

# PfaffModule8L10

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## SUMMARY KEYWORDS

derivative, chain rule, antiderivative, rule, substitution, product, integral, composed, composition, write, equal, identify, examples, function, multiplied, dx, undo, notation, critter, brain

## SPEAKERS

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Welcome, In this lecture, I want to introduce substitution for indefinite integrals. And this is going to be one of the ways that sometimes you can actually handle the integral with product. So, and this is also going to be undoing the chain rule. So let me kind of set up that bit of it, which is that, so one of our goals. So one of our goals is going to be to figure out, so what do you know these two rules, which you've seen before. And I'm purposely writing them in multiple different ways, so that you don't get only used to one kind of notation because people use different notation. Okay, so the product rule, right, which we remember how the product rule goes, maybe hopefully, which is that I'm going to have a product and I'm going to take the derivative. So this is not the way I usually write it, but we're going to write it this way.

So I have the product and I take the derivative of a product, okay? And that equals, so I take the first function, right, without doing anything to it, and then I take the derivative of the second function, and then I add to that, what would happen if I took the second function and multiplied it by the derivative of the first? Okay, and that's the product rule, right? We've known that for a little while now. The other rule, and that one in particular is going to be, did I say this was to undo the product rule? This is to undo the chain rule, in case I said it wrong. Integration by parts will undo the product rule, in a funny kind of way. The other one is the chain rule. So here we go. So we have, I'm going to have, in this case, I'm going to use a slightly different color here. So I have  $F$  composed with  $G$ . And this is composed, so this is not the product this is composed. And maybe I'll make it even bigger and more distinct, so that you can see that that's actually a circle there. And we're taking the derivative of that function composition. And we would get, right, we know that we would get the derivative of the first function. And this is function composition. So I'm going to make a big open circle there so you can see that that's function composition, with the inside left alone, right? And then you have times and then you have the derivative of the inside. Okay? And these are two rules that we know back from differentiation. Always good to remind ourselves though. And we want to know what do these tell us about integrals?

So what do the product rule in the chain rule tell us about integrals? And today, we are going to focus in, or in this particular lecture, we're going to focus in, this is going to be our focus is this one, okay? So let's go ahead. So here's the, so we'll kind of write the chain rule up here. I'm going to have, so

here the chain rule. And then you can kind of see how everything kind of lines up. So I'm going to have that if I have, and also so that the colorings a little bit different than how we're going to do it there. So I'm going to have F composed with G prime here. And now I'm liking even less a decision I made here. So I'm going to slightly change that, things don't have to be the same as my notes. Okay, so I have this and this is going to equal and then I'm going to have we have,

F prime and then this is composed with, we made that composed with distinct enough, that's a big open. It's not a product, that's a composition. Okay, and we have this derivative here with this, and then this is going to be multiplied by, so this is, right, it's just a dot for multiplication, by the derivative. So what does this kind of tell us? So this says that the antiderivative, so this is kind of why I to changed the color scheme a little bit, so the antiderivative of, and then I have the orange one or says an antiderivative of this. And maybe I'll even put the an, to emphasize the anti, an antiderivative of this is actually going to be, so this is actually going to be this critter over here. So F composed of G. Right? So what does that tell me? So that tells me that, so I'm going to have that the integral of, and then I have, so I'm looking at this critter right here. So I have F prime composition of G, multiplied by G prime. And then I have, well let's just suppose it's DX. Of course, it could be DT or something else, it's okay. That this is going to equal, I have, so then I have F, right, it's going to equal this composition. So you can just see this as F of G of X, and then plus some C, because we want to generalize the antiderivative there. Okay? And then we want, right, so I want to write this in another form. So this is kind of one form to see it in. Here's another form, which is the one we'll box. So in other words, so here's the substitution rule.

Now I see there's a problem with changing colors on the fly sometimes is that you ruin your your color scheme that you worked so carefully on. But this is going to be okay. So I have this, so this is going to be F sorry, that's not what I want to write there. So if I have F of U, and we're going to look at examples of this, so don't get scared when this just looks like one disgusting mess that you're not sure what to do with,  $\int F(U) \frac{dU}{dx} dx$  is going to equal the integral of F of U dU. Okay, so this is the substitution rule.

Okay, and what we have, so how do we identify this? Is that you're going to see in here, so here we go. So here is going to be the function on the inside, so function on inside. Right, and then you're going to see next door to it, it's derivative. Okay, I know this is a repeat color of that. But we're just going to deal with that, because I only have so many colors. Okay? So this is the derivative next door.

Door. Okay, and this is how you actually kind of identify, and we'll go through a lot of examples of this so that hopefully, this becomes something that's very comfortable for you. But as with all of these things, the best way to make, you know, the best way for things to get comfortable for you is to practice, practice, practice, practice, practice, okay? And then your brain actually just begins to identify these things. Anybody who's a musician, or actually I feel like it's a little closer to sports, where your brain just kind of is able to identify situations and what to do in them faster and faster, the more you practice them. It's just kind of how our brains develop. So we've undone the chain rule, right? This is what this really does. And it's a way that you will actually see a product of two functions here where you happen to have like an inside here with its derivative next door, you'll see examples. So you'll see what I mean by this. And in that circumstance, there's going to be a way to kind of

simplify what you're integrating. And so it's a method you'll get used to. So I hope that made some sense. And I will see you in the next lecture where I actually go through some of these examples to help give a sense of how this works. Okay, so see you then.