. So we're going to get started on this one, and I put for you the formula up there, so that you can keep track of what's going on. So we know that we're going to need to kind of identify an outside function and an inside function, okay, so if you look at this, right, you can see the outside function is the square root. So this is the outside function. And then we have an inside function here. Right. So X cubed minus 7 is inside. And then I said, we need to have the derivative of the inside function next door. And here we go, it's right here.

So this is like our derivative next door, because you have that, if I took the derivative of that, I would get this. So let's kind of go through this a little bit more carefully. So let's start out. So the first thing that we want to do, so step one is that I actually want to identify what are my outside and inside functions. So I want to identify F of U and U of X .

So identify F of U , and this U of X , which would be my inside and outside functions. Okay, so looking at this one here, we've already kind of done this. So I have that to my outside is going to be so I have F of U is going to equal the square root of U . Okay, because this is going to be my U here, so that'll be my U . Okay, and I'll write that next, so in my inside function is going to be, so I'm going to have that U of X is going to equal X cubed minus 7. Okay, so I have that this function is inside of this function, or it's a composition of those two functions. So now in step two, what I want to do is I want to take the differential, so step two.

Okay, which is going to be that DU is going to equal, and this is going to be U prime of X . So I'm actually going to do this kind of in orange. So this is U prime of X DX . Okay, so in this circumstance here, so what I'm doing is I'm taking the derivative of this, which is actually right up there. So I'm going to get that in this circumstance, DU is going to equal $3X$ squared, because I just took the derivative of this U and then DX . Okay, now in our next step, so step three, I actually want to go through and solve for DX , right, in this formula right here. Okay, so what I'm going to get, well, I just need to bring the $3X$ squared over to the other side. And so I'm just going to get that DU is going,

over $3X$ squared is going to equal DX . So I get DU over this $3X$ squared equals DX . Okay, now, this is the first time, this is, you know, the first part is identifying actually how you do it. And then after that, it's all the steps are more or less straightforward. We've gone through and we've taken, we had to take this differential, which was that I took the derivative of U of X , which would have been $3X$ squared. So that's where that came from. And then I'm going to go through and I'm going to solve for DX . And so I did that. And now what I'm going to do is I'm going to substitute in my U of X and my DX into this integral. Okay, so step four.

So I'm going to sub in, so I'm going to sub U of X and, and DX into the integral that I had from the start. Okay, so I had the integral of $3X$ squared times the square root of X cubed minus $7DX$. Okay, so I want to sub these into there, where we notice that my U of X from here, right, is exactly what's here. The DX , right, is, so this DX here, is the same thing as this one here. So this is going to go into DX . Okay? So let's go ahead and do that. So we get that the integral of $3X$ squared, and then we're going to get, so we're going to get the integral $3X$ squared, we're just replacing that with U . So I keep the square root but I put in my U . And now I'm going to take this piece here and I'm going to put it in right there. Okay, so I just get this DU over $3X$ squared.

Okay, now, the next thing I need to do. Well, okay, so definitely kind of notice that we can't integrate this yet because we have like more than one variable, okay, so can't integrate yet because more than one variable. But in this circumstance, it's they, they cancel out very nicely.

Okay, so step five is just going to be to cancel until everything's in terms of U . Okay, so we're going to go ahead and do that. Because we don't want any of these X 's here, we just want the entire thing to be in terms of U . How do we know that? Because this is DU , okay? So we want to have the integral, and we can see this kind of cancellation happening there. So we're going to get, so when I cancel, I'm going to I'm going to be canceling, right, you can kind of see, okay? So those are going to cancel out and what I'm actually going to get is I'm just going to get the integral of U DU . Okay, now this is, I'm integrating with respect to U , everything's in terms of U , so I'm happy. Now I can just integrate like normal. Okay, so step six, integrate with respect to U .

Okay, so let's go do that, go ahead and do that we know how to integrate this. So the integral of the squared of U DU is equal to, it's always easier for us to just kind of write that as U to the $1/2$ DU equals, and then we get right so we have to do this power thing U to the $1/2$ plus 1 over $1/2$ plus 1 plus C equals, so this is three halves on the bottom, right? This is three halves. Each of these is three halves. Okay, so I'm just going to get, well three halves on the bottom flips over. So I'm going to get two thirds U to the three halves plus C . Okay, so now I feel like I'm like almost done, except that I was doing an integral that involved X and I'm, and I still have U 's left over. Okay, so now everything has to be turned back because I'm supposed to be getting an integral in terms of X . Okay, so the next thing I need to do step seven, is I need to substitute. So in this circumstance, it's U of X equals X cubed minus 7 , back in, so we have U of X equals X cubed minus 7 into this right here, so I want to substitute that in into, so two thirds U to the three halves plus C . Okay, so let's go ahead and do that. So what I'm going to get is, and here is the U in there, right? This is the you that I need to sub in, right? So I'm just going to get out.

