

PfaffModule8L05

📅 Wed, 2/9 1:10AM ⌚ 12:44

SUMMARY KEYWORDS

integral, rectangles, picture, linearity, function, $x \, dx$, constant, area, equal, height, adding, multiply, sound effects, ends, draw, lecture, summing, fact, nice, sense

SPEAKERS

Catherine Pfaff

Okay, so welcome. In this lecture, I want to go through the fact that the definite integral is linear. So you've seen this a lot. This may feel like oh my god, I can't believe I've seen this in every single setting, but it's because they're all related. Their linearity actually, in some sense follows from each other. But I like linearity in this circumstance, because you can kind of draw nice pictures and make yourself believe it. And then you can also have pictures in your mind as what linearity means, in general, instead of just having these ways of computing using it. So let's kind of go through this.

Okay, so here's the first one is that I'm going to have the integral from A to B, just kind of, it's good to put these in another color actually here because sometimes the bounds manner, and sometimes they don't. So I have two functions here. So I have an F of X and a G of X, and maybe I will give these two colors. Don't get at all disturbed by the fact that I'm often using a red for area, but here, it's just going to be a second function. It just kind of highlights what's going on here. So then I have the G of X. So I'm just adding two functions inside of the integral. And then as you know, this is one of these things that you really hope is true, which is that when I take the integral from A to B of F of X, so I can actually just kind of break this apart into the integrals. So I have my F of X DX , and then my, okay, DX was the integral from A to B of G of X DX . Okay, so here's this, this picture, I can draw you a picture for this one. So here we go, okay. So we have X here, and we have Y here. And we can do our first function. This is just a pictorial representation. This is not unfortunately, this is not going to give you, it is actually how you prove it, by the way. But, so you have A, and maybe I'm kind of dot going up and B dot going up, so we know this is kind of the area that we're talking about. And then we draw our F and X. So who knows what our F of X looks like? Well, what happens actually, when you add, this is not what I meant to do.

Okay. So this really here genuinely is the curve Y equals F of X. But let's look at a, if you look at a particular point X here, right, then this area here is going to tell me how far up I went. Right? This is F of X, right? That is how far I up I went via F. Okay, and then if I wanted to add G, like, how would I do this? Well, for each value, I'm just adding them point wise, okay, F of X plus G of X really means that at X to get up to G, I go up whatever G of X is. Right? So what ends up happening, and you know, I don't know exactly what my function is, but each time I'm adding the F of X height, and then the G of X height, right? This is exactly how it works. And so this entire height here, this entire height here is

actually F of X plus G of X , right? This entire height is F of X plus G of X , right? That's my entire height, but half of it came or some portion of it came from the F of X , the other portion of it came from the G of X . And so when you what you end up doing is you end up summing the areas.

I think you can see an S there. Okay, so I have, it's like I get this area from the F of X . Right? This height was like my F of X height. Like we showed over there, this was like my F of X , and my ΔX down here. But then you do the same thing with the other one coming from the G of X . It's supposed to be going straight up, we're just going to live with that. Your professor can't draw, live with it, unfortunately, sorry. And so this whole thing that you're summing here is actually F of X plus G of X , just kind of like emphasized over there.

Okay? so at each step, because when you add F of X and G of X , you're adding these heights together that ends up meaning that you, when you're doing your summation, you're actually summing the areas at each time, which means you're summing the integrals. Okay? So now let's look at another one. It's a nice, it's kind of a nice picture, that it all kind of flows out like that very nicely. I like it, it's, I feel like it's worth giving you. Okay, and now, so now we're going to go, we're still going from A to B . I'm going to, I'm going to pull out a constant C . Okay, so C is any constant of you know that I'm multiplying my function F of X by, and just like we've been able to do in derivatives and everything else, I'm able to pull out the C . And then I get the integral. So I just multiply C times the integral from A to B of exactly that function there. So I was just able to pull out the C from the integral. I could do it because it's a constant. Remember, we can't actually do that in general. Okay, where C as any constant.

Okay, so this one has a nice picture, too, which I'll draw right here for us. The next one doesn't have such a nice picture, so I can use up a little bit of its space. Okay. So again, we have X , and we have Y . And let's say I have my function F of X , and maybe it goes something like this. Then what happens, I have to multiply each of these values by 3. So this gets a little tricky, because if I multiply this by 3, 1, 2, 3, that's all the way up here. Whereas multiplying with this by 3 just goes 1, 2, 3. Okay? So it's like, this is actually $C F$ of X . And then 1, 2, 3 tells me I need to go down to about here. And then 1, 2, oh that's really big. 1, 2, okay, this is just going to explode up. Okay? So, if you look at it, so this is like $3F$ of X . And this is F of X . Right? What what ends up happening, if you want to look at your rectangles at some place on here, they end up getting like 3 times as big. So let's just put a rectangle, we're going to do a rectangle of height right here versus all the way up here. It's supposed to be precisely 3 times as big. I clearly, my drawing wasn't perfect. I try. Okay, so kind of what's going on here is the rectangles are 3 times as tall.

Right, where 3 was actually my constant. Okay? So that's why it would make sense that my area everywhere it gets multiplied by 3 because the area of my rectangles get multiplied by 3. So they're 3 times as tall. So I have 3 times the area. Okay, so here's another one. So this one we're going to go, so the integral from A to B . This is a useful one. It's a combination of two from before, but it's, it's actually kind of worth writing on its own because it's used enough in life. So we're going to take F of X and now we're going to subtract out G of X . And maybe actually, we'll put parentheses around the whole thing. It's usually good to do that just to make sure you kind of understand what's in and

outside the integral. Okay? That this is actually equal to the integral from A to B of F of X DX minus the integral from A to B of G of X DX, right? And this one just follows from one and two. You can combine them to get this, right, where your constant is minus 1.

Okay? And here's a final one that is very, this should feel like this brings you back in life. Okay? Okay, so if I want to take the integral from A to B of C DX, right? So I have any constant, then what is this going to equal? It's just going to equal C, times B of B minus A. Right? And this picture. I feel like mathematics has sound effects. Usually, sound effects are actually used for a change over time, I've discovered. We like to use pictures for visualizations. And then if we want the pictures to change over time I, I found that we often will use sound effects, but I also just kind of use them for fun. Okay. So this is like Y equals C, and then it all makes sense, because this area in here is what we're looking at. So, right, this is C, times B, minus A.

Okay, great. So this is actually where you can, yeah, I kind of wanted to go through linearity in this circumstance where you can actually see it in the picture. So there's really nice pictures here to actually kind of see linearity. And then I'm going to give you a few more properties in the next lecture, because we, there's a lot more that we're actually going to want to use, and they're going to help you a lot in computation. Okay? So thank you for sticking with me, and I hope this all kind of makes sense. And it's nice, you'll have these nice pictures that maybe organizing your mind what's going on or to understand where it comes from. And I'll see you in the next lecture.