

# PfaffModule8L08

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## SUMMARY KEYWORDS

integral, indefinite integral, multiplication, product, properties, chain rule, emphasize, function, summations, tools, compose, rule, compute, imprints, addition, most common, approximation, derivatives, trapezoid rule, pull

## SPEAKERS

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Welcome. So in this lecture, we're going to talk about the first properties of the indefinite integral. And as you can imagine, these are going to come from because the indefinite integral is defined in terms of the antiderivative, it's going to pick up any and all of these properties that the antiderivative has. For example, the first one we'll look at is linearity. Okay? So the indefinite integral is also linear.

Right, so what does that look like? So that looks like, the first thing that looks like is if I take the integral of, and I add two functions here. So I had a function  $F$  of  $X$  and a function  $G$  of  $X$ , that then that's going to be the same thing as I can move the addition to the outside. So that's going to equal the integral, right, we just do this with the sum like this, where I get here  $F$  of  $X$ . And I get here  $G$  of  $X$ . And this is where I'm going to know if this works. For addition, this does not. So this is going to be my, this is like the biggest mistake that people are going to repeatedly make. And don't do this, don't do this, don't do this. So this is like, this is what you do not want to do. Okay, so you do not want to take the integral of a product and just write that as the integral of the product like this. You can't do that. So this is a no, these are not equal. So no, no, no.

It's just me being melodramatic to help you remember. Okay? So here's a little bit of Pfaff melodrama to help you remember, but let's like put that in a big red box. This is like the big, most common integration mistake that students make. So I'm going to, like super emphasize it. So you get this in your mind immediately that we're not going to be allowed to do that, but we'll get some techniques to deal with products. Okay? And then the next part is, so if instead, so if I have an integral, and I have, the only time that I could do anything vaguely like that, but it doesn't even exactly like that, is if I had a constant in front of my function. Then I am allowed to pull this out. And that just actually, because that's, you can do that with summations actually is another way to look at that. Actually, all these things are proved with summation is how you would actually prove them.

Right, and as I kind of noted before, this makes sense, because these work for the, for derivatives and antiderivatives. Okay, so this makes sense since we have that, right, so if I was taking for derivatives I would have, it would be okay to, right, to do the derivatives one at a time. So if this was  $F$  and  $G$

would have, it would be okay to, right, to do the derivatives one at a time. So if this was  $F$  and  $G$ , right, I could add these like that, and then add them like that, and do the derivatives. That would be, that would be okay. And the other thing, so that would be why you would end up with this, and then that also followed through with the antiderivatives, and then this follow through that you had that I had this constant in front I was going to be able to pull that up. Okay, so if I had  $K$  times a function  $F$ , then I could actually do  $K$  times, oops, I'm going to want to have, to emphasize there that it's actually just for that function that I'm taking the derivative, okay? So this should all make sense, because everything we've been dealing with, everything you deal with is linear in this sense, as in summations are linear in this sense, derivatives are linear in this sense, antiderivatives are linear in this sense. And so you can actually prove all these things just using all these facts and properties. But I want to emphasize because I do this danger danger thing here. And we'll kind of get back to this later, like, I'll repeat that this is what we're kind of aiming at when we're doing substitution and so on. And so one of our goals. So one of our goals will be, you know, how do we kind of, you know, what do, we have the product rule.

Right, that stuff all worked with addition, where it could all just kind of go through nicely, but it didn't work with multiplication, right? And this was one of the circumstances where I'm going to multiply, but then it's not going to look exactly the same, right? So if I multiply and I have an  $F$  and a  $G$  here, right, then this would equal, so I would take, I want to write it kind of like I have it written there, so I'd go  $F$  of  $G$ , and then the prime we go with the  $G$ , plus and then the  $G$  times  $F$  prime. Okay, so that looks a lot different than just what I might want to do up here, right? And so what is this? And the other one being the chain rule, right? And these both, though, involve some kind of multiplication. So you could guess that this is part of how we're going to actually developed some techniques. So this is going to be, so here, I'm actually going to compose them. So it's the same deal. But now I'm composing them, this is function composition, where I have  $F$  and  $G$ , and in this one I have to take, so I take, so this is going to equal it's going to be, let's get this, make sure I get this all right. So this is going to be a product there. So I'm going to have a  $G$  prime on the outside, and then I have an  $F$ , an  $F$  prime composed with  $G$ . This is a different way of writing it, than we've been doing it, but it's another way to write it. But I feel like that way of writing it is a little confusing, but that would be another way to write it. So what do these tell us about integrals? And they're going to give us tools to deal with products in particular.

Okay, so we do have all these linearity properties that tell us yes, you can pull out constants, yes, addition kind of follows through as you want. Be very careful that you don't think that these things are going to work with multiplication, because they don't. But what we'll be doing a certain amount is dealing with the like, basically undoing the product rule and the chain rule, which will give us some tools to deal with multiplication. But, you know, I might as well emphasize the fact that most integrals, you can't actually compute using normal techniques. And so when I say that it's important, like approximation tools are almost more important, this is how computers or something would actually compute them, would be something like a Riemann sum is an approximation tool. There's also the trapezoid rule, and the midpoint rule and the Simpson's rule. Like, there's all this stuff, but is actually how anybody would, for the most part, actually compute integrals. But it's good to kind of see, I think it is important a little bit to see these relationships and how they kind of play out. If for no other reason, it starts getting you used to how mathematics works. It's almost like you don't learn something really well until you learn a class that uses it. So it gives you more comfort with mathematics. It gives you an idea of why and what an integral might look like, and why. It kind of imprints into your mind this relationship between these rates of change and actual changes in

quantity and so on. So it's worthwhile spending a little bit of time on it. I'd like to spend a lot more time on kind of understanding how and why these things work and how you might actually use them to solve problems. Okay, so I hope this is all helpful, and I'll see you again in the next lecture.