Two thirds U to the three halves plus C is going to equal two thirds times, and then we just replace, like we like to do, we replace our U with parentheses. And then we go back through and we actually put that in just to make sure we did it carefully. Okay? And at this point, we think we're almost done, but we want to check. Okay, so this is kind of like our potential solution. But I'm going to want to check. So let's, I like to put in a step eight, which involves checking.

Okay, so we want to check that our solution works. Okay, so I want to take the derivative of this and make sure I get the integrand. Okay, so take derivative and hope to get integrand, like the part that's inside the integral. Okay, so let's go ahead and do that. So if I take D DX of, and so we're going to take those as two thirds X cubed minus 7 to the three halves plus C. Okay, so this is going to equal, I can always take the two thirds out. And then I have, and now I have to apply the chain rule here. Okay? So I'm going to take the derivative of the outside, leave in the inside, then take the derivative of the inside. So I'm going to get this is going to be times and then I'm going to get three halves times X cubed minus 7 to the three halves, and then I need to take the derivative with respect to X of the inside. Because we're using the chain rule here.

And I took the derivative of the constant, I got zero, okay? So that I just kind of need to make this nicer. And I get is this is going to equal X cubed minus 7 to the $1/2$ times $3X$ squared. And this is indeed that integrand up there, so I'm happy. Okay, they're written slightly differently, but they're exactly the same thing. Okay? So, so integrand from my original integral, so happy. Okay, so my answer is, and I'll put this kind of like up there. So this was indeed my answer. So my answer is going to be that if I take the integral of $3X$ squared times the square root of X cubed minus 7 DX , then I'm going to get two thirds X cubed minus 7 to the three halves plus C.

Okay, so let's kind of go through what we did. One of the hardest parts is identifying what your, like, F of U and your UR . Your F is like your outside function, your U is your inside function, but you need to have the derivative of your inside function next door. So in this case, our F of U is the square root of U , okay, where my U is my inside function. Okay, so that's what I get here, and then I have to take this differential, all I did was I took the derivative of this U here. Okay. And then once I did that, then I need to solve for DX in here, that looks a little bit funny. But if you do that, and you sub it into here, this is a trick that once upon a time, one of my calculus students actually taught me but it does make it much cleaner and easier, because then also, you don't have to kind of fiddle around too much to figure out what your DU DX is over there, you just whatever's left is left. Let's see, this is kind of a nice clean method. I like this one. Okay, so I replaced my, in my integral, I replaced my DX with my DU DX , sorry, my, I replaced my DX with my DU over this, like this right here. So I that's what I did there, my U has been replaced. And then I can actually do this cancellation here to get everything in terms of U because I'm, I need everything to be in terms of the variable that I'm integrating with respect to. So that's kind of what we did to cancel out. Now everything's in terms of the variable that I'm integrating with respect to so I can integrate. So that's what I'm going to do here is I'm actually going to integrate with respect to U . Okay, and now I've done all my integration, now I'm ready to sub back in my U . Okay, so I've done that. And I sub back in my U , and this should be my answer. I had to sub back in my U so that everything, it was an integral, it was an integral with respect to X originally, so my answer should be a function of X . So I get there. And then I can actually check to make sure that I did this correctly, which is always good. Because I want to be able to take the derivative of this and get the integrand, right, because this is how antiderivatives and integrals work. So I go ahead and I take that derivative, make sure you apply the chain rule properly. And then, then it falls into

place, and it is exactly this, right? Because taking something the $1/2$ power is the same thing as a square root. There's a $3X$ squared, there's a $3X$ squared. So I'm happy that this was indeed actually my answer. Okay, so this is substitution. I'm going to do another example of it just so that you can kind of see. These ones are relatively straightforward runs. There are of course, more tricky examples that one could do. So I hope this all makes sense, and I will see you for another substitution example in the next lecture.