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SPEAKERS

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Welcome. In this lecture, we're going to go through some more definite integral properties. So let's get started. The first one that I want to go through is what happens when you switch the bounds, because so far, we always done the bounds as, you know, the in sequential order. Like the lower bound is the first number, and the upper bound is the second number if we're going like left to right on the number line. But, you know, one of the ways that we deal with this, if you don't, is to show you what happens when you switch the bounds. Okay, so here we go. So here's the first property, which is that I have the integral from A to B. Actually, let's let's do the bounds in different colors so that it's easier to see what I've done here. So I have the integral from A to B of, and then I'm going to take the function F of X DX. So F of X and then DX. So what happens if I switch the bounds? Then what I'm going to do is I'm going to get a negative sign in front. And then, so now, so that B was there, now it's here. That A was there, now it's here. And in that circumstance, I put a minus sign in front, right? So this is F of X DX.

Okay? And then, let's go through an example. Always good to see an example to kind of make sure we see what I mean. Symbols can be a little hard to parse sometimes. So let's go from, if I'm going to go from 1 to 2 of X squared DX, right, then this is going to equal minus if I did the integral from 2 to 1 of X squared DX. Okay, and I'm not sure the picture is particularly going to help here. But I'm going to draw it because we're going to use the same example in the next property anyway. So here's my picture. So I have, here's my X axis, and my Y axis.

Right, and I have Y equals X squared. I think there's also kind of, somehow when we have the picture in our mind, it makes it easier to remember what was going on. So this is Y equals X squared. And this is a Y. And so we're looking at this 1 and 2 are both like, you know, they're, usually we look at things in this order. But sometimes, you know, you might have gotten written them initially, in the opposite order on here, and then you could just swap them and do the sign trick in order to get it in the order you're used to. And just remind you, so what we're looking at is this area here. I'm looking for a color I haven't super used here, let's just go ahead and use blue. So this would be the area we're looking at. And if it went 1 to 2, right, that would give this area. But if we wanted to swap those, or they start out swapped, it would have a minus, it would be a negative area there. So here's kind of, here's another

property. And then we're going to use the same example because you can kind of look at it on there. So if I want to take the integral, and I want to go from the same, right, so I want to go from A to A, so the bounds are the same. And I want to do this for some F of X DX.

Right? Then this is actually going to equal zero. So this is zero. Okay, and looking at this here, so let's look at a particular one just to kind of give you a sensor. So let's say that both of the bounds are one. And we're looking at X squared DX, right? Well, we're going to get zero. My equal sign is a little sloppy there might take a second, take a moment, fix your equal sign. I don't know if that actually helped that much. There we go. Okay, so you could see if, like, if both the bounds are 1 that's like looking at the, you know, there's no width, so when you're multiplying width times height, you're just multiplying zero times your height, right? So that one like kind of should make some sense to you, you should have almost been able to guess that. Okay, so let's do one more. This one, the picture also could really illuminate that you would guess this. So if I had the integral from A to B of F of X DX.

Oops, I wanted to make that red so that you can kind of see how these might match up. So this is a B here, nice and red, okay, of F of X DX. And we added to that the integral from B to C of F of X DX, then we should get the integral from A to C of F of X DX. So A to C of F of X DX. Okay? So let's look at well, let's actually first look at why this is believable. So we'll kind of draw the schematic picture here. So imagine this is your X and your Y. So this is your Y and you have your function. Well, and let's put on our A and our B and our C. So maybe our A is here, and maybe our B is here, and our C is here. Right, and we're looking at this function here, and it's going maybe going down and like this. Okay. So what happens with these areas that are underneath, right? So the first area underneath, so we can look at, I'll make my areas actually yellow. So this is a positive area. So I was looking at this area, and then I would add, this is a minus area. So I add that minus, and then all I'm doing is, the tricky one to think about is just this one here. But you know, areas sum up nicely, when they're, you know, you just kind of cut it in half, then you can actually sum those areas and get the whole, right? So this would be a plus area. And that would have been the first integral, and then we started on the second integral, right, and we're just adding the rest of the areas that you would have gotten if we had in the first place just gone, done the areas from A to C. Okay, so hopefully that makes a little sense. And let's look at an example, like an explicit example. So let's look at, this one is not explicit, it's more of a schematic because I wanted it to have like a lot of pieces, so you can kind of see what's going on. So you have 1 to 2 of X squared DX.

And then if I add to that the integral from 2 to 3 of X squared DX, then this is actually going to equal the integral from 1 to 3 of X squared DX. And you can actually see this. This particular example, we can still use this example here, and imagine that our 3 is right here. I know that that's not exactly even. But then you would write, you would get this area and you'd need to add in that area too, which would be the same thing as if I gone from 1 to 3, right? So you can kind of believe this, I think that pictorially you can really believe this. I do have a caution. So my cautionary tale of the day. So maybe I'll make the caution symbol in red. So I'm going to actually, this isn't just a cautionary tale today. This is the cautionary tale, right, that shows up in a number of our lectures here. Because this is like the most common mistake of students. But then we have ways to work around this and a lot of times you can still compute these. But if I have the integral from A to B of F of X DX, sorry F of X times G of X DX. So I have F of X times G of X DX, then this does not not not equal, you can tell how much I, you'll see me say this like two or three times in the lectures because such a common mistake with students

so this does not, and I understand why, because this works with addition, this works with a lot of things. This does not work with multiplication. Okay, this does not not not equal F of X . You know, you can't just integrate them separately. Okay. I know it would be so nice if you could, but you can't.

Okay, so that's the big caution is that when you have a product, you can't just multiply those separately and then multiply them together. And it's really important that you don't do that, that you actually use, we will have substitution integration by parts, a lot of times give you methods for handling products, and then there will be ones that you can't do. And then you're going to use approximation methods. Okay? So just to kind of go through and summarize each of these. When we switch the bounds, we put a minus sign in front. So I showed that with integral from 1 to 2 of X squared, I would get a minus sign in front if I was going, right from 2 to 1 instead. And then if my bounds are the same, that's like taking zero width, so that's like zero, area. And so you could believe that that integral zero. This one, if it's A to B , and then B to C , and the order that, you know, it does kind of matter that those are the two that match up, which is why I made them the same color. And that really is just you can kind of see that is piecing together the areas, and so that should make a lot of sense. And so you can see this in this example is the one that's here, or inside of this picture here, it works with both signed areas, okay. So, I hope that made some sense, and I will see you in the next lecture.